अखिल भारतीय तकनीकी शिक्षा परिषद्
All India Council for Technical Education

DESIGN OF MACHINE ELEMENTS

A KUMARAVEL
M KATHIRSELVAM

III Year Diploma level book as per AICTE model curriculum
(Based upon Outcome Based Education as per National Education Policy 2020).
The book is **reviewed by Dr. Parlad Kumar**

Design of Machine Elements

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FOREWORD

Engineers are the backbone of any modern society. They are the ones responsible for the marvels as well as the improved quality of life across the world. Engineers have driven humanity towards greater heights in a more evolved and unprecedented manner.

The All India Council for Technical Education (AICTE), have spared no efforts towards the strengthening of the technical education in the country. AICTE is always committed towards promoting quality Technical Education to make India a modern developed nation emphasizing on the overall welfare of mankind.

An array of initiatives has been taken by AICTE in last decade which have been accelerated now by the National Education Policy (NEP) 2020. The implementation of NEP under the visionary leadership of Hon'ble Prime Minister of India envisages the provision for education in regional languages to all, thereby ensuring that every graduate becomes competent enough and is in a position to contribute towards the national growth and development through innovation & entrepreneurship.

One of the spheres where AICTE had been relentlessly working since past couple of years is providing high quality original technical contents at Under Graduate & Diploma level prepared and translated by eminent educators in various Indian languages to its aspirants. For students pursuing 3rd year of their Engineering education, AICTE has identified 48 books, which shall be translated into 12 Indian languages - Hindi, Tamil, Gujarati, Odia, Bengali, Kannada, Urdu, Punjabi, Telugu, Marathi, Assamese & Malayalam. In addition to the English medium, books in different Indian Languages are going to support the students to understand the concepts in their respective mother tongue.

On behalf of AICTE, I express sincere gratitude to all distinguished authors, reviewers and translators from the renowned institutions of high repute for their admirable contribution in a record span of time.

AICTE is confident that these outcomes based original contents shall help aspirants to master the subject with comprehension and greater ease.

T. G. Sitharam

(Prof. T. G. Sitharam)

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We sincerely acknowledge the valuable contributions of Dr. Parlad Kumar, Professor, Department of Mechanical Engineering, Punjabi University, Punjab, for reviewing the book. His expertise and insights have been immensely helpful in shaping this book into its final form.

We express our sincere gratitude to Ph.D. supervisors, Prof. (Late) N. Ganesan, and Prof. Raju Sethuraman, Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai, for their continuous inspiration and learning opportunities. We are grateful to Thiru. R. Srinivasan, Chairman, K.S.R. Educational Institutions for providing the facilities to write this book. We are also thankful to Dr. R. Gopalakrishnan, Principal of K.S. Rangasamy College of Technology for providing support in completing the book on time.

This book is an outcome of various suggestions of AICTE members, experts and authors who shared their opinion and thought to further develop the engineering education in our country. Acknowledgements are due to the contributors and different workers in this field whose published books, review articles, papers, photographs, footnotes, references and other valuable information enriched us at the time of writing the book.

We are grateful to many individuals and organizations who have supported us throughout the process of writing this book. We sincerely thank our friends, colleagues and family members for their patience while we wrote this book.

Dr. A. Kumaravel
Dr. M. Kathirselvam

PREFACE

The field of mechanical engineering is continuously evolving, driven by advancements in technology and the growing demands of modern industry. One of the fundamental aspects of this discipline is the design of machine elements, which forms the backbone of creating robust, efficient, and reliable machinery. This book, "Design of Machine Elements," aims to provide a comprehensive and accessible introduction to the principles and applications involved in this critical area.

The primary objective of this book is to equip diploma students with the knowledge and skills necessary to design various machine elements that are commonly used in engineering practice. The content has been meticulously structured to cover both the theoretical foundations and practical considerations of machine element design, ensuring that readers can not only understand the concepts but also apply them effectively in real-world scenarios.

Each chapter is organized to progressively build on the previous one, starting from basic concepts and moving towards more complex topics. This systematic approach helps students develop a clear and coherent understanding of the subject matter. Numerical examples are included throughout the book to illustrate the application of design principles. The book covers the topics, including the design of cotter and knuckle joints, levers, shafts, keys, couplings, bearings, spur gears, springs, fasteners, and other critical machine elements. This extensive coverage ensures that students are well-prepared to tackle various design challenges. Detailed diagrams and illustrations are provided to enhance the understanding of complex concepts and design processes.

We have referred a number of books, NPTEL videos and online materials while we were writing to make sure the information is current and relevant. In addition, we have added QR codes to each chapter, which students can scan to get more details and resources on a particular topic.

It is our genuine desire that the book will encourage students to study and debate the concepts underlying the fundamentals of machine elements design, thereby helping to build a strong foundation in the field. Any constructive criticism and recommendations that will help to make the next editions of the book even better are welcome and appreciated. It is with great pleasure that we present the teachers and students this book.

Dr. A. Kumaravel
Dr. M. Kathirselvam

OUTCOME BASED EDUCATION

For the implementation of an outcome based education the first requirement is to develop an outcome based curriculum and incorporate an outcome based assessment in the education system. By going through outcome based assessments, evaluators will be able to evaluate whether the students have achieved the outlined standard, specific and measurable outcomes. With the proper incorporation of outcome based education there will be a definite commitment to achieve a minimum standard for all learners without giving up at any level. At the end of the program running with the aid of outcome based education, a student will be able to arrive at the following Program Outcomes (POs):

PO1. **Basic and Discipline Specific Knowledge:** Apply knowledge of basic mathematics, science and engineering fundamentals and engineering specialization to solve the engineering problems.

PO2. **Problem Analysis:** Identify and analyze well-defined engineering problems using codified standard methods.

PO3. **Design/Development of Solutions:** Design solutions for well-defined technical problems and assist with the design of systems components or processes to meet specified needs.

PO4. **Engineering Tools, Experimentation and Testing:** Apply modern engineering tools and appropriate technique to conduct standard tests and measurements.

PO5. **Engineering Practices for Society, Sustainability and Environment:** Apply appropriate technology in context of society, sustainability, environment and ethical practices.

PO6. **Project Management:** Use engineering management principles individually, as a team member or a leader to manage projects and effectively communicate about well-defined engineering activities.

PO7. **Life-long Learning:** Ability to analyze individual needs and engage in updating in the context of technological changes.

COURSE OUTCOMES

After completion of the course, the students will be able to:

CO-1: Design and draw simple machine components used in industries

CO-2: Analyze and interpret engineering problems, applying the fundamental principles and philosophy of machine design

CO-3: Analyze the modes of failures of machine components and decide the design criteria and equations

CO-4: Determine the appropriate dimensions of components based on the analysis of loads, forces, and stresses

CO-5: Apply analytical techniques to evaluate and solve engineering design problems

Mapping of Course Outcomes with Program Outcomes to be done according to the matrix given below:

Course Outcomes	**Expected Mapping with Program Outcomes** *(1- Weak Correlation; 2- Medium Correlation; 3- Strong Correlation)*						
	PO-1	**PO-2**	**PO-3**	**PO-4**	**PO-5**	**PO-6**	**PO-7**
CO-1	3	3	3	3	-	-	2
CO-2	3	3	3	3	2	-	2
CO-3	3	3	3	3	-	-	2
CO-4	3	3	3	3	-	-	2
CO-5	3	3	3	3	-	-	2

GUIDELINES FOR TEACHERS

To implement Outcome Based Education (OBE) knowledge level and skill set of the students should be enhanced. Teachers should take a major responsibility for the proper implementation of OBE. Some of the responsibilities (not limited to) for the teachers in OBE system may be as follows:

- Within reasonable constraint, they should manoeuvre time to the best advantage of all students.
- They should assess the students only upon certain defined criterion without considering any other potential ineligibility to discriminate them.
- They should try to grow the learning abilities of the students to a certain level before they leave the institute.
- They should try to ensure that all the students are equipped with the quality knowledge as well as competence after they finish their education.
- They should always encourage the students to develop their ultimate performance capabilities.
- They should facilitate and encourage group work and team work to consolidate newer approach.
- They should follow Bloom's taxonomy in every part of the assessment.

Bloom’s Taxonomy

Level	Teacher should Check	Student should be able to	Possible Mode of Assessment
Create	Students ability to create	Design or Create	Mini project
Evaluate	Students ability to justify	Argue or Defend	Assignment
Analyze	Students ability to distinguish	Differentiate or Distinguish	Project/Lab Methodology
Apply	Students ability to use information	Operate or Demonstrate	Technical Presentation/ Demonstration
Understand	Students ability to explain the ideas	Explain or Classify	Presentation/Seminar
Remember	Students ability to recall (or remember)	Define or Recall	Quiz

GUIDELINES FOR STUDENTS

Students should take equal responsibility for implementing the OBE. Some of the responsibilities (not limited to) for the students in OBE system are as follows:

- Students should be well aware of each Unit Outcome (UO) before the start of a unit in each and every course
- Students should be well aware of each CO before the start of the course
- Students should be well aware of each PO before the start of the program
- Students should think critically and reasonably with proper reflection and action
- Learning of the students should be connected and integrated with practical and real life consequences
- Students should be well aware of their competency at every level of OBE

ABBREVIATIONS AND SYMBOLS

List of Abbreviations

Abbreviations	Full form	Abbreviations	Full form
AISI	American Iron and Steel Institute	ASTM	American Society for Testing and Materials
AWS	American Welding Society	BHN	Brinell Hardness Number
CG	Centre of Gravity	CI	Cast Iron
DIN	Deutches Institut Fuer Normung	GP	Geometric Progression
FOS	Factor of Safety	PPE	Personal Protective Equipment
SAE	Society of Automotive Engineers		

List of Units

Unit Symbol	Unit	Unit Symbol	Unit
kN	kilonewton	kW	kilowatts
MPa	Megapascal	m/s	metre per second
mm	millimeter	N	Newton
rpm	revolution per minute		

List of Symbols

Symbols	Description	Symbols	Description
α	Helix angle	γ	Shear strain
ΔL	Change in length	ε	Normal strain
θ	Angle of twist	μ	Coefficient of friction
v	Pitch line velocity	σ_b	Bending stress
σ_c	Compressive stress	σ_x	Normal stress in x direction
σ_t	Tensile stress	σ_y	Normal stress in y direction
σ_1, σ_2	Maximum principal stresses	τ	Shear stress
ϕ	Friction angle	φ	Angle of deformation
Υ	Axial load factor	a	Distance from slot end to rod end
A	Area	Al	Aluminium
b	Breadth	Be	Beryllium
c	Socket collar thickness	Co	Cobalt
Cr	Chromium	Cu	Copper
C_v	Velocity factor	d	Diameter

Symbols	Description	Symbols	Description
d_m	Mean diameter	D	Pitch diameter
e	Eccentric distance	F	Force
F_a	Axial or thrust load	F_r	Radial load
F_w	Wear strength	G	Shear modulus
h	Depth	I	Moment of inertia
J	Polar moment of inertia	k	Radius of gyration
k_θ	Reduction factor of angle of twist	k_b	Stiffness constant for the bolt
k_c	Stiffness constant for compressed member	K_o	Service factor
K_m	Shock factor	K_t	Stress concentration factor
K_t	Fatigue factor	K_w	Load stress factor
l	Length	l_b	Effective length of the bush
L_{10}	Rated bearing life	L_o	Original length
m	Module	M_b	Bending moment
M_t	Twisting torque	Mn	Manganese
Mo	Molybdenum	n	number of threads
N	Speed	Nb	Niobium
Ni	Nickel	p_b	Bearing pressure
P	Power transmitted	Pb	Lead
q	Number of balls	r	Distance from neutral axis
R	Radius	R_f	Reaction at fulcrum
S_a	Ultimate tensile strength	S_b	Beam strength of gear tooth
S_{uc}	Ultimate stress in compression	S_{ut}	Ultimate stress in tension
S_{yc}	Yield point stress in compression	S_{yt}	Yield point stress in tension
Si	Silicon	t	Thickness
Ta	Tantalum	Ti	Titanium
V	Vanadium	w	Width
W	Load	W	Tungsten
W_a	Actual load	W_d	Design load
X	Radial load factor	Y	Lewis form factor
z	Number of teeth	Z	Section modulus
Zr	Zirconium		

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1 Introduction of Design

UNIT SPECIFICS

Through this unit, the following aspects are discussed:

- Design procedure and general considerations
- Stress – strain behavior of materials
- Types of stresses
- Creep, fatigue and factor of safety
- Properties of engineering materials
- Designation of materials
- Use of standards and preferred numbers
- Theories of elastic failures

RATIONALE

The first unit of this book will help the students to have good understanding of the behavior of the materials with respect to applied loads. After completing this course, a student will be qualified to choose a suitable material according to its properties and applications. The various stresses induced in the materials by applying the different loads are discussed. The mechanical properties of engineering materials are discussed to understand in selecting materials for specific engineering applications. The importance of designation and standardization of materials to ensure consistency, interoperability, safety, and quality across different components, systems, and processes are presented in this chapter. This unit also enables the student to understand the various theories of failure that they provide insight into how and why materials and components may fail under certain conditions.

PRE-REQUISITES

Strength of materials, Engineering mechanics

UNIT OUTCOMES

List of outcomes of this unit is as follows:

On the successful completion of the unit, students will be able to

U1-O1: Apply the design steps, its considerations, and standards in designing of simple machine elements.

U1-O2: Apply and analyze fundamental mechanical principles such as force, torque, stress, and strain to design machine elements.

U1-O3: Select suitable materials, for different machine elements depending on their physical and mechanical properties.

U1-O4: Use and apply the materials standards to design machine elements.

U1-O5: Apply the various theories of failure to design machine elements.

Unit 1 Outcomes	**Mapping with Course Outcomes** (1 – weak correlation, 2 – medium correlation, 3 – strong correlation)				
	CO-1	**CO-2**	**CO-3**	**CO-4**	**CO-5**
U1-O1	3	3	-	3	3
U1-O2	1	3	2	3	3
U1-O3	3	-	-	-	3
U1-O4	3	3	-	3	3
U1-O5	-	-	3	-	3

1.1. INTRODUCTION

Machine design is a multidisciplinary field that involves the creation of mechanical systems and devices, ranging from simple mechanisms to complex machinery. The goal of machine design is to develop systems that efficiently perform desired functions while meeting various constraints, such as safety, reliability, cost, and environmental impact. This process typically involves a systematic approach that includes concept generation, analysis, detailed design, prototyping, testing, and manufacturing.

Machine design requires a multidisciplinary approach, often involving collaboration among mechanical engineers, electrical engineers, and control system engineers. The field continually evolves with technological advancements, materials innovation, and changing industrial needs. Successful machine design integrates engineering principles with creativity and problem-solving skills to deliver machines that meet or exceed expectations.

1.2. PHILOSOPHY

The philosophy of machine design encompasses the underlying principles and guiding beliefs that influence the approach and decisions made during the design process. It involves a set of considerations and values that engineers and designers adhere to as they create mechanical systems and devices. Here are some key aspects of the philosophy of machine design:

Design Lec 1

(i) Functionality and purpose: Prioritize the understanding of the intended purpose and functions of the machine. Design decisions should align with the primary objectives and desired outcomes.

(ii) User-Centered design: Emphasize the importance of designing machines with the end user in mind. Consider user experience, ergonomics, and ease of operation to enhance usability and efficiency.

(iii) Safety first: Make safety a paramount concern in machine design. Integrate features and mechanisms to mitigate risks and protect operators, users, and the environment.

(iv) Reliability and performance: Strive for high reliability and performance. Design machines that consistently meet or exceed performance requirements, ensuring their effectiveness and longevity.

(v) Efficiency and optimization: Optimize the design for efficiency, minimizing energy consumption, material usage, and waste. Consider the life cycle of the machine, from manufacturing to operation and eventual disposal.

(vi) Cost-Effectiveness: Balance performance requirements with cost considerations. Optimize material selection, manufacturing processes, and maintenance procedures to achieve cost-effectiveness without compromising quality.

(vii) Sustainability: Consider the environmental impact of the machine throughout its life cycle. Design with sustainability in mind, incorporating features that reduce resource consumption and promote eco-friendly practices.

(viii) Innovation and creativity: Encourage innovative thinking and creativity in the design process. Explore new technologies, materials, and approaches to solve engineering challenges and enhance machine functionality.

(ix) Adaptability and modularity: Design machines that can adapt to changing requirements. Consider modularity to facilitate easier maintenance, upgrades, and repairs, extending the machine's life and reducing downtime.

(x) Interdisciplinary collaboration: Foster collaboration among different engineering disciplines, such as mechanical, electrical, and control systems. Integrating diverse expertise ensures a holistic approach to machine design.

(xi) Compliance with standards: Ensure that the machine design adheres to relevant industry standards and regulations. Addressing legal and safety requirements is essential for the successful deployment and operation of the machine.

(xii) Testing and validation: Implement thorough testing procedures to validate the design and ensure it meets performance and safety criteria. Continuous testing and validation help identify and address potential issues early in the design process.

(xiii) Documentation and communication: Maintain clear and comprehensive documentation throughout the design process. Effective communication is crucial for conveying design intent and facilitating collaboration among team members.

The philosophy of machine design is dynamic and responsive to advancements in technology, materials, and societal needs. Designers should approach each project with a commitment to delivering solutions that not only meet technical specifications but also align with ethical, environmental, and user-centric considerations.

1.3. MACHINE DESIGN PROCEDURES

Machine design procedures involve a systematic series of steps and processes to develop efficient and effective mechanical systems. While specific procedures may vary depending on the complexity of the machine and the industry, the following general steps outline a typical approach to machine design:

(i) Problem definition and requirements analysis: Clearly define the problem or need that the machine is intended to address. Identify and analyze the requirements and constraints, including performance specifications, environmental conditions, safety considerations, and any relevant industry standards.

(ii) Conceptualization and idea generation: Generate creative ideas and concepts to fulfill the identified requirements. Brainstorm potential solutions and explore different design alternatives.

(iii) Preliminary analysis and feasibility study: Conduct a preliminary analysis of the proposed concepts to assess their feasibility. Evaluate the potential solutions in terms of technical, economic, and operational feasibility.

(iv) Selection of design concept: Evaluate the conceptual designs based on criteria such as functionality, safety, cost, and ease of manufacturing. Choose the most suitable design concept for further development.

(v) Detailed design: Develop detailed drawings and specifications for the chosen design concept. Specify dimensions, materials, tolerances, and manufacturing processes.

(vi) Analysis and simulation: Perform detailed engineering analysis using mathematical models, simulations, and finite element analysis (FEA) to validate the design and identify potential issues. Refine the design based on analysis results.

(vii) Prototyping: Build a prototype or a scaled-down model of the machine. Test the prototype to verify its functionality and performance.

(viii) Testing and validation: Conduct comprehensive testing to ensure the machine meets the specified requirements. Test for performance, safety, reliability, and any other relevant criteria. Iterate on the design based on test results.

(ix) Final design and documentation: Incorporate feedback from testing and make final adjustments to the design. Create detailed documentation, including assembly drawings, operation manuals, and maintenance guides.

(x) Materials selection and manufacturing: Choose appropriate materials based on the mechanical properties required for the machine components. Select manufacturing processes that are cost-effective and suitable for the chosen materials.

(xi) Assembly and integration: Develop a plan for assembling the machine components into the final product. Ensure proper integration of mechanical, electrical, and control systems if applicable.

(xii) Quality control and inspection: Implement quality control measures during the manufacturing process. Conduct inspections to ensure that each component meets the specified quality standards.

(xiii) Installation and commissioning: Install the machine at the intended location. Perform commissioning activities to ensure the machine operates as intended.

(xiv) Training and operation: Provide training for operators and maintenance personnel. Ensure that users are familiar with the operation, safety procedures, and maintenance requirements.

(xv) Monitoring and maintenance: Implement a monitoring and maintenance plan to ensure the ongoing performance and reliability of the machine. Establish regular inspection and maintenance schedules.

1.4. GENERAL CONSIDERATIONS IN MACHINE DESIGN

Machine design involves a multitude of considerations to ensure that the resulting mechanical systems meet specified requirements, perform reliably, and adhere to safety standards. Here are some general considerations in machine design:

(i) Functional requirements: Clearly define the purpose and intended functions of the machine. Understand the tasks it needs to perform and the desired outcomes.

(ii) Performance specifications: Establish quantitative performance criteria such as speed, accuracy, load capacity, and efficiency that the machine must meet.

(iii) Material selection: Choose materials based on their mechanical properties, durability, and suitability for the intended application. Consider factors like strength, stiffness, corrosion resistance, and cost.

(iv) Safety standards and regulations: Ensure compliance with relevant safety standards and regulations. Design safety features and mechanisms to protect operators, users, and the environment.

(v) Environmental considerations: Evaluate the environmental impact of the machine throughout its life cycle. Consider factors such as energy efficiency, recyclability, and emissions.

(vi) Cost constraints: Balance performance requirements with cost constraints. Optimize material selection, manufacturing processes, and design complexity to achieve cost-effectiveness.

(vii) Ergonomics and user interface: Design with user comfort and usability in mind. Consider ergonomics for operator interfaces, controls, and maintenance access.

(viii) Reliability and maintenance: Aim for high reliability to minimize downtime. Design for ease of maintenance, with accessible components and clear documentation for troubleshooting and repairs.

(ix) Manufacturability: Choose manufacturing processes that are suitable for the selected materials. Optimize designs for ease of fabrication, assembly, and scalability.

(x) Modularity and scalability: Consider modularity to facilitate easier maintenance, upgrades, and repairs. Design systems that can be easily scaled or adapted to changing requirements.

(xi) Integration of mechanical and electrical systems: If applicable, integrate mechanical, electrical, and control systems seamlessly. Ensure compatibility and communication between different subsystems.

(xii) Testing and validation: Implement thorough testing procedures to validate the design. Test for performance, safety, and reliability. Use simulations and prototypes to identify and address potential issues.

(xiii) Documentation: Create comprehensive documentation including detailed drawings, assembly instructions, operation manuals, and maintenance guides. Clear documentation is crucial for manufacturing, operation, and maintenance.

(xiv) Interdisciplinary collaboration: Foster collaboration among different engineering disciplines (mechanical, electrical, control systems) to ensure a holistic approach to design.

(xv) Adherence to industry standards: Follow industry standards and best practices relevant to the specific application and sector. Adherence to standards ensures compatibility and compliance with regulations.

(xvi) Lifecycle considerations: Assess the entire lifecycle of the machine, from manufacturing and operation to maintenance and disposal. Consider the impact on the environment and plan for sustainable practices.

These considerations highlight the importance of a multidisciplinary approach, collaboration, and careful planning in the machine design process. The successful integration of these factors leads to the creation of machines that are not only technically sound but also safe, efficient, and cost-effective.

1.5. TYPES OF LOADS

In the context of machine design and structural analysis, various types of loads act on components and structures. Understanding these types of loads is crucial for designing components that can withstand the forces they will encounter. Here are some common types of loads:

(i) Dead load (Static load): The weight of the structure or component itself. It remains constant and does not change with time. Dead loads include the weight of materials, equipment, and any other permanent fixtures.

(ii) Live load (Dynamic load): Temporary or moving loads that can vary in magnitude and location. Examples include the weight of people, vehicles, or equipment on a structure. Live loads are dynamic and can change over time.

(iii) Point load: A concentrated load applied at a specific point on a structure or component. It is often represented as a single force acting at a particular location.

(iv) Distributed load: A load spread over an area rather than concentrated at a single point. It is often represented as a force per unit length or area.

(v) Uniform load: A type of distributed load that is evenly distributed over a specific length or area. The intensity of the load remains constant.

(vi) Concentrated load: A load applied at a specific point, as opposed to being distributed over an area. It can be a point load or a concentrated force.

(vii) Torsional load: A twisting or torque load applied to a component, causing it to rotate around its axis. Torsional loads are common in shafts and other rotating elements.

(viii) Axial load: A load applied along the axis of a structure, causing compression (pushing) or tension (pulling). Axial loads can be present in columns, rods, or cables.

(ix) Bending load: A load that induces bending in a structure, causing it to deform. Bending loads result in a combination of tensile and compressive stresses.

(x) Shear load: A force applied parallel to the surface of a material, causing one part of the material to slide or deform relative to an adjacent part. Shear loads are common in beams and connections.

(xi) Thermal load: Changes in temperature that cause thermal expansion or contraction of materials. Thermal loads can lead to stresses and deformations in structures.

(xii) Impact load: A sudden and intense force applied to a structure. Impact loads can result from events like collisions, drops, or other abrupt actions.

(xiii) Hydrostatic load: The pressure exerted by a fluid (usually water) on a surface. This load is often encountered in structures submerged in fluids or in hydraulic systems.

(xiv) Wind Load: The force exerted by the wind on structures. Wind loads can vary based on factors such as wind speed, direction, and the shape of the structure.

It is crucial for engineers to understand the types of loads and their effects in order to design structures and components that can safely and efficiently withstand the forces they will encounter in their intended applications.

1.6. CONCEPTS OF STRESS AND STRAIN

In the field of materials science and engineering, stress and strain are fundamental concepts that describe how materials respond to external forces.

These concepts are essential for understanding how materials behave under different conditions, which helps in the design and analysis of structures and components. Let's explore the concepts of stress and strain:

Types of Stresses

1.6.1. Stress

The force exerted on a material per unit area, which measures the material's internal resistance to deformation when subjected to an external force.

1.6.1.1. Types of Stress

(i) Tensile stress (σ_t): The stress that occurs when a material is subjected to a pulling or stretching force as shown in Fig. 1.1. It is positive when the material is under tension.

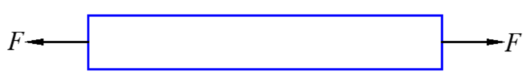


Fig.1.1: Tensile stress

(ii) Compressive stress (σ_c): The stress that occurs when a material is subjected to a compressing or squashing force as shown in Fig. 1.2. It is negative when the material is under compression.

Fig. 1.2: Compressive stress

For tensile and compressive stress: $\sigma_t = \sigma_c = \dfrac{F}{A}$ (1.1)

Where σ_t and σ_c are tensile and compressive stresses $\left(N/mm^2\right)$, F is the applied force (N), A is the cross-sectional area $\left(mm^2\right)$.

(iii) Shear stress (τ): When a body experiences two equal and opposing forces acting tangentially across a resistive section, which causes the body to shear off the section, then the stress induced is called shear stress.

Consider the two plates connected by a rivet joint as shown in Fig. 1.3. The tangential force (F) leads to shear the rivet at the cross section as shown in Fig. 1.3 (a). When the tangential force is resisted at cross-section of the rivet, then the rivets are said to be in shear.

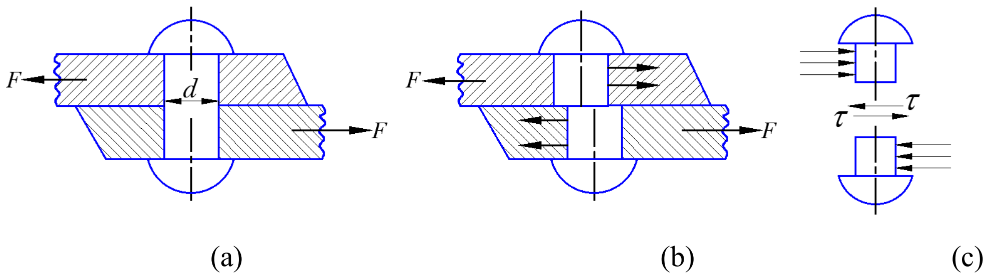


Fig.1.3: (a) Riveted joint (b) Shear deformation (c) Shear stress

The shear stress can be written as,

Shear stress $\tau = \dfrac{F}{A} = \dfrac{F}{\frac{\pi}{4}d^2} = \dfrac{4F}{\pi d^2}$ (1.2)

Where F is tangential force $\left(N/mm^2\right)$, A is resisting area $\left(mm^2\right)$, d is diameter of the rivet (mm).

1.6.2. Strain (ε)

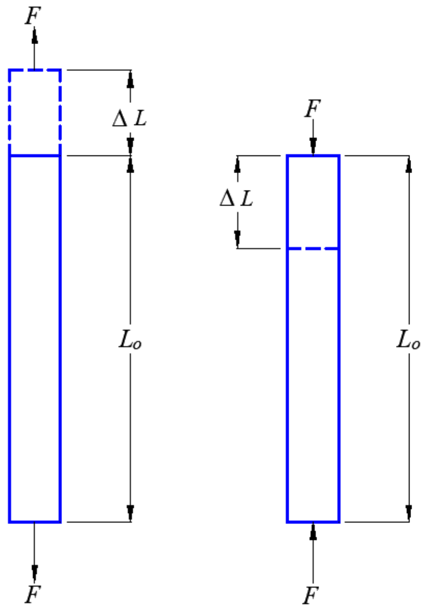
The measure of deformation or change in size or shape experienced by a material in response to an applied stress.

1.6.2.1. Types of strain

(i) Tensile strain (ε)**:** The strain that occurs when a material is subjected to a pulling or stretching force as shown in Fig. 1.4 (a). It is positive for elongation and negative for contraction.

(ii) Compressive strain (ε)**:** The strain that occurs when a material is subjected to a compressing or squashing force as shown in Fig. 1.4. (b). It is positive for contraction and negative for elongation.

For tensile and compressive strain: $\varepsilon = \dfrac{\Delta L}{L_o}$ (1.3)



(a) (b)

Strain Lec 6

Fig.1.4: (a) Tensile strain (b) Compressive strain

Where ε is the strain, ΔL is the change in length (mm), L_o is the original length (mm).

(iii) Shear strain (γ)**:** The strain that occurs due to the change in shape when parallel layers of a material slide past each other and it is shown in Fig. 1.5.

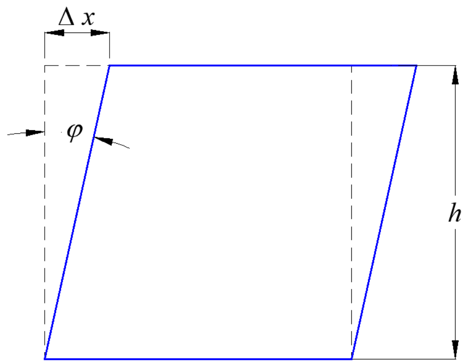


Fig. 1.5: Shear strain

For shear strain: $\gamma = \dfrac{\Delta x}{h} = \tan(\varphi)$ (1.4)

Where γ is the shear strain, φ is the angle of deformation.

1.7. STRESS – STRAIN DIAGRAM FOR DUCTILE AND BRITTLE MATERIALS

The stress-strain diagram is a graphical representation of how a material responds to applied forces, providing insights into its mechanical properties. Ductile and brittle materials exhibit distinct behaviors in their stress-strain curves. Let's explore the typical stress-strain diagrams for ductile and brittle materials:

1.7.1. Ductile materials

1.7.1.1. Stress-Strain curve characteristics

For ductile materials, the stress – strain curve is shown in Fig. 1.6.

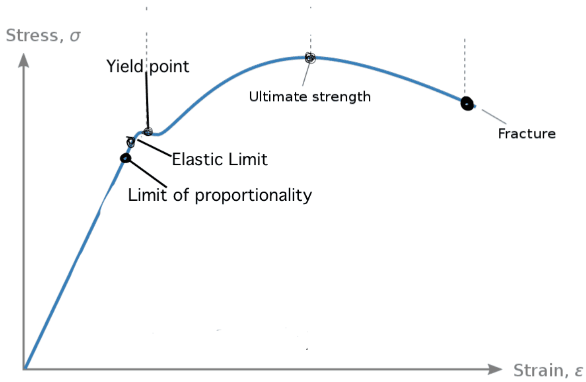


Fig.1.6: Stress – strain curve for ductile materials

(i) Elastic region (Linear deformation): Initially, the material undergoes elastic deformation, where stress and strain are proportional (Hooke's Law). This region is characterized by a linear stress-strain relationship.

(ii) Yield point: Beyond the elastic limit, the material enters the plastic deformation phase. The yield point is the stress at which plastic deformation begins, and the material undergoes permanent deformation.

(iii) Plastic deformation: In this region, the material continues to deform plastically without a significant increase in stress. Ductile materials exhibit necking, where localized deformation occurs in a reduced cross-sectional area.

(iv) Ultimate tensile strength: The maximum stress the material can withstand is reached at the point of ultimate tensile strength. Necking becomes more pronounced in this phase.

(v) Fracture point: The material eventually fractures after reaching the ultimate tensile strength. The stress decreases rapidly, leading to failure.

1.7.2. Brittle materials

1.7.2.1. Stress-Strain curve characteristics

(i) Elastic region (Linear deformation): Similar to ductile materials, brittle materials initially undergo elastic deformation with a linear stress-strain relationship. For brittle materials, the stress – strain curve is shown in Fig. 1.7.

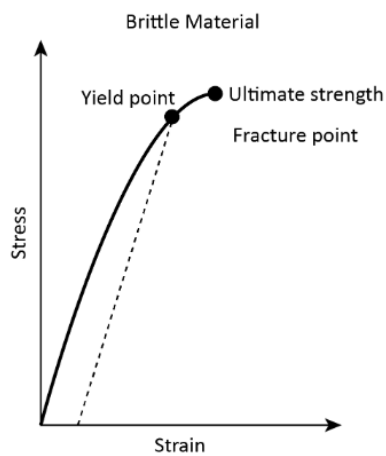


Fig.1.7: Stress – strain curve for brittle materials

(ii) Yield point: Unlike ductile materials, brittle materials lack a distinct yield point. Elastic deformation continues until the material reaches its breaking point.

(iii) Fracture point (Catastrophic failure): Brittle materials exhibit little or no plastic deformation. Once the applied stress exceeds the material's ultimate strength, it experiences rapid and catastrophic failure without significant necking or deformation.

1.7.3. Key differences

(i) Ductility: Ductile materials exhibit significant plastic deformation and necking before failure. Brittle materials fail suddenly and without appreciable plastic deformation.

(ii) Yield point: Ductile materials have a distinct yield point, marking the onset of plastic deformation. Brittle materials lack a clear yield point.

(iii) Fracture behaviour: Ductile materials undergo necking and gradual deformation before fracture. Brittle materials experience sudden and catastrophic failure.

Understanding the stress-strain behavior of materials is crucial for engineering design and material selection, ensuring that structures are designed to withstand loads while considering the material's response to stress.

1.8. BEARING PRESSURE INTENSITY OR CRUSHING

Bearing pressure intensity refers to the pressure exerted on a bearing surface, typically by a load or force acting on the surface. The bearing pressure intensity can be calculated using the formula as given in Eq. (1.5):

$$p_b = \frac{F}{A} \tag{1.5}$$

Where: p_b is the bearing pressure intensity (N/mm^2), F is the force applied on the bearing surface (N), A is the area over which the force is distributed (mm^2).

It's important to note that bearing pressure intensity is a measure of the force per unit area and is expressed in units such as Pascal (Pa) in the International System of Units (SI). In engineering design and analysis, it's essential to ensure that bearing pressure does not exceed the allowable limits for the materials involved. Excessive bearing pressure can lead to structural failure, deformation, or other undesirable effects. Engineers often consider factors like safety margins, material properties, and load distribution to determine appropriate bearing pressure limits for a given application.

"Crushing" typically refers to the process of applying a force or pressure to break or compress materials. In the context of structures or materials, crushing can lead to deformation, failure, or collapse. The connection between the two concepts lies in the potential consequences of excessive bearing pressure intensity. If the bearing pressure intensity becomes too high, it can result in localized crushing of the material, leading to structural failure. Therefore, engineers must carefully consider and analyze the bearing pressure to avoid scenarios where the applied forces may exceed the material's or structure's capacity, causing crushing or other forms of failure.

1.9. BENDING

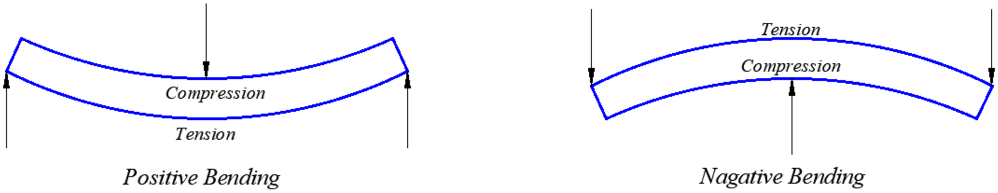
Bending refers to the deformation of a structural component or material due to the application of external forces, typically moments or torques. This deformation occurs in response to forces that cause the material to bend or flex rather than break. Bending is a common phenomenon in engineering and structural design, and it is a crucial consideration in the analysis of beams, columns, bridges, and other structural elements.

(i) Bending Moment: Bending is often associated with the presence of a bending moment. A bending moment is a force that tends to cause a structural element to bend. It is the result of a force acting at a distance from a reference point (usually a support). The farther the force is from the point of support, the

greater the bending moment. There are two types of bending moment, based on which way the bending occurs:

Sagging or positive bending: When a material experiences compression on top fiber and tension (stretching) in the bottom fiber as shown in Fig. 1.8 (a).

Hogging or negative bending: When a material experiences tension (stretching) on top fiber and compression in the bottom fiber as shown in Fig. 1.8 (b).



(a) (b)

Fig. 1.8: Types of bending (a) Sagging or positive bending (b) Hogging or negative bending

(ii) Bending stress: Bending stress is the stress within a material caused by the applied bending moment. It is calculated using the formula:

$$\sigma_b = \frac{M_b \times r}{I} \tag{1.6}$$

Where: σ_b is the bending stress $\left(N/mm^2\right)$, M_b is the bending moment $(N-m)$, r is the distance from the neutral axis to the outermost fiber (mm) and I is the moment of inertia of the cross-sectional shape $(kg.m^2)$.

Fig. 1.9 shows the variation of bending stress at section $X-X$. The bending stress varies linearly across the cross section.

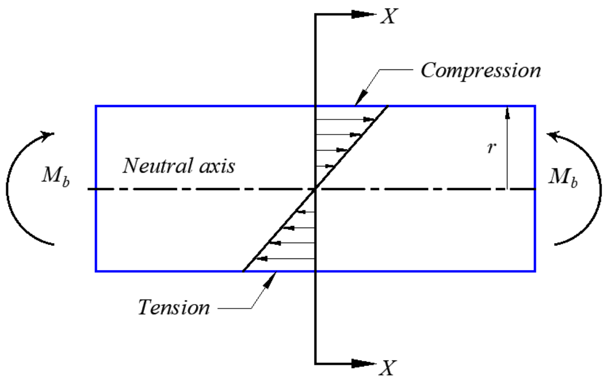


Fig.1.9: Variation of bending stress

(iii) Neutral Axis: The neutral axis is an imaginary line within the cross-section of a bent structure where there is no change in length during bending. Fig. 1.10 shows the neutral axis for cantilever and simply supported beam. It is a critical concept in understanding how materials respond to bending stresses. Understanding bending is crucial in designing structures that can withstand various loads and forces

without experiencing failure. Engineers use principles of bending to design beams, columns, and other structural elements to ensure they can support the applied loads and resist bending without compromising their integrity.

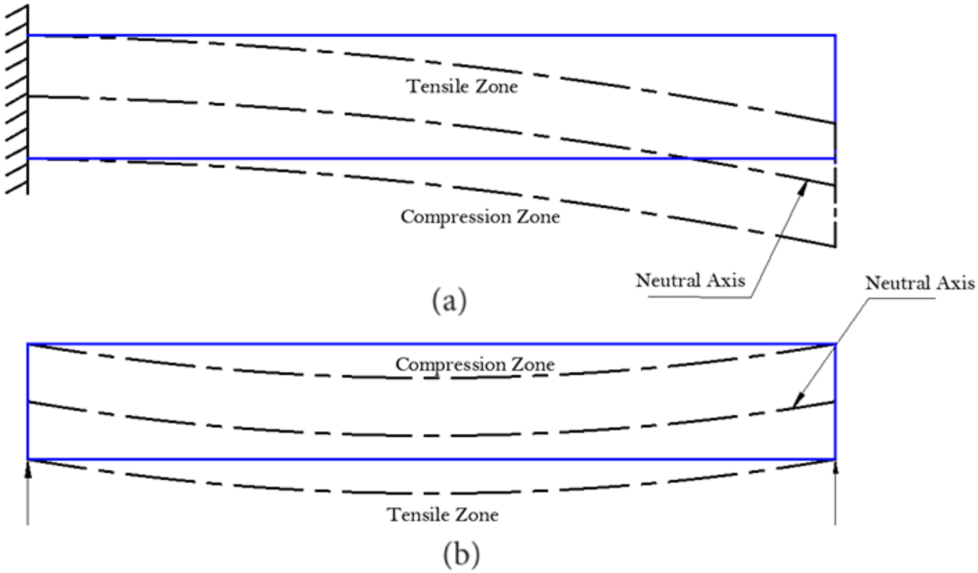


Fig.1.10: Neutral axis in a (a) Cantilever beam (b) Simply supported beam

1.10. TORSION

Torsion refers to the twisting or rotation of a structural element when subjected to a torque or twisting moment. This phenomenon is common in engineering and can occur in various applications, such as shafts, beams, and other structural components. Torsional deformation results in the angular displacement of the ends of a structure relative to each other.

(i) Torque: Torsion is induced by the application of torque, which is a force that causes rotation. Torque is typically applied at one end of a structural element and resisted by the material's internal resistance to shearing deformation.

(ii) Torsional stress: Torsional stress is the stress induced within a material due to the applied torque. It is calculated by using the Eq. (1.7):

$$\tau = \frac{M_t \times r}{J} \tag{1.7}$$

Where τ is the torsional stress (N/mm^2), M_t is the applied torque or torsional moment ($N-m$), r is the radial distance from the center of the shaft to the point where the stress is being calculated (known as the radius) (mm) and J is the polar moment of inertia of the cross-sectional area of the shaft ($kg.m^2$).

The polar moment of inertia (J) depends on the geometry of the cross-section and is a measure of the resistance of the shaft to torsional deformation. Torsional stress is significant in the design of components that experience rotational motion. Engineers must ensure that the material and geometry of a structure can withstand the applied torsional loads without exceeding the material's allowable torsional stress, which could lead to failure. In practical applications, materials are characterized by their shear modulus (G), and torsional stress can also be expressed in terms of shear modulus:

$$\tau = \frac{M_t \times r}{J} = \frac{M_t \times r}{G \times \theta} \tag{1.8}$$

Where θ is the angle of twist along the length of the shaft.

1.11. PRINCIPAL STRESSES

Principal stresses are the normal stresses ie. maximum and minimum normal stresses that occur on particular planes ie. principal plane where the shear stress is zero.

For a 2D stress state (plane stress), the principal stresses can be determined using the following equations:

Maximum principal stress $$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{1.9}$$

Minimum principal stress $$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{1.10}$$

Where σ_x and σ_y are the normal stresses in the x and y directions. τ_{xy} is the shear stress on the xy plane.

Principal plane $$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \tag{1.11}$$

Maximum shear stress $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{1.12}$$

Example 1.1: A point in a strained material is subjected to the stress as shown in Fig. 1.11. Locate the principal planes and find the principal stresses with respect to plane YZ.

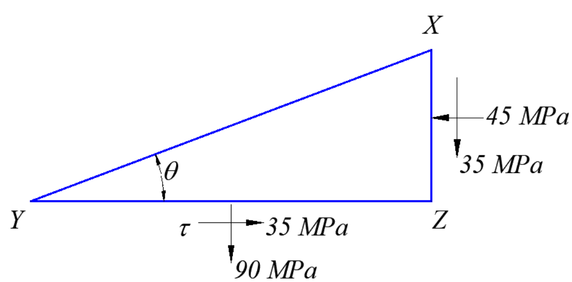


Fig.1.11: Point in a strained material subject to the stress

Given data:

Normal stress $\sigma_x = 90\ MPa = 90 N/mm^2$

Normal stress $\sigma_y = -45\ MPa = -45 N/mm^2$

Shear stress $\tau_{xy} = -35\ MPa = -35 N/mm^2$

Find:

1. Principal planes
2. Principal stresses with respect to plane YZ

Solution:

Consider the reference plane YZ,

Normal stress σ_x =+90 MPa

Shear stress τ_{xy} = - 35 MPa

On perpendicular plane XZ,

Normal stress σ_y = - 45 MPa

Shear stress τ_{xy} = 35 MPa

Let the principal plane make anticlockwise angle $=\theta$, Then

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times 35}{90-(-45)}$$

$$2\theta = \tan^{-1}\left(\frac{70}{135}\right) = 27.4 \text{ and } 27.4+180$$

$$\theta = 13^o\ 42' \text{ and } 103^o\ 42'$$

Maximum principal stress $\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{90-45}{2} + \sqrt{\left(\frac{90-(-45)}{2}\right)^2 + 35^2}$$

$$= 22.5 + 76.03$$

Maximum principal stress $\sigma_1 = 98.53 MPa$

Minimum principal stress $\sigma_2 = \dfrac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{90-45}{2} - \sqrt{\left(\frac{90-(-45)}{2}\right)^2 + 35^2}$$

$$= 22.5 - 76.03$$

Minimum principal stress $\sigma_2 = -53.53\,MPa$

Example 1.2: Given the two-dimensional stress state at a point as depicted in Fig. 1.12, determine the principal planes, principal stresses, and maximum shear stress.

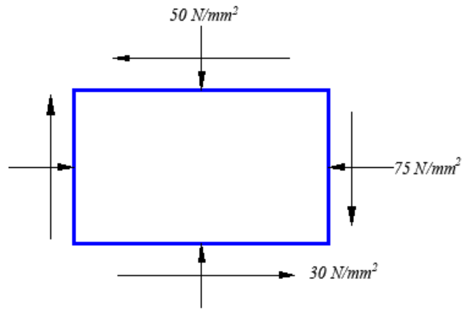


Fig.1.12: Stresses in two dimensions at a point in a deformed material

Given data:

Normal stress $\sigma_x = -75\,N/mm^2$

Normal stress $\sigma_y = -50\,N/mm^2$

Shear Stress $\tau_{xy} = -30\,N/mm^2$

Find:

1. Principal planes, principal stresses, and maximum shear stress

Solution:

Let the principal plane make anticlockwise angle $= \theta$, Then

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times(-30)}{-75-(-50)}$$

$$2\theta = \tan^{-1}\left(\frac{-70}{-25}\right) = 67.38 \text{ and } 67.38+180$$

$$\theta = 33^o\,41' \text{ and } 247^o\,22'$$

Maximum principal stress $\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{-75-50}{2} + \sqrt{\left(\frac{-75-(-50)}{2}\right)^2 + (-30)^2}$$

$$= -62.5 + 32.5$$

Maximum principal stress $\sigma_1 = -30\ N/mm^2$

Minimum principal stress $\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{-75-50}{2} - \sqrt{\left(\frac{-75-(-50)}{2}\right)^2 + (-30)^2}$$

$$= -62.5 - 32.5$$

Minimum principal stress $\sigma_2 = -95 N/mm^2$

Maximum shear Stress $\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \sqrt{\left(\frac{-75-(-50)}{2}\right)^2 + (-30)^2}$$

Maximum Shear Stress $\tau_{\max} = 32.5 N/mm^2$

1.12. CREEP STRAIN AND CREEP CURVE

Creep strain refers to the gradual and time-dependent deformation that occurs in a material when it is subjected to a constant load or stress at elevated temperatures. This deformation is plastic in nature, meaning that it is permanent and results in a change in the shape or dimensions of the material. The creep curve is a graphical representation of the relationship between strain and time under constant stress and elevated temperatures. The curve typically exhibits the three stages of creep—primary, secondary, and tertiary and it is shown in Fig. 1.13.

(i) *Primary creep:* In the initial stage, primary creep, the strain rate is relatively high but gradually decreases over time. This stage is marked by a rapid increase in strain, but the rate of deformation slows down as the material undergoes restructuring.

(ii) *Secondary creep:* The secondary creep stage is characterized by a relatively constant strain rate. During this phase, the rate of deformation becomes more uniform and predictable. The material undergoes steady-state creep, and the strain accumulates at a more constant rate.

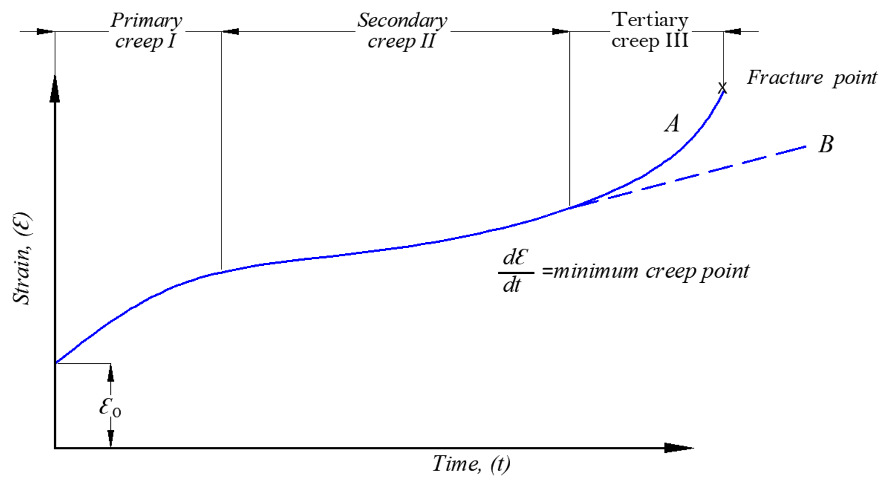


Fig.1.13: Creep curve

(iii) *Tertiary creep:* In the final stage, tertiary creep, the strain rate accelerates again, leading to the eventual failure of the material. Tertiary creep is often associated with the development of cracks, voids, or other modes of failure, culminating in the material reaching its ultimate limit.

The creep curve provides valuable information about a material's behavior under specific temperature and stress conditions. Engineers and materials scientists use this curve to analyze and predict the long-term deformation and potential failure of materials subjected to high-temperature and constant load conditions.

1.13. FATIGUE

Fatigue refers to the progressive and localized structural damage that occurs when a material undergoes cyclic loading or repeated stress over time. Unlike creep, which involves time-dependent deformation under a constant load, fatigue failure occurs due to the repeated application of loads, even if they are below the material's ultimate strength. Fatigue failure occurs in three stages – crack initiation; slow, stable crack growth; and rapid fracture. When it comes to engineering materials, fatigue failures often fall into three categories. Three categories are: high-cycle fatigue, low-cycle fatigue, and thermal fatigue.

The American Society for Testing and Materials (ASTM) defines fatigue life, N_f, as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs. Fatigue life is affected by cyclic stresses, residual stresses, material properties, internal defects, grain size, temperature, design geometry, surface quality, oxidation, corrosion, etc. For some materials, notably steel and titanium, there is a theoretical value for stress amplitude below which the material will not fail for any number of cycles, called a fatigue limit, endurance limit, or fatigue strength.

Engineers use a number of methods to determine the fatigue life of a material. One of the most useful is the stress-life method, is commonly characterized by an S-N curve, also known as a Wöhler curve. This method is illustrated in the Fig. 1.14. The number of cycles till failure is displayed on a logarithmic scale along the horizontal axis. The stress amplitude of the cycle is given on the vertical axis, which can be either linear or logarithmic. Fatigue tests are used to create S-N curves. Until the specimen fails, tests are conducted by applying a cyclic stress with constant amplitude on it. In some cases, the test is terminated after an excessively high number of cycles $\left(N > 10^6\right)$. The outcome is then understood to represent infinite life.

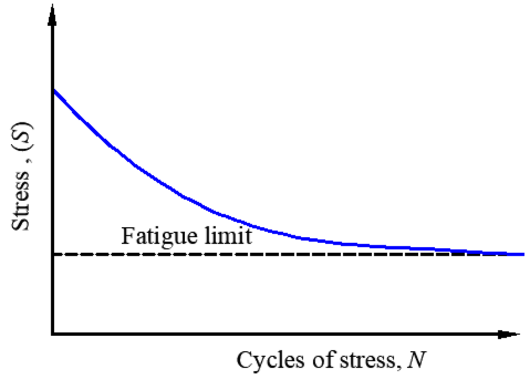


Fig.1.14: S – N Curve (Fatigue Limit)

The following terms are defined for S-N curve:

(i) Fatigue Limit: Fatigue limit (also sometimes called the endurance limit) is the stress level, below which fatigue failure does not occur. This limit exists only for some ferrous (iron-base) and titanium alloys, for which the S–N curve becomes horizontal at higher N values. Other structural metals, such as aluminium and copper, do not have a distinct limit and will eventually fail even from small stress amplitudes. Typical values of the limit for steels are 1/2 the ultimate tensile strength, to a maximum of 290 MPa.

(ii) Fatigue Strength: The ASTM defines fatigue strength, S_{Nf} , as the value of stress at which a material fails after specified number of cycles loading and unloading.

(iii) Fatigue Life: Fatigue life characterizes a material's fatigue behavior. It is the number of cycles to cause failure at a specified stress level, as taken from the S–N plot.

1.14. FACTOR OF SAFETY

The ability of a machine component to carry loads over and beyond what it can support is known as a factor of safety. The factor of safety is calculated using the following formula: Factor of Safety (FOS) = Ultimate strength or maximum load / applied load or stress.

$$FOS = \frac{Ultimate\ strength\ or\ \text{maximum load}}{Applied\ \text{load}\ or\ stress} \tag{1.13}$$

By having a factor of safety greater than 1, the design ensures that the machine component can handle loads well beyond what it is expected to encounter in regular service conditions.

For example, if a bridge is designed with a factor of safety of 2, it means that the bridge can theoretically handle twice the maximum load it is anticipated to carry during normal usage. This additional margin of safety helps prevent unexpected failures and enhances the reliability and durability of the structure.

The following factors should be taken into account by a design engineer before choosing an appropriate factor of safety:

1. The characteristics of the material and the potential changes to these characteristics during operation
2. Type of applied load, whether it is gradual or impact
3. Initial stresses set up during manufacturing of component
4. The extent of localized stresses
5. The degree of simplification in the assumptions
6. Mode of failure

Factor of Safety

The unnecessary risk of failure can be avoided by choosing high value of factor of safety. The safety factor values for various materials and load types are listed in Table 1.1, with consideration given to ultimate strength.

Table 1.1: Factor of safety for various materials and type of loads

Material	Steady load	Live load	Shock load
Cast iron	5 to 6	8 to 12	16 to 20
Wrought iron	4	7	10 to 15
Steel	4	8	12 to 16
Soft materials	6	9	15
Alloys	6	9	15
Leather	9	12	15
Timber	7	10 to 15	20

1.15. STRESS CONCENTRATION

The localized high stresses induced in the machine component due to presence of irregularities and abrupt variations in the cross section is defined as stress concentration. In a machine component, it is impossible to avoid the holes, grooves, notches, keyways, shoulders and sudden change in the cross section etc.

The discontinuity in a machine component affects the stress distribution in the nearer regions and it acts as one of the factor to raise the stress values. The stress concentration factor is used to consider the effect of stress concentration and find out localized stresses.

Stress Con. Lec 8

Consider a plate with a central hole and a uniform thickness subjected to uniform tensile force at the ends. The hole diameter (d), is significantly less than the plate width (w). The stress distribution at section $X_2 - X_2$ passing through the hole is depicted in Figure 1.15.

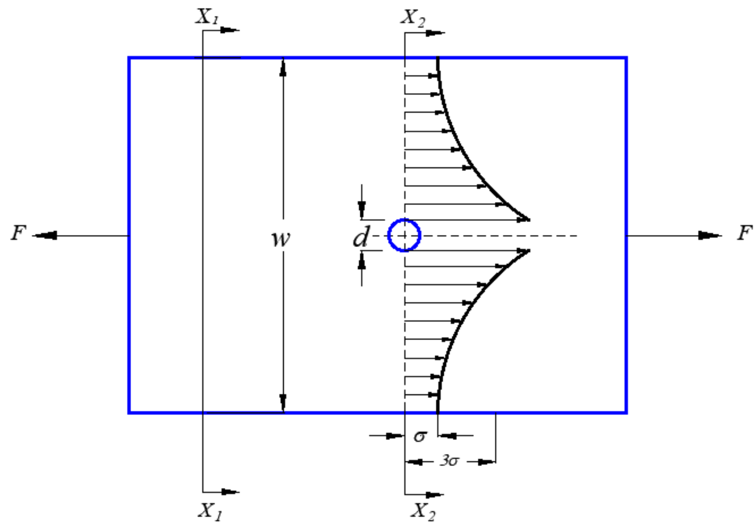


Fig.1.15: Stress concentration in a plate with a hole in the centre

It is observed that the high stress values in the vicinity of the hole. By using the strength of material approach, the tensile stress on any cross-section $X_1 - X_1$ may be determined as $\frac{F}{tw}$, where t is the thickness, under a tensile force F that is applied perpendicular to breadth on the plate's plane. The tensile stress would be $\frac{F}{t(w-d)}$, if the same computation technique were used on the cross-section $X_2 - X_2$ through the hole. However, the Theory of Elasticity solution to this problem provides tw with a stress distribution across-section $X_2 - X_2$, as illustrated in Fig. 1.15. This solution reduces parabolically to a constant value of $\frac{F}{tw}$, which is nominal stress, and demonstrates that the stress is maximum at the hole's edge on either side. In this instance, the real stress at the hole's edge is $\frac{3F}{tw}$. As a result, it is evident that simple calculations using the strength of the material approach may not provide the actual stress value in situations when the cross-section changes suddenly.

The term "stress concentration" refers to this type of cross-sectional change. The stress concentration factor—often represented by (K_t) is the ratio of actual stress to nominal stress (calculated stress).

(i) Causes of stress concentration:

1. The properties of the material change from one end to the other, leading to discontinuities in the component and resulting in stress concentration.
2. Stress concentration occurs when a relatively small area is subjected to a concentrated load.
3. A transmission shaft is designed with stepped cuts and shoulders to facilitate the attachment of gears, sprockets, pulleys, and ball bearings. Although these features are essential, they modify the shaft's cross-section, leading to concentrated stress at these points.
4. Discontinuities in machine components, such as keyways, oil holes, splines, oil grooves, and screw threads, create areas where stress is concentrated.
5. Surface imperfections such as machining scratches, stamp marks, or inspection marks lead to stress concentration.

(ii) Remedies of stress concentration:

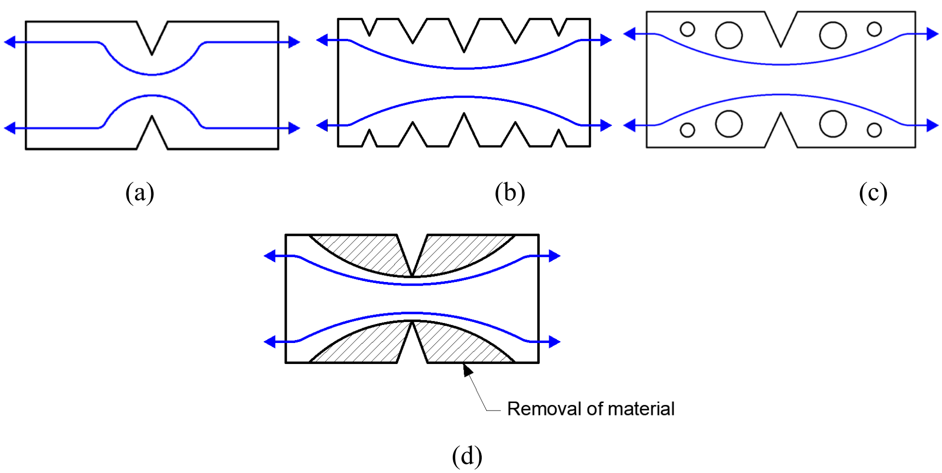


Fig. 1.16: Decrease in the concentration of stress in V-Notches (a) Initial notch (b) Multiple notches (c) Drilled holes (d) Elimination of undesired material

1. Stress concentration can be reduced in three ways: (a) by incorporating multiple notches, (b) by drilling additional holes, and (c) by removing excess material. Fig. 1.16 depicts the reduction of stress concentration in V-Notches.
2. To address abrupt changes in cross-section, as shown in Fig. 1.17, a fillet radius, undercutting, and notch can be introduced to the member.
3. A keyway introduces a discontinuity that reduces torsional shear strength and creates stress concentrations at its corners. To mitigate this, a fillet radius can be added to the inner corners of the keyway, or two symmetrical drilled holes can be placed on the sidewalls, as depicted in Fig. 1.18.
4. Reduction of stress concentration in threaded members – The stress concentration may be reduced by providing undercut between threaded portion and shank. Also the fillet radius may be added to the sharp edges of the undercut as shown in Fig. 1.19.

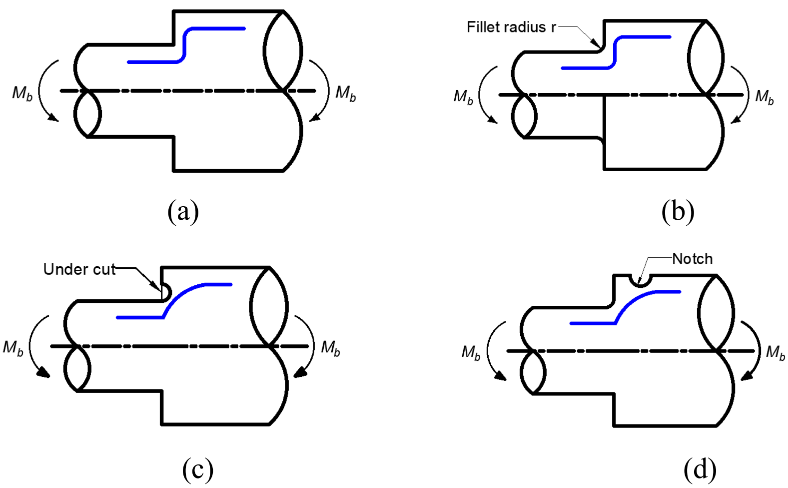


Fig. 1.17: Minimizing stress concentration in areas with abrupt cross-sectional changes: (a) Original component, (b) Fillet radius, (c) Undercut, (d) Addition of notch

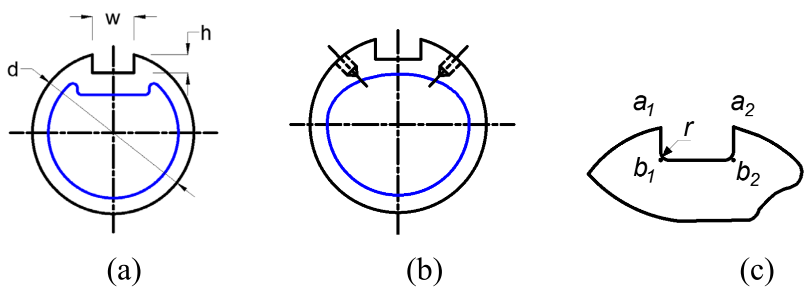


Fig. 1.18: Stress concentration reduction in a shaft with a keyway

(a) Original shaft (b) Drilled holes (c) Fillet radius

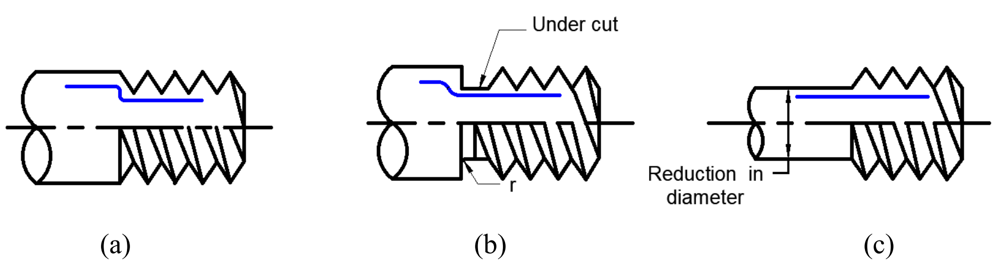


Fig. 1.19: Stress concentration reduction in threaded components

(a) Original part (b) Undercutting (c) Decreasing the shank diameter

1.16. CONVERTING ACTUAL LOAD OR TORQUE INTO DESIGN LOAD OR TORQUE

The design factors such as the velocity factor, factor of safety, and service factor are used to evaluate the design load or torque from actual load or torque. These factors account for various uncertainties and conditions that might affect the performance of the machine component.

(i) Service factor (K_o)*:* The service factor is defined as the ratio between the maximum torque M_t, i.e. design torque, transmitted between two gears and the rated torque $(M_t)_{mean}$ i.e. actual torque.

$$M_t = K_o \times (M_t)_{mean} \tag{1.14}$$

This factor considers the conditions under which the component will operate, including the nature of the load (steady, fluctuating, shock), the environment (temperature, corrosion), and other operational conditions. The service factor is selected based on the operational conditions, which may be provided as given in Table 1.2.

Table 1.2: Service factor (K_o) for speed reduction gear boxes

Working condition of source of power	**Working condition of driven machine**		
	Uniform	**Moderate shock**	**Heavy shock**
Uniform	1.00	1.25	1.75
Low shock	1.25	1.5	2.0
Medium shock	1.5	1.75	2.25

(ii) Velocity factor (C_v): The velocity factor takes into consideration the effect of speed on the load or torque. The dynamic force is produced between mating teeth when gears rotate at significant speeds. This force arises due to inaccuracies in the teeth profile, errors in tooth spacing, misalignment between the bearings, and the elasticity of the gears. In order to account this effect, the velocity factor is considered in the early stage of the design. The empirical relationship for velocity factor is used to calculate the design load between two meshing teeth.

$$W_d = \frac{K_o Wa}{C_v} \tag{1.15}$$

Where W_d is design load; W_a is actual load, C_v is velocity factor

The velocity factor is provided below for a variety of conditions:

$C_v = \dfrac{3}{3+v}$ for ordinary and cut gears using form cutters with $v < 10 \ m/s$

$C_v = \frac{6}{6+v}$ for accurately hobbed and generated gears with $v < 20\ m/s$

$C_v = \frac{5.6}{5.6+\sqrt{v}}$ for precision gears involving lapping, grinding and shaving processes with $v < 20\ m/s$

Here, v is pitch line velocity $= \frac{\pi DN}{60}$ in m/s,

D is pitch diameter of the gear in (m) and N is speed in rpm.

(iii) Factor of safety (FOS): This is a multiplier that accounts for uncertainties in material properties, load estimations, and potential flaws in design or manufacturing. The factor of safety is assumed based on the criticality of the machine component and the level of uncertainty in the design and operating conditions. The design load is calculated using Eq. (1.16) considering the factor of safety.

$$W_d = FOS \times W_a \qquad (1.16)$$

The values of factor of safety are listed in Table 1.1 for various materials and type of loads.

1.17. PROPERTIES OF ENGINEERING MATERIALS

Engineering materials can be either ductile or brittle, depending on their properties. Mechanical properties describe how materials behave elastically and plastically when subjected to forces or loads. These properties are essential in assessing a material's suitability for different engineering applications. The commonly used mechanical properties are:

(i) Strength: According to material mechanics, a material's strength is determined by its capacity to bear a certain weight without breaking down or deforming plastically. Strength is assessed through different metrics depending on the types of stresses caused by applied loads. It can be quantified as tensile strength, compressive strength, shear strength, and so on.

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(ii) Elasticity: Elasticity is the property of a material that allows it to deform under stress and return to its original shape once the stress is removed. While all engineering metals exhibit elasticity, the extent of their elasticity varies between metals. Steel, for example, is considered a perfectly elastic metal. It can experience minor deformations under applied loads, but its atoms shift from their original positions during the deformation and revert to those positions, restoring the metal to its initial shape when the loads are removed.

(iii) Plasticity: Plasticity is defined as the ability of a material, to undergo permanent deformation or changes in shape when subjected to applied forces. Unlike elastic deformation, which is reversible and temporary, plastic deformation involves a permanent alteration in the material's structure. After the load is removed, metals will experience significant permanent deformations when the stress reaches critical value,

called the yield stress. The movement of dislocations and the migration of grain boundaries on the micro-scale cause plastic deformations in metals and other crystalline materials.

(iv) Stiffness: Stiffness, or rigidity, refers to a material's ability to resist deformation when exposed to external forces. While all materials deform to some extent under stress, those that exhibit minimal deformation within their elastic limits are considered the stiffest. Stiffness can be quantified by the modulus of elasticity. For example, the modulus of elasticity for aluminum alloy is 71,000 N/mm^2, while for carbon steel it is 207,000 N/mm^2. As a result, carbon steel is more rigid than aluminum alloy, which makes stiffness a crucial consideration in the design of transmission shafts.

(v) Resilience: Resilience refers to a material’s ability to absorb and store energy during elastic deformation and then release that energy when the load is removed. A resilient material can handle energy within its elastic range without permanent deformation. This property is especially important for materials used in springs. Resilience is measured as the modulus of resilience, which indicates the strain energy per unit volume needed to stress the material up to its elastic limit in a tension test. This value is determined by the area under the stress-strain curve from the origin to the elastic limit.

(vi) Toughness: Toughness is defined as a material's capacity to absorb energy prior to experiencing fracture, essentially representing the energy required for failure due to fracture. This attribute is particularly vital for machine components that need to endure impact loads. Materials characterized as tough possess the ability to undergo bending, twisting, or stretching before reaching the point of failure. Structural steels, for instance, fall into the category of tough materials. Toughness is quantified by the modulus of toughness, which represents the total area under the stress-strain curve during a tension test and indicates the amount of work required to fracture the specimen. Practically, toughness is commonly measured with Izod and Charpy impact testing machines. It is important to note that toughness generally decreases as temperature increases.

The difference between resilience and toughness can be understood through the following points aspects:

1. Resilience refers to a material's capacity to absorb energy within its elastic range, whereas toughness involves absorbing energy both within the elastic and plastic ranges.
2. The modulus of resilience is measured by the area under the stress-strain curve up to the yield point in a tension test, while the modulus of toughness includes the entire area beneath the stress-strain curve.
3. Resilience is important for applications like springs, while toughness is crucial for components that experience bending, twisting, stretching, or impact loads. Resilience is exemplified in spring steels, while toughness is a characteristic of structural steels.

(vii) Malleability: Malleability refers to a material's ability to undergo significant deformation before showing signs of cracking when subjected to compressive forces. The term "malleability" comes from a word meaning "hammer," and it specifically describes a material's capability to be hammered into thin sheets. Malleable metals can be shaped through processes such as rolling, forging, or extrusion, which involve compressive forces. Examples of malleable metals include low carbon steels, copper, and

aluminum. Malleability generally increases with temperature, which is why processes like forging and rolling are known as hot working techniques, involving the shaping of heated ingots or slabs to achieve the desired form.

(viii) Ductility: Ductility is a material's ability to deform significantly before cracking under tensile stress. It specifically measures the permanent deformation that occurs before fracture during a tension test, reflecting how much a material can undergo plastic deformation before breaking. Examples of ductile materials include mild steel, copper, and aluminum. These materials can be shaped, drawn, or bent since these processes involve stretching under tension. Ductility is particularly valuable for machine components that might experience unexpected overloads or impacts. It is typically measured by percentage elongation or percentage reduction in area during a tension test. However, as temperatures rise, a metal's ductility generally decreases because the material becomes weaker at higher temperatures.

It is important to note that all ductile materials are also malleable, the reverse is not necessarily true. Some metals, despite being soft, can be weak under tension and may tear apart when stretched. Both malleability and ductility can be negatively impacted by impurities in the metal. Key differences between these properties include:

1. Malleability refers to a material's ability to deform under compressive forces, while ductility describes its capacity to deform under tensile forces.
2. Malleability generally increases with temperature, whereas ductility tends to decrease as temperature rises.
3. Although all ductile materials are malleable, not all malleable materials are ductile.
4. Malleability is crucial for processes like forging, rolling, or extrusion, whereas ductility is important for forming, drawing, or handling shock loads.

(ix) Brittleness: Brittleness is a property exhibited by materials that undergo minimal plastic deformation before reaching the point of fracture. It stands in direct contrast to ductility, with brittle materials showing little plastic deformation prior to fracture, as exemplified by cast iron. Unlike ductile materials, where failure occurs through yielding, brittle components experience abrupt fracture. The boundary between ductile and brittle materials in a tension test is a tensile strain of 5% at fracture. The main differences between ductility and brittleness are as follows:

1. In a tension test, brittle materials show minimal plastic deformation while ductile materials show considerable plastic deformation prior to breaking.
2. Certain materials are ductile, such as steel, copper, and aluminum, but cast iron is brittle.
3. Under a tension test, ductile specimens absorb more energy before breaking, whereas brittle fractures absorb the least amount of energy.
4. In brittle materials, failure happens immediately by fracture, but in ductile materials, failure happens gradually through yielding.

(x) Hardness: The ability of a material to resist abrasion, scratching, cutting, or shaping is indicated by its hardness, which is defined as its resistance to penetration or permanent deformation. This property is

crucial when selecting materials for components that undergo friction, such as pinions and gears, cams and followers, rails and wheels, and ball bearing parts. Improved wear resistance in these components is accomplished by increasing surface hardness through techniques such as case hardening. The four primary methods for measuring hardness are the Brinell hardness test, the Rockwell hardness test, the Vickers hardness test, and the Shore scleroscope.

In the first three methods, an indenter, typically made of diamond, carbide, or hardened steel and shaped like a ball, pyramid, or cone, is pressed onto the surface with a specified force. The hardness is then determined as an empirical value, like the Brinell hardness number, based on the cross-sectional area and depth of the indentation. The Shore scleroscope measures hardness based on the rebound height from the surface being tested. Unlike tension tests, hardness tests are simpler and non-destructive, as a small indentation may not adversely affect product performance. The material's hardness is contingent upon its resistance to plastic deformation, meaning that an increase in hardness correlates with an increase in strength. Empirical relationships between strength and hardness have been established for certain metals, such as steels, as shown in Eq. (1.17),

$$S_a = 3.45(BHN) \tag{1.17}$$

where S_a is the ultimate tensile strength in N/mm^2.

1.18. DESIGNATION OF MATERIALS AS PER IS

Various types of steel are used to manufacture machine parts. Steels are identified by a set of letters or numbers representing one of the following three properties: (i) tensile strength, (ii) carbon content, or (iii) the composition of alloying elements.

Steels are standardized according to their tensile strength. For instance, $Fe\,360$ denotes a steel with a minimum tensile strength of $360\,N/mm^2$. $Fe\,E\,250$ denotes a steel with a minimum yield strength of $250\,N/mm^2$.

The following three quantities are used to identify plain carbon steel:

1. A number that indicates the hundred times the typical carbon proportion
2. A letter C to denote Carbon
3. A number that denotes the ten times the typical manganese proportion

For instance, a simple carbon steel with 0.55% carbon and 0.4% manganese is designated as $55C4$.

'Alloy' steels are defined as low- and medium-grade alloy steels whose total alloying elements do not exceed 10%. The following quantities used for the designation of alloy steels:

1. A number that represents the hundred times the typical carbon proportion
2. Chemical symbols for the alloying elements, each accompanied by a factor multiplied by the value representing the element's average percentage content.

The chemical symbols and their percentages are listed in descending order of composition. The multiplying factor for different alloying elements are given in Table 1.3. For instance, $25Cr4Mo2$ is an alloy steel with an average of 0.2% molybdenum, 1% chromium, and 0.25% carbon. The typical carbon percentage is 0.25%, which is designated by the number (0.25×100) or 25. Average percentage of chromium is 1%. From Table 1.3, the multiplying factor for chromium is 4 and (1×4) is 4 or $Cr4$. Average percentage of molybdenum is 0.2%. The multiplying factor for molybdenum is 10 and (0.2×10) is 2 or $Mo2$. Therefore, the complete designation of steel is $25Cr4Mo2$.

Table 1.3: Multiplying factor for different alloying elements

Elements	**Factor of multiplication**
Cr, Co, Ni, Mn, Si and W	4
Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr and Mo	10
P, S and N	100

Steels containing over 10% alloying elements are known as 'high alloy steels'. The following quantities are used to designate high alloy steels:

1. A letter $'X'$
2. A number that represents the hundred times the typical carbon proportion
3. The symbol for each alloying element's average % content, rounded to the closest integer, and the chemical symbol for that element;
4. Chemical symbol to denote an element that has been added specifically to achieve desired qualities

As an example, $X15Cr25Ni12$ is high alloy steel with 0.15% carbon, 25% chromium, and 12% nickel.

Steel and cast iron are necessary components of any product. A wide range of cast iron and steel are produced for various uses. Collaborations with foreign companies have led to the use of several foreign standards and designations in our nation. The following are some significant names for ferrous materials:

(i) Grey cast iron has been categorized by the American Society for Testing Materials (ASTM) using a number. The minimum tensile strength is indicated by this class number in ksi. The minimum ultimate tensile strength of ASTM Class No. 20, for instance, is $20,000\ psi$ or $137.90\ N/mm^2$. Likewise, ASTM Class No. 50 refers to cast iron i.e. grey cast iron that has a minimum ultimate tensile strength of $50,000\ psi$ or $344.74\ N/mm^2$.

(ii) Grey cast iron is defined in Germany by the Deutches Institut Fuer Normung (DIN), with a minimum ultimate tensile strength in kgf/mm^2. Grey cast iron, for instance, has a minimum ultimate tensile strength of $12 kgf/mm^2$, as indicated by the designation $CG-12$. The DIN standard states that the common kinds of grey cast iron are $CG-12$, $CG-14$, $CG-18$, $CG-22$, $CG-26$, and $CG-30$.

(iii) The American Iron and Steel Institute (AISI) and the Society of Automotive Engineers (SAE) of the United States of America have established a numbered system for carbon and alloy steels. It is determined by the chemical makeup of the steel. There are either four or five digits in the number. The type or alloy classification is indicated by the first two digits. The carbon content is indicated by the last two numbers.

The SAE number and the AISI number for steel are the same. A capital letter prefixed to the number, such as A, B, C, D, or E, is also present. These uppercase letters stand for the steel-making process. These letters have the following meanings:

A - Alloy, Basic open-hearth alloy steel

B - Acid Bessemer carbon steel

C - Basic open-hearth carbon steel

D - Acid open-hearth carbon steel

E - Electric furnace alloy steel

For example, B1020 means non modified carbon steel, produced in acid Bessemer and containing 0.20% of carbon. The last two digits indicate the percentage of carbon in 0.01% (20×0.01) is 0.20%

1.19. USE OF DESIGN DATA BOOK

A Machine Design Data Book is a comprehensive reference guide that engineers, designers, and students use during the process of designing mechanical components and systems. It serves as a valuable resource for quick access to essential information, formulas, and data relevant to machine design. Design Data Book – Data Book for Engineers published by PSG College of Technology, Coimbatore is used for solving the problems related to machine design in this book. Here are some key uses of a Machine Design Data Book:

(i) Material Properties: Provides information about the mechanical properties of various materials, including tensile strength, yield strength, hardness, and thermal conductivity. This data is crucial for selecting the right material for a specific application.

(ii) Standard Dimensions and Tolerances: Offers standardized dimensions, tolerances, and fits for common machine elements such as shafts, bearings, gears, and fasteners. Ensures compatibility and interchangeability of components.

(iii) Design Formulas: Contains design formulas and equations for calculating stresses, deflections, and other critical parameters in machine components. Helps engineers in the design and analysis of machine elements.

(iv) Fastener Selection: Provides guidelines for selecting appropriate bolts, nuts, and screws based on load requirements, material properties, and environmental conditions. Ensures reliable and safe fastening.

(v) Power Transmission Elements: Offers information on gears, belts, chains, and other power transmission elements, including design considerations and selection criteria.

(vi) Bearings and Lubrication: Covers bearing types, selection criteria, and lubrication guidelines to ensure proper functioning and longevity of rotating components.

(vii) Safety Factors: Includes recommended safety factors for different materials and applications, helping engineers determine the margin of safety in their designs.

(viii) Conversion Tables: Provides conversion factors for units commonly used in machine design, facilitating easy conversion between different measurement systems.

(ix) Manufacturing Processes: Gives insights into various manufacturing processes and their impact on the design, helping engineers make informed decisions about material selection and fabrication methods.

(x) Environmental Considerations: Offers guidance on designing components that can withstand specific environmental conditions, such as corrosion resistance or temperature extremes.

1.20. STANDARDIZATION

The adoption of specified rules and specifications regarding materials, techniques, and equipment can be characterized as standardization in the engineering field. Various machine parts are utilized in several machines, including couplings, belts, ropes, chains, pumps, motors, lubrication devices, handles, seals, and fasteners. Because a standard item costs significantly less than a custom-made part, the sizes and ratings of these parts should be standardized. We should only maintain the necessary variety of sizes to ensure cost-effectiveness, and these sizes should be as standardized as possible. For this reason, systems, or "preferred numbers," or sizes, have been formed. Standardization has the benefits of lower costs, easier replacement, and lower amounts of material needed in stock. To save a great deal of work, the designer should be well-versed in the current standards. This will allow him to utilize the information in his designs. If standards are not maintained, the product may fail to compete internationally.

National and international organizations carry out the role of standardizing; in India, this responsibility is given to the Indian Standards Institution. For example, IS 4600 - 1968 relates to Indian Standards No. 4600, which was developed in 1968 and covers "Specification of flexible shafts". Standardization simplifies component specification for both manufacturers and customers.

1.21. PREFERRED NUMBERS

Standardization aims to establish preferred numbers or sizes. Preferred numbers are used to determine the appropriate number of sizes for a given product. The word "size" might apply to a product's linear dimensions, volume, weight, speed, power, or any other attribute. It has been discovered that when the range of sizes follows a Geometric Progression (GP), where each term is greater than the previous one by a fixed common ratio, the needs of the customers are largely addressed.

Preferred Numbers

Five series—R5, R10, R20, R40, and R80—are used to categorize the preferred numbers. Each series has its own series factor. The series factor referred to as an R5 series is $\sqrt[5]{10}$. The series factors for each series are listed in Table 1.4.

Table 1.4: Series factors for five series

Series	Series factors
Series R5	$\sqrt[5]{10} = 1.58$
Series R10	$\sqrt[10]{10} = 1.26$
Series R20	$\sqrt[20]{10} = 1.12$
Series R40	$\sqrt[40]{10} = 1.06$
Series R80	$\sqrt[80]{10} = 1.03$

1.22. THEORIES OF ELASTIC FAILURES

Different machine components are subjected to various loads at the same time. For example, a power screw experiences both torsional moments and axial forces. Similarly, an overhang crank is exposed to a mix of bending and torsional moments. Bolts that hold a bracket in place face forces that create both tensile and shear stresses. Components such as crankshafts, propeller shafts, and connecting rods are examples of parts subjected to complex loads. When a component experiences different types of loads, it encounters combined stresses. For instance, torsional moments lead to torsional shear stress, while bending moments produce bending stresses in transmission shafts. Failures in these components are typically categorized into two primary types: elastic failure, yielding, and fracture. Elastic failure leads to excessive deformation, impairing the component's ability to function properly. Yielding results in significant plastic deformation beyond the yield point, whereas fracture involves the component breaking into several pieces. The failure theories examined in this chapter are relevant to the elastic failure of machine parts.

Theories of Failure

The design of machine parts subjected to combined loads should rely on material properties determined experimentally under 'similar' conditions. However, testing every possible load combination to derive mechanical properties is impractical. Typically, mechanical properties are obtained through a basic tension test, which includes yield strength, ultimate tensile strength, and percentage elongation. In a tension test, the specimen is subjected to axial loading without any bending, torsion, or mixed loads. Elastic failure theories link the strength of a machine component under complex stress conditions to the mechanical properties measured in tension tests. These theories allow for the use of tension test data to determine component dimensions, regardless of the complex stress conditions the component might face.

Many theories have been proposed, each offering a different hypothesis about failure. The main theories related to elastic failure include:

1. Maximum principal stress theory (Rankine's theory)
2. Maximum shear stress theory (Coulomb, Tresca, and Guest's theory)
3. Distortion energy theory (Huber von Mises and Hencky's theory)
4. Maximum strain theory (St. Venant's theory)
5. Maximum total strain energy theory (Haigh's theory)

In this chapter, we will explore the initial three theories. By applying the loads, the principle stresses induced at a point on the machine part are denoted as σ_1, σ_2 and σ_3. The application of these failure theories aims to establish the relationship between σ_1, σ_2 and σ_3 on one hand and material properties such as S_{yt} or S_{ut} on the other.

1. Maximum principal stress theory (Rankine's theory)

According to this theory, the failure of the machine component takes place when the component undergoes combined loading and maximum principal stress reaches the yield or ultimate strength of the material.

The British engineer W.J.M. Rankine provided this theory of failure, so it is also known as Rankine's Theory or maximum primary stress theory.

The three principal stresses are σ_1, σ_2 and σ_3 at a point on the component and

$\sigma_1 > \sigma_2 > \sigma_3$, the failure occurs whenever

$$\sigma_1 = S_{yt} \text{ or } \sigma_1 = S_{ut} \tag{1.18}$$

whichever is applicable.

The theory exclusively accounts for the maximum principal stresses, overlooking the impact of other principal stresses. The dimensions of the component are established by incorporating a factor of safety.

For tensile stresses: $\sigma_1 = \dfrac{S_{yt}}{FOS}$ for ductile materials

$\sigma_1 = \dfrac{S_{ut}}{FOS}$ for brittle materials

For compressive stresses: $\sigma_1 = \dfrac{S_{yc}}{FOS}$ for ductile materials;

$\sigma_1 = \dfrac{S_{uc}}{FOS}$ for brittle materials

Where FOS is the factor of safety, S_{yt} is yield point stress in tension, S_{ut} is ultimate stress in tension, S_{yc} is yield point stress in compression, S_{uc} is ultimate stress in compression.

As the maximum principal or normal stress theory relies on failure occurring in tension or compression, neglecting the potential for failure due to shearing stress, it is not applicable to ductile materials.

2. Maximum shear stress theory (Coulomb, Tresca, and Guest's theory)

According to this theory, failure or yielding occurs at a specific point within a member when the maximum shear stress in a bi-axial stress system equals the shear stress at the yield point in a simple tension test.

$$\tau_{\max} = \frac{\tau_{yt}}{FOS} \tag{1.19}$$

Where $\tau_{\max}$ is maximum shear stress in biaxial stress system, τ_{yt} is shear stress at yield point, FOS is the factor of safety.

As the shear stress at the yield point in a simple tension test is equivalent to half the yield stress in tension, therefore the Eq. (1.20) may written as,

$$\tau_{\max} = \frac{\sigma_{yt}}{2 \times FOS} \tag{1.20}$$

This theory finds predominant use in the design of components made from ductile materials.

3. Distortion energy theory (Huber von Mises and Hencky's theory)

In accordance with this theory, failure or yielding takes place at a specific point within a member when the distortion strain energy (also referred to as shear strain energy) per unit volume in a bi-axial stress system attains the limiting distortion energy per unit volume, as determined from simple tension test.

Strain energy per unit volume in a bi-axial stress system is,

$$U_1 = \frac{1}{2E}\left[(\sigma_1)^2 + (\sigma_2)^2 - \frac{2\sigma_1\sigma_2}{m}\right] \tag{1.21}$$

Limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left[\left(\frac{\sigma_{yt}}{FOS}\right)^2\right] \tag{1.22}$$

According to the distortion energy theory, $U_1 = U_2$

$$\frac{1}{2E}\left[(\sigma_1)^2 + (\sigma_2)^2 - \frac{2\sigma_1\sigma_2}{m}\right] = \frac{1}{2E}\left[\left(\frac{\sigma_{yt}}{FOS}\right)^2\right] \tag{1.23}$$

Therefore, $(\sigma_1)^2 + (\sigma_2)^2 - \frac{2\sigma_1\sigma_2}{m} = \left(\frac{\sigma_{yt}}{FOS}\right)^2$ (1.24)

Where $\frac{1}{m}$ is poison's ratio.

This theory is commonly employed for ductile materials as a substitute for the maximum strain energy theory.

UNIT SUMMARY

- Stress: The force applied per unit area on a material. It is a measure of the internal resistance of a material to deformation under an applied force
- Types of stress: tensile stress, compressive stress, shear stress
- Strain: The measure of deformation or change in size or shape experienced by a material in response to an applied stress
- Types of strain: Tensile strain, Compressive strain, Shear strain
- Bearing pressure intensity: Bearing pressure intensity refers to the pressure exerted on a bearing surface, typically by a load or force acting on the surface
- Bending: Bending refers to the deformation of a structural component or material due to the application of external forces, typically moments or torques
- Types of bending moment: Sagging or positive bending, Hogging or negative bending
- Bending Stress: Bending stress is the stress within a material caused by the applied bending moment. It is calculated using the formula: $\sigma = \dfrac{M \times c}{I}$
- Maximum principal stress $\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Minimum principal stress $\sigma_2 = \dfrac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Maximum shear stress $\tau_{\max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Principal plane $\theta = \dfrac{1}{2}\tan^{-1}\left(\dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$
- Creep strain: Creep strain refers to the gradual and time-dependent deformation that occurs in a material when it is subjected to a constant load or stress at elevated temperatures
- Fatigue: Fatigue refers to the progressive and localized structural damage that occurs when a material undergoes cyclic loading or repeated stress over time
- Factor of safety: The ability of a machine component to carry loads over and beyond what it can support is known as a factor of safety
- Stress concentration: The localized high stresses induced in the machine component due to presence of irregularities and abrupt changes in the cross section is defined as stress concentration

- Service factor: The service factor is defined as the ratio between the maximum torque M_t, i.e. design torque, transmitted between two gears and the rated torque $(M_t)_{mean}$ i.e. actual torque
- Velocity factor: The velocity factor takes into consideration the effect of speed on the load or torque
- Standardization: The adoption of specified rules and specifications regarding materials, techniques, and equipment can be characterized as standardization in the engineering field

EXERCISES

Multiple Choice Questions

1. Shear stress on mutually perpendicular planes are

   (a) zero (b) maximum
   (c) equal (d) minimum

2. Which one of the following is dimensionless?

   (a) deformation (b) Young's modulus
   (c) stress (d) strain

3. ________________ parameters can be obtained by tension test of a standard specimen.

   (a) Proportional limit (b) Endurance limit
   (c) Percentage reduction in area (d) Fatigue limit

4. ___________ is measure of stiffness.

   (a) Young's modulus (b) Plasticity modulus
   (c) Resilience (d) Toughness

5. Principal stress is the magnitude of _________ stress acting on the principal plane.

   (a) normal stress (b) shear stress
   (c) both (a) and (b) (d) state of stress

6. Which of the following is not the method to reduce stress concentration?

   (a) additional notches and holes (b) fillet radius
   (c) additional discontinuities (d) sharp corners

7. According to Indian standard specifications, a plain carbon steel designated by 40C8 means that

   (a) carbon content is 0.04% and manganese is 0.08%
   (b) carbon content is 0.4% and manganese is 0.8%
   (c) carbon content is 4% and manganese is 8%
   (d) carbon content is 0.004% and manganese is 0.008%

8. Steels are standardized according to their ______________.

   (a) tensile strength (b) compressive strength
   (c) hardness (d) brittleness

9. Preferred numbers are arranged in

(a) arithmetic series
(b) hyperbolic series
(c) geometric series
(d) random number

10. _________ of failure is mostly used for ductile materials.

(a) maximum principal stress theory
(b) maximum shear stress theory
(c) distortion energy theory
(d) haigh's theory

11. The designation of steel for an alloy steel with an average of 0.2% molybdenum, 1% chromium, and 0.25% carbon.

(a) $25Cr4Mo2$
(b) $25Cr4Mo0.2$
(c) $25Cr1Mo2$
(d) $0.25Cr4Mo2$

12. ___________ is defined as a material's capacity to absorb energy prior to experiencing fracture, essentially representing the energy required for failure due to fracture.

(a) malleability
(b) toughness
(c) fatigue
(d) creep strain

13. The ability of a machine component to carry loads over and beyond what it can support is known as a __________.

(a) velocity factor
(b) toughness
(c) factor of safety
(d) service factor

14. The value of bending stress at neutral axis is___________.

(a) 0
(b) 1
(c) 0.1
(d) 2

15. The maximum bending stress occurs at

(a) 1/3rd distance from neutral axis
(b) neutral axis
(c) outer fibre
(d) none of these above

Answers to Multiple Choice Questions

1. (c) 2. (d) 3. (a) 4. (a) 5. (a) 6. (d) 7. (b) 8. (a) 9. (c) 10. (b) 11. (a) 12. (b) 13. (c) 14. (a) 15. (c)

Short and Long Answer Type Questions

1. Define load.
2. Define principal planes and principal stresses.
3. Define endurance limit.
4. What is the use of standardization in machine design?
5. What are preferred numbers?
6. Define factor of safety. What is its importance in design?

7. Mention various types of design considerations of machine elements.
8. What are various types of loads that are subjected to machine elements?
9. Distinguish between ductile and brittle materials with the help of a stress- strain diagram.
10. List the various types of materials used in machine design.
11. What is S-N diagrams? Draw S-N diagram for ductile material from that define endurance limit.
12. Discuss the various factors to be considered in deciding the factor of safety.
13. List the basic series of preferred numbers. How will you denote them?
14. State the advantages and limitations of standardization.
15. Discuss the basic series for preferred numbers.
16. List the advantages of preferred numbers to a machine designer.
17. Name the important mechanical properties of materials used in design.
18. Define "Stress concentration".
19. Show how to reduce stress concentration in a component.
20. Name various theories of failures and explain.
21. At a point in a strained material the principal stresses are $100\ N/mm^2$ (tensile) and $60\ N/mm^2$ (compressive). Determine normal stress, shear stress, resultant stress on a plane inclined at 50^o to the axis of the major principal stress. Also determine the maximum shear stress at the point.
22. The state of stress (in N/mm^2) acting at a certain point of the strained material are $75N/mm^2$ (tensile), $45\ N/mm^2$ (compressive) and $45\ N/mm^2$ (shear). Compute (i) The magnitude and nature of principal stresses and (ii) The orientation of principal planes.

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NPTEL VIDEOS

1. Lecture No 1 - Design Philosophy - Design of Machine Elements I by Prof. B. Maiti, Indian Institute of Technology, Kharagpur

Design Philosophy

2. Lectures 3&4 - Engineering Materials - Design of Machine Elements I by Prof. B. Maiti, Indian Institute of Technology, Kharagpur

Engg. Materials 1 | Engg. Materials 2

Lecture 3 | **Lecture 4**

3. Lectures 5&6 - Simple Stresses in Machine Elements - Design of Machine Elements I by Prof. B. Maiti, Indian Institute of Technology, Kharagpur.

Simple Stresses 1 | Simple Stresses 2

Lecture 5 | **Lecture 4**

4. Lectures 10 – Design for strength - Design of Machine Elements I by Prof. B. Maiti, Indian Institute of Technology, Kharagpur.

Stress Conc. Factor

5. Lecture 9 - Design for strength - Design of Machine Elements I by Prof. B. Maiti, Indian Institute of Technology, Kharagpur.

Principle Stresses

6. Lecture 1 – Types of stresses - Strength of materials by Dr. Suraj Prakash Harsha, Indian Institute of Technology, Roorkee.

Types of Stresses

7. Lecture 2 – Strength of materials - Strength of materials by Dr. Suraj Prakash Harsha, Indian Institute of Technology, Roorkee.

Types of Stresses

8. Lecture 6 – Principal stresses - Strength of materials by Dr. Suraj Prakash Harsha, Indian Institute of Technology, Roorkee.

Principal Stresses

9. Lecture 8 – Analysis of strains - Strength of materials by Dr. Suraj Prakash Harsha, Indian Institute of Technology, Roorkee.

Analysis of Strains

10. Lecture 12 – Stress – strain curve - Strength of materials by Dr. Suraj Prakash Harsha, Indian Institute of Technology, Roorkee.

Stress_Strain Curve

11. Lecture 19 – Bending equation - Strength of materials by Dr. Suraj Prakash Harsha, Indian Institute of Technology, Roorkee.

Bending Equation

12. Lecture 21 – Springs - Strength of materials by Dr. Suraj Prakash Harsha, Indian Institute of Technology, Roorkee.

Springs

2 Design of Simple Machine Parts & Antifriction Bearings

UNIT SPECIFICS

Through this unit, the following aspects are discussed:

- Design of cotter & knuckle joint
- Design of levers
- Design of C-clamp
- Classification of bearings
- Sliding and rolling contact bearings
- Terminology of ball bearings
- Life load relationship
- Selection of ball bearings using manufacturer's catalogue

RATIONALE

This unit focuses on the fundamental building blocks of machines: simple machine parts and bearings. By understanding the design principles of components like levers, cotter joints, and pulleys, a student will gain the knowledge to create efficient mechanisms that transmit forces effectively. Additionally, exploring different bearing types (sliding vs. rolling contact) and their selection process equips them with the tools to minimize friction and ensure smooth operation in the machine designs. By understanding the relationship between life load and basic static/dynamic load ratings for ball bearings, the student can able to interpret manufacturer catalogs and select the optimal bearing for a particular application. This knowledge can help the students to design machine components that achieve efficient force transmission, minimize friction losses, and ensure extended operational lifespans.

PRE-REQUISITES

Strength of materials, Engineering mechanics

UNIT OUTCOMES

List of outcomes of this unit is as follows:

On the successful completion of the unit, students will be able to

U2-O1: Identify and explain the functions of simple machine parts

U2-O2: Design simple machine parts with effective force transmission

U2-O3: Choose appropriate rolling contact bearings specific to applications

U2-O4: Interpret ball bearing terminology

U2-O5: Utilize manufacturer catalogue to select appropriate ball bearings

Unit 2 Outcomes	Mapping with Course Outcomes (1 – weak correlation, 2 – medium correlation, 3 – strong correlation)				
	CO-1	CO-2	CO-3	CO-4	CO-5
U2-O1	3	3	-	2	1
U2-O2	3	3	3	3	3
U2-O3	3	3	3	3	3
U2-O4	3	3	-	2	1
U2-O5	3	3	3	1	3

2.1. COTTER JOINTS

A cotter joint is a mechanical fastener used to connect two coaxial rods or bars, typically under tensile or compressive load. A cotter joint uses the wedge action principle. Cotters are usually wedge-shaped pieces of steel plates and by using the wedge action, the joint is tightened and adjusted.

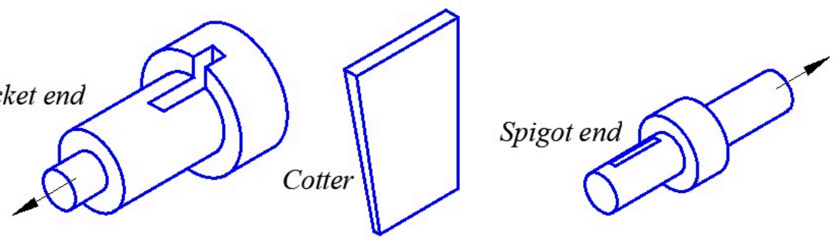


Fig. 2.1: Cotter joint parts

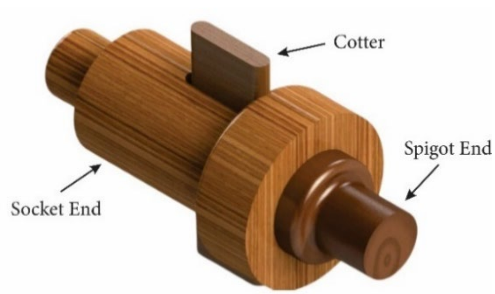


Fig. 2.2: Cotter joint assembly

Fig. 2.1 shows the components of a cotter joint that can connect two different rods A and B instantly. Usually, the socket end is provided at one end of the rod, while the another connecting rod is provided with a spigot end. Ideally, the socket end of one rod should fit over the spigot end of the other rod as shown in Fig. 2.2. A narrow rectangular slot is provided in both the socket and spigot. In the slot, a cotter is tightly fitted between the socket and the spigot. There is a slight taper in the width dimension b of the cotter; usually a taper of 1 in 24. There are two reasons for providing a taper:

- With the cotter inserted in the slot and hammered into the spigot, wedge action causes the cotter to tighten. As a result, the joint remains tight while operating and the parts don't loosen
- The taper shape of the cotter makes it easy to remove and dismantle

Between the slots and the cotter, there is a clearance of 2 to 3 mm as shown in Fig. 2.4. Cotters are driven into slots until the socket collar rests on the spigot collar so that the two rods are drawn

together. In cotters, draw is defined as the distance by which the cotter is driven into the slot to tighten the joint and secure the connection between the two components.

2.1.1. Advantages of cotter joint

- A cotter joint can be assembled and dismantled in a short time. During assembly, the spigot end is inserted into the socket end and the cotter will be placed in their common slots. As the cotter is pressed, the rods are brought closer and tightened. By hammering the cotter out of the slot, the cotter can be dismantled
- With the wedge action, a very high tightening force is developed, preventing parts from slipping during service
- Designing and manufacturing the joint is simple

2.1.2. Applications

It is used to connect machine parts, such as crossheads or base plates, on one side with rods on the other. Cotter joints are not suitable to connect two shafts that transmits torque.

- Steam engine piston rod-crosshead joint
- Valve mechanism joint between slide spindle and fork
- Piston rod-tail rod joint
- Foundation bolt

2.2. TYPES OF COTTER JOINTS

The following are three cotter joints used:

1. Socket and spigot cotter joint
2. Sleeve and cotter joint, and
3. Gib and cotter joint.

2.3. DESIGN OF SOCKET AND SPIGOT COTTER JOINT

To design a cotter joint, the following assumptions are made for the stress analysis

- The applied force (F) acts purely along the axis of the joint (either tensile or compressive).
- The effect of the slot on stress concentration is not considered
- Stress induced during initial cotter pin tightening is ignored

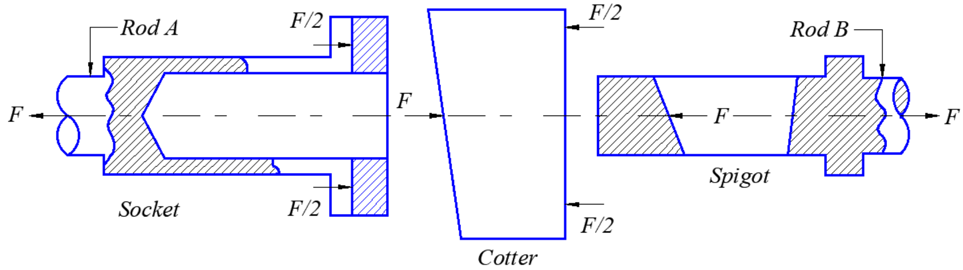


Fig.2.3: Free body diagram of forces

1. Analysis of cotter joint forces:

This section analyzes the forces exerted on the different parts of a cotter joint: the socket, the cotter, and the spigot (refer to Fig. 2.3).

Design of Cotter Joint

The forces depicted on the cotter represent the reaction forces that correspond to the interaction between the socket end of Rod A and the spigot end of Rod B. These forces are equal in magnitude but opposite in direction.

2. Rod A (socket end):

- Rod A experiences a horizontal force F acting towards the left.
- To maintain equilibrium, the sum of all horizontal forces acting on Rod A must be zero.

3. Rod B (spigot end):

- Rod B experiences a horizontal force F acting towards the right.
- Similar to Rod A, the sum of all horizontal forces acting on Rod B must be zero.

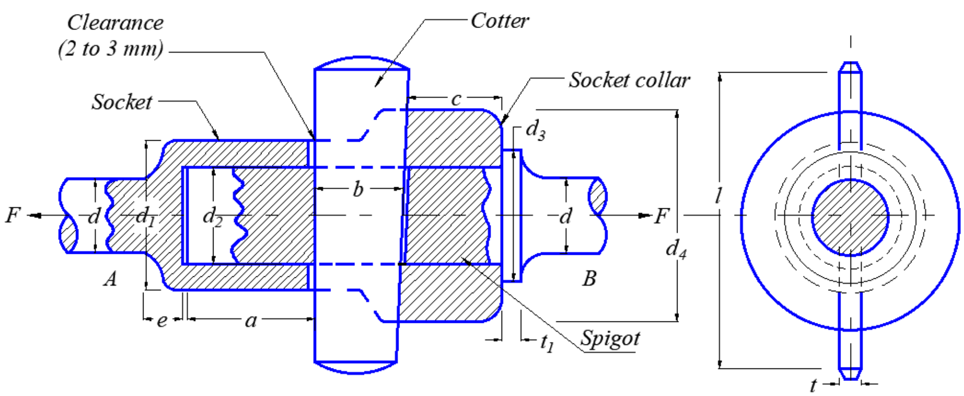


Fig.2.4: Sectional view of socket and spigot cotter joint

A typical socket and spigot cotter joint is depicted in Fig.2.4.

(i) Diameter of rod:

The diameter of rod can be calculated by assuming that rod fails in tension,

Tensile force = Resisting cross sectional area x Permissible stress

Cotter Joint

$$F = \frac{\pi}{4} \times d^2 \times \sigma_t \tag{2.1}$$

where F is applied force (N), d is rod diameter (mm) and σ_t is permissible tensile stress in rod (N/mm^2)

(ii) Diameter of spigot:

The spigot diameter or inner diameter of socket (d_2) can be calculated by assuming that spigot fails in tension,

$$F = \left\{ \frac{\pi}{4}(d_2)^2 - (d_2 \times t) \right\} \sigma_t \tag{2.2}$$

If the rod or cotter fails under crushing, then diameter of spigot can be calculated by

Area of the rod subjected to crushing = $d_2 \times t$

$$F = d_2 \times t \times \sigma_c \tag{2.3}$$

where d_2 is Spigot diameter (or) socket internal diameter (mm), t is thickness of cotter (mm) $(t = d_2/4)$ and σ_c is permissible crushing stress in cotter (N/mm^2)

(iii) Diameter of socket:

The socket diameter can be calculated by assuming that socket fails in tension around the slot

Area of the socket subjected to load around the slot = $\frac{\pi}{4}\left[d_1^2 - d_2^2\right] - \left[(d_1 - d_2) \times t\right]$

$$F = \left\{ \frac{\pi}{4}\left[d_1^2 - d_2^2\right] - \left[(d_1 - d_2) \times t\right] \right\} \sigma_t \tag{2.4}$$

where d_1 is socket external diameter (mm)

(iv) Width of cotter:

The cotter width can be calculated by assuming that socket fails under shear (double shear)

Area of cotter subjected to shear $= 2(b \times t)$

$$F = 2b \times t \times \tau \tag{2.5}$$

where b is average width of the cotter (mm) and τ is permissible shear stress (N/mm^2)

(v) Diameter of socket collar:

The socket collar diameter can be calculated by assuming that socket collar fails under crushing, as failure occurs primarily due to the compressive stresses created around the hole; tension forces are not accounted for the calculation.

Area of socket collar subjected to crushing $=(d_4-d_2)\times t$

$$F=\left[(d_4-d_2)\times t\right]\times\sigma_c \tag{2.6}$$

where d_4 is Socket collar outer diameter (mm)

(vi) Thickness of socket collar:

The socket collar thickness can be calculated by assuming that socket end fails under double shear

Shearing Area $=2(d_4-d_2)c$

$$F=(d_4-d_2)c\times\tau \tag{2.7}$$

where c is socket collar thickness (mm)

(vii) Distance between slot end and rod end:

The distance can be calculated by assuming that rod end fails under double shear

Shearing Area $=2(a\times d_2)$

$$F=2a\times d_2\times\tau \tag{2.8}$$

where a is distance from slot end to rod end (mm)

(viii) Diameter of spigot collar:

The spigot collar diameter can be calculated by assuming that spigot collar fails under crushing. Since, the spigot collar primarily fails due to tightening of the cotter i.e., compressive loads, the tensile force acting on the joint may not be responsible for it. Hence, only the compressive load is taken into account,

Area of collar subjected to crushing = $\frac{\pi}{4}\left[d_3^2-d_2^2\right]$

$$F=\frac{\pi}{4}\left[d_3^2-d_2^2\right]\times\sigma_c \tag{2.9}$$

Where d_3 is spigot collar diameter (mm)

(ix) Thickness of spigot collar:

The spigot collar thickness can be calculated by assuming that spigot collar fails due to shear.

Since tensile or crushing forces are not directly responsible for the failure of spigots in terms of thickness, only shear stress is taken into account.

Area of collar subjected to shear = $\pi d_2\times t_1$

$$F=\pi d_2\times t_1\times\tau \tag{2.10}$$

where t_1 in spigot collar thickness (mm)

(x) Bending stress:

Maximum bending moment is at the centre of cotter

$$\left(M_b\right)_{\max}=\frac{F}{2}\left(\frac{1}{3}\times\frac{d_4-d_2}{2}+\frac{d_2}{2}\right)-\frac{F}{2}\times\frac{d_2}{4}=\frac{F}{2}\left(\frac{d_4-d_2}{6}+\frac{d_2}{4}\right)$$

Hence, bending stress in cotter is given by,

$$\sigma_b=\frac{\left(M_b\right)_{\max}}{Z}=\frac{\frac{F}{2}\left(\frac{d_4-d_2}{6}+\frac{d_2}{4}\right)}{t\times b^2/6}=\frac{F\left(d_4+0.5d_2\right)}{2t\times b^2} \tag{2.11}$$

where is Z is section modulus of the cross section (rectangle)

(xi) Length of cotter:

Cotter length is usually taken as 4 times the rod diameter i.e., $l=4d$

Where l is cotter length (mm)

(xii) Cotter taper:

The taper in the cotter should not greater than 1 in 24.

(xiii) Cotter draw:

The Draw of cotter is usually between 2 and $3\ mm$.

Example 2.1: A varying load of $50\ kN$ compression to $50\ kN$ tension is applied to a cotter joint made of carbon steel. Take tensile/ compressive, shear, and crushing stress $70\,MPa$, $50\,MPa$, and $100\,MPa$, respectively. Assuming the load is applied axially, design the cotter joint.

Given data:

Load, $F=50\,kN=50{,}000N$

Allowable tensile / compressive stress, $\sigma=70\,MPa=70\,N/mm^2$

Shear stress, $\tau=50\,MPa=50\,N/mm^2$

Crushing stress, $\sigma_c=100\,MPa=100\,N/mm^2$

Find:

1. Design the cotter joint

Solution:

(i) Rod diameter (d)**:**

The diameter of rod can be calculated by assuming that rod fails in tension,

Tensile Load = Resisting Area $\times$ Stress

$$50\times10^3 = \frac{\pi}{4}\times d^2\times\sigma_1 = \frac{\pi}{4}\times d^2\times70 = 54.95d^2$$

$$d^2 = 50\times10^3/54.95 = 910 \quad or \quad d = 30.12 = 31mm$$

(ii) Spigot diameter (d_2)**:**

The spigot diameter or inner diameter of socket (d_2) can be calculated by assuming that spigot fails in tension,

$$F = \left\{\frac{\pi}{4}(d_2)^2 - (d_2\times t)\right\}\sigma_t$$

$$d_2^2 = 50\times10^3/25 = 2000 \; or \; d_2 = 44.7 = 45mm$$

$$50\times10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2\times t\right]\sigma_1 = \left[\frac{\pi}{4}(d_2)^2 - d_2\times\frac{d_2}{4}\right]70 = 38(d_2)^2$$

$$(d_2)^2 = 50\times10^3/38 = 1315.8 \quad or \quad d_2 = 36.27 = 37\,mm$$

(iii) Cotter thickness (t)**:**

Thickness of cotter is usually $d_2/4$.

$$t = \frac{d_2}{4} = \frac{37}{4} = 9.5\,mm$$

Validating calculated thickness against crushing stress.

$$F = d_2\times t\times\sigma_c$$

$$50\times10^3 = d_2\times t\times\sigma_c = 37\times9.5\times\sigma_c = 351.5\sigma_c$$

$$\sigma_c = 50\times10^3/351.5 = 142.24\,N/mm^2$$

Since calculated crushing stress is greater than permissible crushing stress, it is not advisable to use (d_2) as $37\,mm$ and t as $9.5\,mm$.

Hence, calculate the value of spigot diameter and cotter thickness again from the crushing stress $\sigma_c = 100\,N/mm^2$.

$$50\times10^3 = d_2\times\frac{d_2}{4}\times100 = 25\times d_2^2$$

$$d_2^2 = 50\times10^3/25 = 2000 \; or \; d_2 = 44.7 = 45mm$$

Cotter thickness, $t = d_2/4 = 45/4 = 12\,mm$

(iv) Socket outside diameter (d_1):

The socket diameter can be calculated by assuming that socket fails in tension around the slot

$$F = \left\{ \frac{\pi}{4}\left[d_1^2 - d_2^2\right] - \left[(d_1 - d_2)\times t\right] \right\}\sigma_t$$

$$50\times10^3 = \left[\frac{\pi}{4}\left[d_1^2 - d_2^2\right] - (d_1 - d_2)t\right]\sigma_t$$

$$50\times10^3 = \left[\frac{\pi}{4}\left\{(d_1)^2 - (45)^2\right\} - (d_1 - 45)12\right]70$$

$$50\times10^3/70 = 0.7854(d_1)^2 - 1590 - 12d_1 + 540$$

$$0.7854(d_1)^2 - 12d_1 - 1764.2 = 0 \qquad (d_1)^2 - 15.3d_1 - 2246.2 = 0$$

$$d_1 = \frac{15.3 \pm \sqrt{(15.3)^2 + 4\times2246.2}}{2} = \frac{15.3\pm96}{2}$$

$d_1 = 55.6 = 56\,mm$ (Talking +ve sign)

(v) Cotter width (b):

The cotter width can be calculated by assuming that socket fails under shear (double shear)

$$F = (2b\times t)\times\tau$$

$$50\times10^3 = (2b\times t)\times\tau = 2b\times12\times50 = 1200b$$

$$b = 50\times10^3/1200 = 42\,mm$$

(vi) Socket collar diameter (d_4):

The socket collar diameter can be calculated by assuming that socket collar fails under crushing

$$F = \left[(d_4 - d_2)\times t\right]\times\sigma_c$$

$$50\times10^3 = (d_4 - d_2)t\times\sigma_c = (d_4 - 45)12\times100 = (d_4 - 45)1200$$

$$d_4 - 45 = 50\times10^3/1200 = 41.6 \;\; or \;\; d_4 = 41.6 + 45 = 86.6 = 87\,mm$$

(vii) Socket collar thickness (c):

The socket collar thickness can be calculated by assuming that socket end fails under double shear

$$F = (d_4 - d_2)c\times\tau$$

$$50\times10^3=\left(d_4-d_2\right)c\times\tau=\left(87-45\right)c\times50=1950c$$

$$c=50\times10^3/1950=25.6=26\,mm$$

(viii) Distance between slot end and rod end (a)**:**

The distance can be calculated by assuming that rod end fails under double shear

$$F=2a\times d_2\times\tau$$

$$50\times10^3=2a\times d_2\times\tau=2a\times45\times50=4500a$$

$$a=50\times10^3/4500=11.1=11.5\,mm$$

(ix) Spigot collar diameter (d_3)**:**

The spigot collar diameter can be calculated by assuming that spigot collar fails under crushing

$$F=\frac{\pi}{4}\left[d_3^2-d_2^2\right]\times\sigma_c$$

$$50\times10^3=\frac{\pi}{4}\left[(d_3)^2-(d_2)^2\right]\sigma_c=\frac{\pi}{4}\left[(d_3)^2-(45)^2\right]100$$

$$\left(d_3\right)^2-\left(45\right)^2=\frac{50\times10^3\times4}{100\times\pi}=636$$

$$\left(d_3\right)^2=636+\left(45\right)^2=2661\quad or\quad d_3=51.5=52\,mm$$

(x) Spigot collar thickness (t_1)**:**

The spigot collar thickness can be calculated by assuming that spigot collar fails due to shear

$$F=\pi d_2\times t_1\times\tau$$

$$50\times10^3=\pi d_2\times t_1\times\tau=\pi\times45\times t_1\times50=7065t_1$$

$$t_1=50\times10^3/7065=7.07=8\,mm$$

(xi) Cotter length (l)**:**

$$l\ =\ 4\times d\ =\ 4\times31\ =\ 124\ mm$$

(xii) Dimension (e)**:**

$$e\ =\ 1.2\times d\ =1.2\times31\ =\ 37.2\ =\ 38\ mm$$

2.4. DESIGN OF SLEEVE AND COTTER JOINT

A sleeve and cotter joint shown in Fig 2.5 & Fig 2.6 uses a wedged component, known as a cotter, to induce frictional forces between a hollow cylinder (sleeve) that slides over the aligned rods and their corresponding ends. These joints are capable of withstanding both tensile (pulling) and compressive (pushing) loads applied along the longitudinal axis of the rods. Typically, cotters have a taper of 1 in 24.

In general, the dimensions of different parts with respect to rod diameter (d) as follows,

1. Sleeve external diameter (d_1) is 2.5 times the rod diameter (d)
2. Larger end rod diameter (d_2) (or) sleeve internal diameter is 1.25 times the rod diameter (d)
3. Sleeve length (L) is 8 times the rod diameter (d)
4. Cotter thickness (t) is $d_2/4$ or 0.31 times the rod diameter (d)
5. Cotter width (b) is 1.25 times the rod diameter (d)
6. Cotter length (l) is 4 times the rod diameter (d)
7. Length between the rod end and the cotter hole (a) and the length between rod end and the cotter hole (c) is 1.25 times the rod diameter (d)

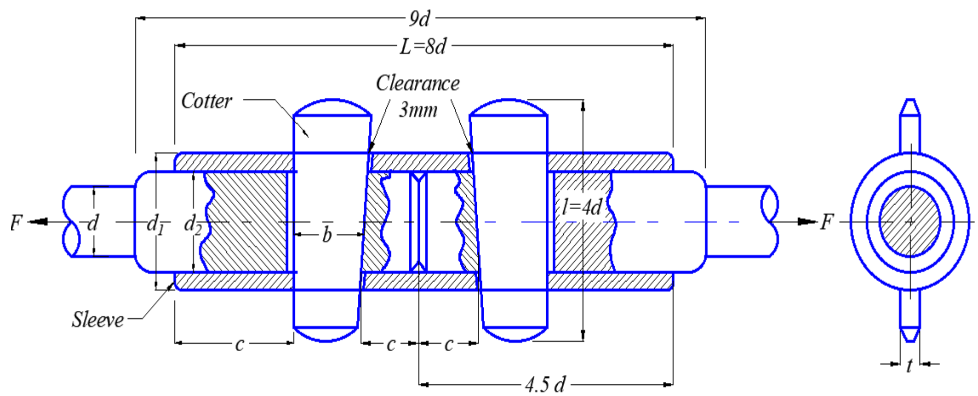


Fig. 2.5: Sectional view of sleeve and cotter joint

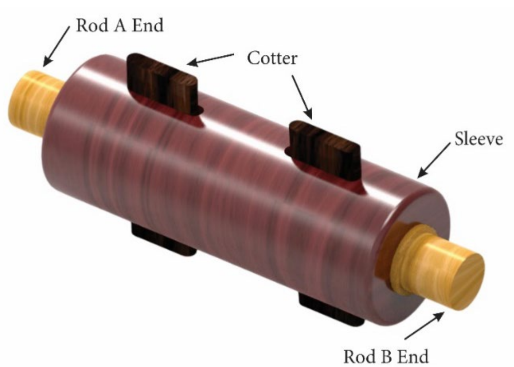


Fig. 2.6: 3D view of sleeve and cotter joint

The detailed design procedure on the basis of load is given below:

(i) Diameter of rod:

The diameter of rod can be calculated by assuming that rod fails in tension,

$$F=\left(\frac{\pi}{4}\times d^2\right)\times\sigma_t \tag{2.12}$$

where F is applied force (N), d is rod diameter (mm) and σ_t is permissible tensile stress in rod (N/mm^2)

(ii) Diameter of larger end of rod:

The diameter of larger end of rod (d_2) can be calculated by assuming that slotted area of cotter fails under the tension,

$$F=\left\{\frac{\pi}{4}(d_2)^2 - (d_2\times t)\right\}\sigma_t \tag{2.13}$$

where d_2 is diameter of larger end of rod (mm) and t is thickness of the cotter (mm) $(t=d_2/4)$

If the cotter joint is assumed to fail under the crushing load, then

$$F=(d_2\times t)\times\sigma_c \tag{2.14}$$

where σ_c is permissible crushing stress in cotter (N/mm^2)

The above equation can help to check the failure of cotter joint with induced crushing stress.

(iii) Outer diameter of sleeve:

The outer diameter of sleeve (d_1) can be calculated by assuming that sleeve across the slot fails under the tension,

$$F=\left\{\frac{\pi}{4}\left[d_1^2-d_2^2\right]-\left[(d_1-d_2)\times t\right]\right\}\sigma_t \tag{2.15}$$

where (d_1) is outer diameter of sleeve (mm)

(iv) Width of cotter:

The cotter width can be calculated by assuming that cotter fails under shear (double shear)

$$F=(2b\times t)\times\tau \tag{2.16}$$

where b is cotter width (mm) and τ is permissible shear stress (N/mm^2)

(v) Distance between beginning of rod end and cotter hole:

The distance can be calculated by assuming that rod end fails under double shear

$$F=(2a\times d_2)\times\tau \tag{2.17}$$

where a is distance between beginning of rod and cotter hole

(vi) Distance between end of rod end and cotter hole:

The distance can be calculated by assuming that sleeve end fails under double shear

$$F = 2\left(d_1 - d_2\right) c \times \tau \tag{2.18}$$

where c is distance between end of rod and cotter hole

2.5. GIB AND COTTER JOINT

A gib and cotter joint is a mechanical fastening device used to connect two rectangular or square cross-sections that are subjected to axial forces, such as in the case of connecting a piston rod to the crosshead in a steam engine. This type of joint is particularly useful when the connection must withstand both tensile and compressive forces. The gib and cotter joint shown in Fig. 2.7 consists of three primary components: the gib, the cotter, and the rods or bars to be joined. The gib is a flat, tapered piece of metal that provides additional strength and support to the joint, preventing the cotter from loosening under load.

The cotter is a wedge-shaped piece of metal that fits through slots in the connected rods and the gib, effectively locking the components together. The slots are typically tapered to match the cotter, ensuring a tight fit and the capability to handle substantial axial loads.

A joint without gib results in friction between the cotter and the strap, causing the cotter to deform outwards. This can be avoided by using Gib to hold the ends of the straps together.

2.5.1. Types of gib and cotter joint

(i) Single gib and cotter joint:

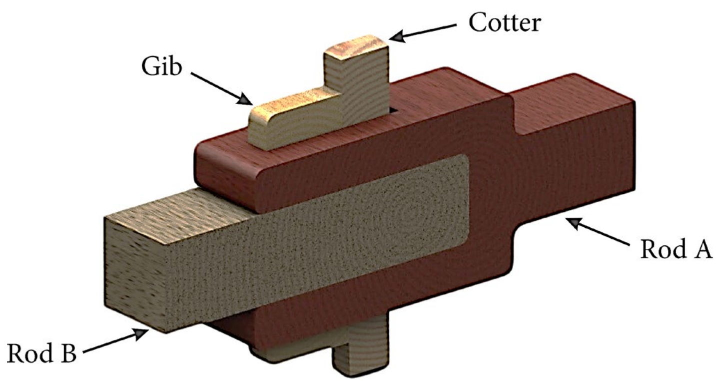


Fig. 2.7: Single gib and cotter joint

In this type, a single gib is used along with a cotter to secure the connection between two rods as shown in Fig. 2.7. The gib is a wedge-shaped piece that helps in preventing lateral movement of the cotter and distributes the load more evenly.

(ii) Double gib and cotter joint:

This type uses two gibs placed on either side of the cotter as shown in Fig. 2.8 to enhance the stability and load distribution of the joint. The double gib and cotter joint provides better alignment and is more robust than the single gib and cotter joint.

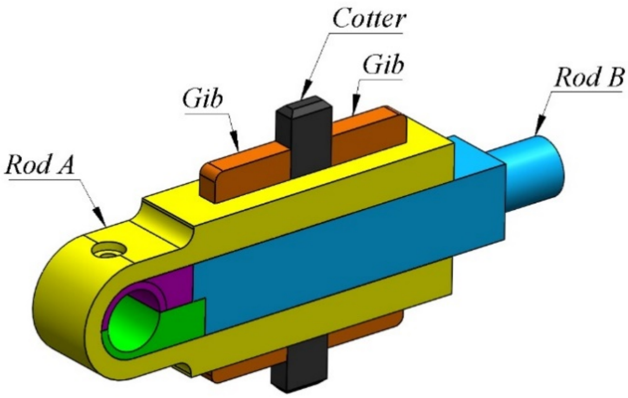


Fig. 2.8: 3D view of gib and cotter joint

2.6. SELECTION OF COTTER JOINT

When selecting a suitable cotter joint for a specific application, several critical factors must be evaluated to ensure optimal performance and durability.

The socket and spigot cotter joint are ideal for applications where precise alignment and ease of assembly and disassembly are essential, such as in connecting piston rods to crossheads in engines or securing tie rods in mechanical linkages. This type ensures a reliable connection with straightforward construction, making it suitable for moderate loads and regular maintenance routines.

The sleeve and cotter joint are better suited for applications involving larger diameters and higher axial loads, common in heavy machinery and structural frameworks. The sleeve provides additional reinforcement, distributing the load more evenly and reducing stress concentrations, which enhances the joint's ability to handle greater forces without compromising integrity.

For applications requiring a highly secure and durable connection with minimal play, the gib and cotter joint is the optimal choice. The gib adds an extra layer of stability and reduces wear over time, making it ideal for scenarios where the joint must endure constant or fluctuating loads, such as in agricultural machinery or heavy-duty mechanical linkages.

2.7. KNUCKLE JOINT

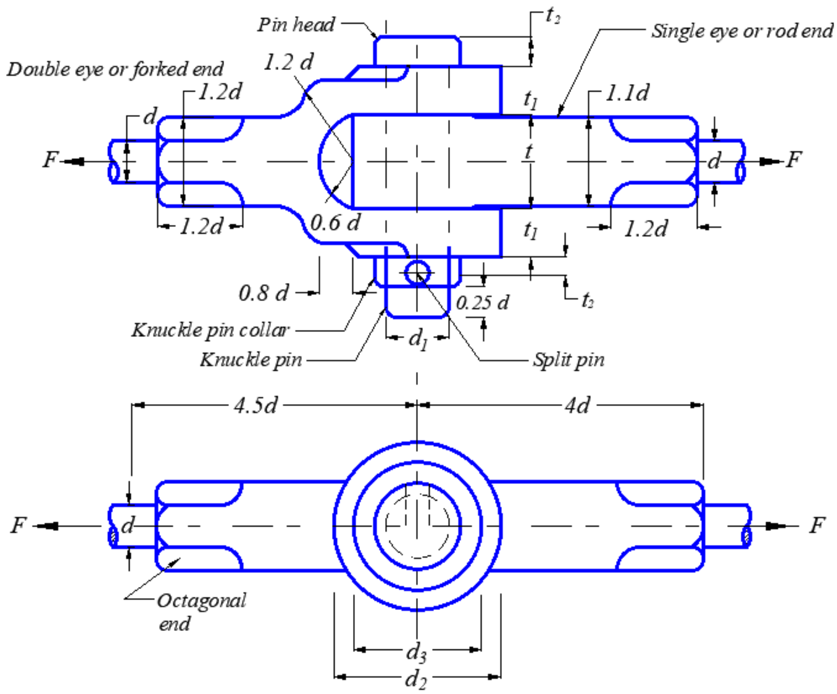


Fig. 2.9: Knuckle joint

A knuckle joint shown in Fig. 2.9 is used to link two rods that transmit tensile loads and can also allow a certain degree of angular movement between them.

This type of joint is commonly found in applications where slight misalignment or angular movement is necessary, such as in the linkage mechanisms of vehicles, cranes, and structural frameworks. The knuckle joint consists of three primary components: the eye, the fork, and the pin as shown in Fig. 2.10.

Knuckle Joint

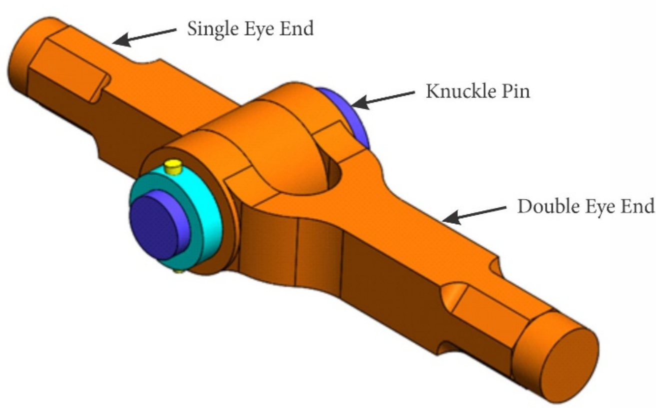


Fig. 2.10: 3D view of knuckle joint

The eye is formed at the end of one rod, featuring a single cylindrical hole that serves as the pivot point. This hole is carefully machined to ensure a precise fit with the pin, which is crucial for maintaining the joint's integrity under load. The fork is created at the end of the other rod, having two arms with holes aligned to accommodate the eye. The arms of the fork are designed to be robust enough to handle the applied forces without bending or breaking. The fork's holes are also machined to match the diameter of the pin, ensuring a secure and tight fit.

The pin passes through the holes of both the eye and the fork, securing the connection while allowing for rotational movement around the pin axis. The pin is typically held in place with a locking mechanism such as a split pin, a nut, or a washer to prevent it from slipping out during operation.

Design of Cotter Joint 2

The design of the pin is critical, as it must be strong enough to withstand shear forces while also being able to rotate within the holes to allow for the necessary angular movement. This rotational capability is essential in applications where the connected rods may not remain perfectly aligned during operation, such as in machinery that experiences dynamic loads or variable positions.

2.7.1. Dimension of knuckle joint parts

In general, the dimensions of different parts with respect to rod diameter (d) as follows,

1. Pin diameter (d_1) is equal to the rod diameter (d)
2. Eye outer diameter (d_2) is 2 times the rod diameter (d)
3. Knuckle pin head and collar diameter (d_3) is 1.5 times the rod diameter (d)
4. Single eye thickness (t) is 1.25 times the rod diameter (d)
5. Fork thickness (t_1) is 0.75 times the rod diameter (d)
6. Pin head thickness (t_2) is 0.5 times the rod diameter (d)

The detailed design on the basis of given load and permissible stresses is given below:

(i) Diameter of rod:

The diameter of rod can be calculated by assuming that rod fails in tension,

$$F = \left(\frac{\pi}{4} \times d^2\right) \times \sigma_t \tag{2.19}$$

where F is applied force (N), d is rod diameter (mm) and σ_t - tensile stress in rod (N/mm^2)

After finding the diameter of rod, calculate other dimensions on the basis of section 2.7.1 and check the design values against induced stresses.

(ii) Diameter of pin:

The diameter of rod can be calculated by assuming that pin fails in shear.

$$F = 2 \times \frac{\pi}{4} \times (d_1)^2 \tau \tag{2.20}$$

where (d_1) is pin diameter (mm) and τ is shear stress (N/mm^2)

2.7.2 Calculation of the induced stresses in the knuckle joint

(i) Tensile stress in single eye or rod end:

The tensile stress induced in the eye or rod end can be calculated by assuming that eye or rod end fails i.e., tear off under the tensile force.

$$F = (d_2 - d_1) t \times \sigma_t \tag{2.21}$$

where t is single eye thickness (mm) and σ_t is induced tensile stress of single eye (N/mm^2)

If the calculated tensile stress is greater than the allowable stress, then the diameter of the eye (d_2) should be increased.

(ii) Shear stress in single eye or rod end:

The shear stress induced in the eye or rod end can be calculated by assuming that eye or rod end fails under shearing with the tensile force.

$$F = (d_2 - d_1) t \times \tau \tag{2.22}$$

where τ is induced shear stress of single eye (N/mm^2)

(iii) Crushing stress in single eye or rod end:

The crushing stress induced in the eye or rod end can be calculated by assuming that eye or rod end fails under crushing with the tensile force.

$$F = d_1 \times t \times \sigma_c \tag{2.23}$$

where σ_c is induced crushing stress of single eye (N/mm^2)

If the calculated crushing stress is greater than the allowable stress, then the thickness of the eye (t) should be increased.

(iv) Tensile stress in forked end:

The tensile stress induced in the double eye or forked end can be calculated by assuming that eye or rod end fails in tension i.e., tear off under the tensile force.

$$F = (d_2 - d_1) \times 2t_1 \times \sigma_t \tag{2.24}$$

where σ_t is induced tensile stress of forked end (N/mm^2)

(v) Shear stress in forked end:

The shear stress induced in the double eye or forked end can be calculated by assuming that eye or rod end fails under shearing with the tensile force.

$$F = (d_2 - d_1) \times 2t_1 \times \tau \tag{2.25}$$

where τ is induced shear stress of forked end (N/mm^2)

If the calculated shear stress is greater than the allowable stress, then the thickness of the fork (t_1) should be increased.

(vi) Crushing stress in forked end:

The crushing stress induced in the double eye or forked end can be calculated by assuming that eye or rod end fails under crushing with the tensile force.

$$F = d_1 \times 2t_1 \times \sigma_c \tag{2.26}$$

where σ_c is induced crushing stress of forked end (N/mm^2)

Example 2.2: A knuckle joint has to transfer a load of $200\,kN$. Take tension, shear, and compression stress as $95\,MPa,\ 80\,MPa\ and\ 175\,MPa$ respectively. Design the knuckle joint.

Given data:

Load = $200\,kN = 200 \times 10^3\ \ N$

Permissible Tensile Stress = $95\,MPa = 95\,N/mm^2$

Permissible Compressing Stress = $175\,MPa = 175\,N/mm^2$

Shear Stress = $80\,MPa = 80\,N/mm^2$

Find:

1. Design Knuckle joint

Solution:

(i) Rod diameter (d)**:**

Load, F= Area × Stress

$$200 \times 10^3 = \left[\frac{\pi}{4} \times d^2\right] \times 95 = 74.5d^2$$

$$d^2 = 200 \times 10^3 / 74.5 = 2684 \quad or \quad d = 51.8 = 52\,mm$$

As discussed in section 2.6,

(ii) Knuckle pin diameter (d_1)**:**

$$d_1 = d = 52\,mm$$

(iii) Eye outer diameter (d_2)

$$d_2 = 2d = 2\times 52 = 104\ mm$$

(iv) Knuckle pin head and collar diameter (d_3)

$$d_3 = 1.5d = 1.5\times 52 = 78\,mm$$

(v) Single eye thickness (t)

$$t = 1.25d = 1.25\times 52 = 65\,mm$$

(vi) Fork thickness (t_1)

$$t_1 = 0.75d = 0.75\times 52 = 39 = 40mm$$

(vii) Pin head thickness (t_2)

$$t_2 = 0.5d = 0.5\times 52 = 26\,mm$$

These design values will be tested to calculate the induced stresses with the given load and if the stresses are within the permissible limits there is no need of modification of any dimension.

Testing of design:

(a) Induced shear stress in the knuckle pin:

The shear stress induced in the knuckle pin can be calculated by assuming that knuckle pin fails under shearing (Double shear)

$$F = 2\times\left[\frac{\pi}{4}\times d^2\right]\times\tau$$

$$200\times 10^3 = 2\times\left[\frac{\pi}{4}\times(d_1)^2\right]\tau = 2\times\left[\frac{\pi}{4}\times(52)^2\right]\tau = 4248\tau$$

$$\tau = 200\times 10^3 / 4248 = 47\,N/mm^2 = 47\,MPa$$

(b) Tensile stress in single eye or rod end:

The tensile stress induced in the eye or rod end can be calculated by assuming that eye or rod end fails i.e., tear off under the tensile force.

$$F = (d_2 - d_1)t\times\sigma_t$$

$$200\times 10^3 = (d_2 - d_1)t\times\sigma_t = (104-52)65\times\sigma_t = 3380\,\sigma_t$$

$$\sigma_t = 200\times 10^3 / 3380 = 59.17\,N/mm^2 = 60\,MPa$$

(c) Shear stress in single eye or rod end:

The shear stress induced in the eye or rod end can be calculated by assuming that eye or rod end fails under shearing with the tensile force.

$$F=(d_2-d_1)t\times\tau$$

$$200\times10^3=(d_2-d_1)t\times\tau=(104-52)65\times\tau=3380\tau$$

$$\tau=200\times10^3/3380=59.17N/mm^2=60\,MPa$$

(d) Crushing stress in single eye or rod end:

The crushing stress induced in the eye or rod end can be calculated by assuming that eye or rod end fails under crushing with the tensile force.

$$F=d_1\times t\times\sigma_c$$

$$200\times10^3=d_1\times t\times\sigma_c=52\times65\times\sigma_c=3380\sigma_c$$

$$\sigma_c=200\times10^3/3380=59.17N/mm^2=60\,MPa$$

(e) Tensile stress in forked end:

The tensile stress induced in the double eye or forked end can be calculated by assuming that eye or rod end fails in tension i.e., tear off under the tensile force.

$$F=(d_2-d_1)\times2t_1\times\sigma_t$$

$$200\times10^3=(d_2-d_1)2t_1\times\sigma_t=(104-52)2\times40\times\sigma_t=4160\sigma_t$$

$$\sigma_t=200\times10^3/4160=48\,N/mm^2=48\,MPa$$

(f) Shear stress in forked end:

The shear stress induced in the double eye or forked end can be calculated by assuming that eye or rod end fails under shearing with the tensile force.

$$F=(d_2-d_1)\times2t_1\times\tau$$

$$200\times10^3=(d_2-d_1)2t_1\times\tau=(104-52)2\times40\times\tau=4160\tau$$

$$\tau=200\times10^3/4160=48\,N/mm^2=48\,MPa$$

(g) Crushing stress in forked end:

The crushing stress induced in double eye or forked end can be calculated by assuming that eye or rod end fails under crushing with the tensile force.

$$F=d_1\times2t_1\times\sigma_c$$

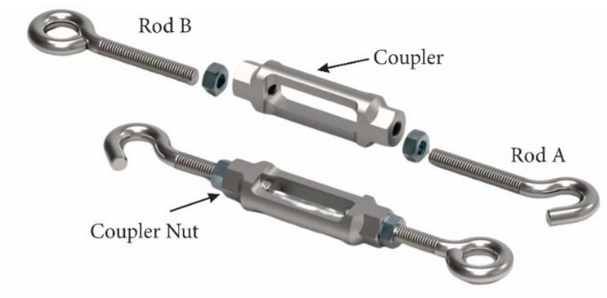
$$200\times10^3 = d_1\times2t_1\times\sigma_c = 52\times2\times40\times\sigma_c = 4160\,\sigma_c$$

$$\sigma_c = 200\times10^3 / 4160 = 48.07\,N/mm^2 = 48\,MPa$$

It can be noticed that the calculated induced stress values are less than permissible stress and hence the design is safe.

2.8. DESIGN OF TURNBUCKLE

A turnbuckle helps to adjust the tension or length of ropes, cables, tie rods, and other tensioning systems. It consists of two threaded end fittings in the form of eye bolts, hooks, or jaws, depending on the specific application as shown in Fig. 2.11. One of the rods has a left-hand thread and the other has a right-hand thread. These end fittings are screwed into either end of an internally threaded central body to match the threads on the end fittings. The central body, often referred to as the turnbuckle body or frame, typically has a hexagonal or rectangular shape to facilitate easy turning with a wrench or by hand.



Turnbuckle Design

Fig. 2.11: Turnbuckle

By rotating the central body, the threaded end fittings either move closer together or farther apart. When the central body is turned in one direction, the end fittings move towards each other, increasing the tension in the attached ropes, cables, or rods. Conversely, turning the body in the opposite direction causes the end fittings to move apart, decreasing the tension. This mechanism allows for precise adjustment of the tension without disconnecting the tensioning elements.

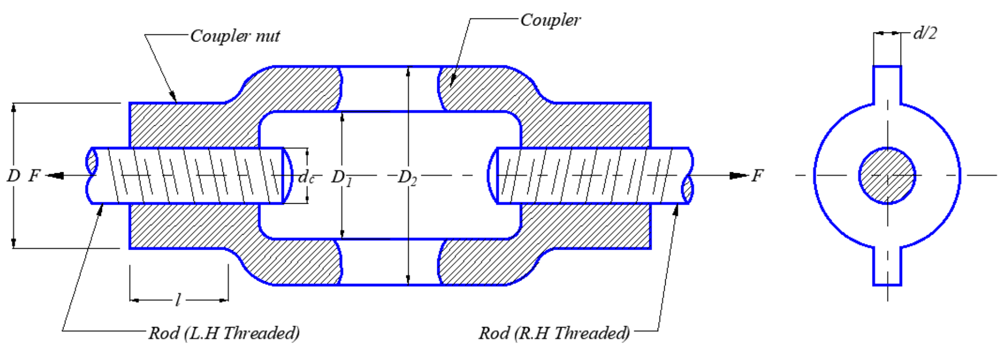


Fig. 2.12: Sectional view of a turnbuckle

Fig. 2.12 shows a turnbuckle under tensile load $'F'$ and it in turn creates tensile stress in the threaded rod,

Tensile Stress, $\sigma_t = \dfrac{F}{A} = \dfrac{F}{\dfrac{\pi}{4}(d_c)^2}$ (2.27)

where F is force in N and d_c is core diameter of the threaded rod in mm.

Torque, $M_t = F\tan(\alpha+\phi)\dfrac{d_p}{2}$ (2.28)

where d_p is pitch diameter in mm, α is helix angle and $\tan\phi$ is coefficient of friction

Shear stress, $\tau = \dfrac{M_t}{J} \times \dfrac{d_p}{2}$

$$\tau = \frac{8F}{\pi(d_p)^2}\left(\frac{\tan\alpha+\tan\phi}{1-\tan\alpha\times\tan\phi}\right) \quad (2.29)$$

The values of $\tan\alpha = 0.03$, $\tan\phi = 0.02$ and $d = 1.08d$

$$\tau = \frac{8F}{\pi(1.08d_c)^2}\left(\frac{0.03+0.2}{1-0.03\times0.2}\right) = \frac{\sigma_t}{2} \quad (2.30)$$

Maximum tensile stress, $(\sigma_t)_{\max} = \dfrac{\sigma_t}{2} + \dfrac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2} = 1.207\sigma_t$ (2.31)

Design load, $F_d = 1.3$ times the normal load F (2.32)

2.8.1 Design of turnbuckle

(i) Diameter of rod:

The diameter of threaded rod can be calculated by assuming that threads fail due to tearing at their roots

$$F_d = \frac{\pi}{4}(d_c)^2\sigma_t \quad (2.33)$$

where F_d is design load (N), d_c is core diameter of threads (mm) and σ_t is tensile stress (N/mm^2)

(ii) Length of coupler nut:

The length of coupler nut can be calculated by assuming that threads fail due to shearing at their roots

$$F_d = \frac{\pi}{4}d_c \times l \times \tau \quad (2.34)$$

Note: For steel, $l = 1-1.25d$ and for cast iron and softer material, $l = 1.5-2d$

The length of coupler must be checked against crushing of threads,

$$F_d = \frac{\pi}{4}\left[(d)^2 - (d_c)^2\right] \times n \times l \times \sigma_c \qquad (2.35)$$

where d is diameter of tie rod in mm, n is number of threads per mm length, l is coupler nut length in mm, τ is shear stress of the coupler nut in N/mm^2 and σ_c is induced crushing stress in coupler nut in N/mm^2

Turn Buckle

(iii) Outside diameter of coupler nut:

The outer diameter of coupler nut can be calculated by assuming that coupler nut fails due to tearing

$$\text{F} = \frac{\pi}{4}\left(D^2 - d^2\right)\sigma_t \qquad (2.36)$$

where D is external diameter of coupler nut in mm and σ_t is tensile stress of the coupler nut in N/mm^2

Note: $D = 1.25d \text{ to } 1.5d$

(iv) Outside diameter of coupler:

The outer diameter of coupler can be calculated by assuming that coupler fails due to tearing

$$\text{F} = \frac{\pi}{4}\left(D_2^2 - D_1^2\right)\sigma_t \qquad (2.37)$$

where D_2 is coupler external diameter in mm, D_1 is coupler internal diameter in mm and σ_t is permissible tensile stress in N/mm^2

Note: $D_1 = d + 6\,mm$ & D_2 is between $1.5d$ and $1.7d$

(v) Length of coupler between the nuts (L):

Distance between nuts in the coupler, $L = 6d$ (2.38)

(vi) Coupler thickness:

Coupler thickness, $t = 0.75d$ (2.39)

Coupler nut thickness, $t_1 = 0.5d$ (2.40)

Example 2.3: Design a turnbuckle to attach in the tie rod subjected to pull force of $65\,kN$. Take tension, shear, and compression stress as $90\,MPa,\ 50\,MPa \text{ and } 110\,MPa$ respectively.

Given data:

Load $F = 65\,kN$

Permissible tensile stress = $90\,MPa$

Permissible crushing stress = $110\,MPa$

Shear stress = $50\,MPa$

Find:

1. Design turnbuckle

Solution:

Design load $F_d = 1.3 \times F = 1.3 \times 65 \times 10^3 = 84.5 \times 10^3\,N$

(i) Failure of the rod in tension:

The diameter of threaded rod can be calculated by assuming that threads fail due to tearing at their roots

Design load, $F_d = \frac{\pi}{4}(d_c)^2\,\sigma_t$

$$84.5 \times 10^3 = \frac{\pi}{4} d_c^2 \times \sigma_t$$

$$84.5 \times 10^3 = \frac{\pi}{4} d_c^2 \times 90$$

$$84.5 \times 10^3 = 70.65 \times d_c^2$$

$$d_c^2 = 84.5 \times 10^3 / 70.65 = 1196$$

$$d_c = 34.6\,mm$$

From Design Data Book or refer Appendix 1, the standard core diameter is $36.416\,mm$ and the respective diameter of tie rod is $d = 42\,mm$

(ii) Length of coupler nut (l)*:*

$l = d = 42\,mm$

(iii) Outside diameter of the coupler nut (D)*:*

$D = 1.25d = 52.5\,mm$

(iv) Outside diameter of the coupler (D_2)*:*

$D_2 = 1.5d = 63\,mm$

(v) Length of the coupler between nuts (L)*:*

$L = 6d = 252\,mm$

(vi) Coupler thickness (t_1)*:*

$t_1 = 0.75d = 31.5\,mm$

(vii) Coupler nut thickness (t) :

$t = 0.5d = 21mm$

Testing of design

(i) Length of coupler nut

The length of coupler nut can be calculated by assuming that threads fail due to shearing

Design load, $F_d = (\pi d_c\, l)\tau$

$$84.5\times10^3 = \pi\times36.416\times l\times50 = 5717.3l$$

$$l = 84.5\times10^3 / 5717.3 = 14.77mm$$

From Design Data Book or refer Appendix 1, the pitch of the threads in $39mm$, diameter rod is $4.5mm$, hence $n = 1/4.5 = 0.222$

Checking the length of coupler against crushing of threads,

$$F_d = \frac{\pi}{4}\left[d^2 - d_c^2\right]n\times l\times\sigma_c$$

$$84.5\times10^3 = \frac{\pi}{4}\left[42^2 - 36.416^2\right]0.222\times42\times\sigma_c$$

$$\sigma_c = 26.36\,MPa$$

Since the induced crushing stress is less than the permissible stress, design is safe.

(ii) Outside diameter of the coupler nut:

Coupler nut outer diameter can be calculated by assuming that coupler nut fails due to tearing

Axial load, $\mathrm{F} = \frac{\pi}{4}\left(D^2 - d^2\right)\sigma_t$

$$65\times10^3 = \frac{\pi}{4}\left(D^2 - d^2\right)\sigma_t$$

$$65\times10^3 = \frac{\pi}{4}\left(D^2 - (42)^2\right)90$$

$$65\times10^3 = 70.65\left[D^2 - (42)^2\right]$$

$$D^2 - (42)^2 = 65\times10^3 / 70.65 = 920$$

$$D^2 = 920 + (42)^2 = 2684$$

$D = 51.8mm = 52mm$

Since the value of D is greater than the design value, design is safe.

(c) Outside diameter of the coupler:

The outer diameter of coupler can be calculated by assuming that coupler fails due to tearing

$$F = \frac{\pi}{4}\left(D_2^2 - D_1^2\right)\sigma_t$$

$$65\times10^3 = \frac{\pi}{4}\left[\left(D_2^{\,2} - D_1^{\,2}\right)\right]\sigma_t$$

$$65\times10^3 = \frac{\pi}{4}\left[\left(D_2^{\,2} - 48^2\right)\right]90$$

$$65\times10^3 = \,= 70.65\left[\left(D_2^{\,2} - 48^2\right)\right]$$

$$\left(D_2\right)^2 = \left(65\times10^3 / 70.65\right) + \left(48\right)^2 = 3224$$

$$D_2 = 56.78mm$$

Since the value of D_2 is greater than the design value, design is safe.

2.9. LEVERS

Levers are rigid rods or bars capable of rotating about a fixed point known as the fulcrum. The fundamental principle behind a lever is based on torque and mechanical advantage. When one end of the lever is subjected to force, it generates a turning effect, or torque, around the fulcrum. By adjusting the distances from the fulcrum to where the input force is applied and where the output force is exerted, levers can multiply the input force, allowing a smaller force to lift a larger load.

2.10. HAND LEVERS

Hand levers mechanisms can be found in the hand-operated presses, brake levers on bicycles, and the controls of machinery. These levers are designed to maximize the efficiency of the force applied by the operator, often featuring ergonomic handles to ensure a comfortable grip and reduce fatigue during extended use. Fig. 2.13 shows a typical hand lever.

Design of Levers

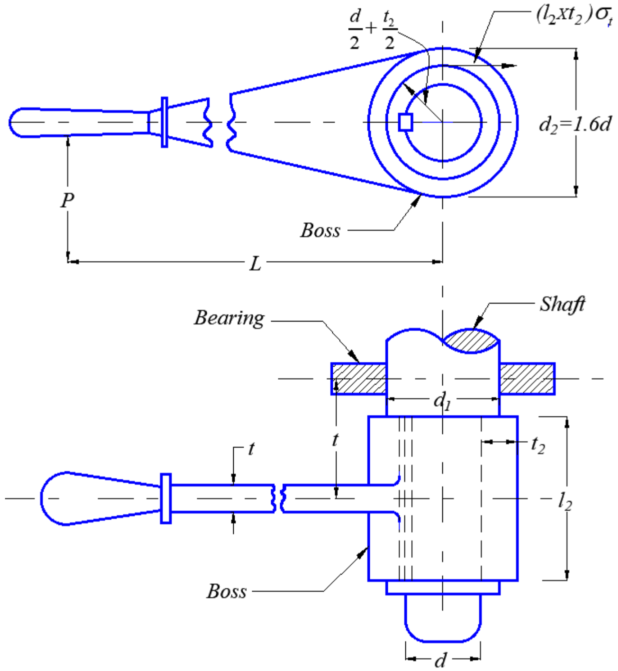


Fig. 2.13: Hand lever

2.10.1. Design of hand levers

(i) Diameter of the shaft:

The shaft diameter can be calculated by assuming that shaft is under pure torsion,

Twisting moment (M_t) = Force (F) × Distance (L)

Resisting torque, $M_t = \frac{\pi}{16} \times \tau \times d^3$ (2.41)

$F \times L = \frac{\pi}{16} \times \tau \times d^3$ $\quad d = \sqrt[3]{\frac{16 \times F \times L}{\pi \times \tau}}$

where d is shaft diameter in mm, F is force applied in N, L is length of the lever in mm and τ is permissible shear stress in N/mm^2

(ii) Diameter and thickness of the boss:

$d_2 = 1.6d$ & $t_2 = 0.3d$ (2.42)

where d_2 is boss diameter in mm and t is boss thickness in mm

(iii) Length of the boss:

$l_2 = 1.25d$ (2.43)

where l_2 is length of the boss in mm

(iv) Shaft diameter at bearing center:

Consider the shaft is under combined bending and twisting moment

Bending moment is given by, $M_b = P \times l$

Twisting moment is given by, $M_t = P \times L$

∴ Net twisting moment can be calculated from the following relation

$$M_t = \sqrt{\left((M_b)^2 + (M_t)^2\right)} = \sqrt{(P \times l)^2 + (P \times L)^2} \tag{2.44}$$

$$= P\sqrt{\left(l^2 + L^2\right)}$$

The net twisting moment can also be calculated by

$$M_t = \frac{\pi}{16} \times \tau (d_1)^3$$

Equating both $P\sqrt{\left(l^2 + L^2\right)} = \frac{\pi}{16} \times \tau (d_1)^3$ (2.45)

From Eq. (2.45), the shaft diameter at centre (d_1) of bearing can be calculated.

Note: Length, l may be taken as $2l_2$

(v) Lever cross-section (width and thickness of lever):

Bending moment on the lever, $M_b = P \times L$ (2.46)

Section modulus, $Z = \frac{1}{6} \times t \times B^2$ (2.47)

We know that, Bending stress, $\sigma_b = \frac{M_b}{Z} = \frac{P \times L}{\frac{1}{6} \times t \times B^2} = \frac{6P \times L}{t \times B^2}$ (2.48)

Thickness of the lever can be calculated from Eq. (2.48) meanwhile the lever width B may be taken as $4\,to\,5$ times of lever thickness (t). $B = 4\,to\,5t$ (2.49)

Note: The lever's width decreases gradually (tapered section), however its thickness (t) remains same. The lever width near the handle end may be taken as $B/2$.

2.11. FOOT LEVER

Foot levers, commonly used in various machinery, convert human effort into mechanical work. They are pivotal in devices like sewing machines, vehicles, and industrial equipment. The foot lever is as same as the hand lever except that the force is applied on the foot plate instead of handle. The schematic view of the foot lever is shown in Fig 2.14.

Example 2.4: A $500\,N$ load is applied on the foot lever which is located at a distance of $800\,mm$ form the shaft mid-point. Calculate the diameter of shaft, key dimensions and the cross-section dimensions of the arm at a distance of $40\,mm$ from shaft centre. Assume its width as three times the arm thickness. Take tensile and shear stress as $85\,MPa$ and $65\,MPa$ respectively.

Given data:

Load $F = 500\,N$

Tensile Stress $\sigma_t = 85\,MPa = 85N/mm^2$

Shear Stress $\tau = 68\,MPa = 68N/mm^2$

Width = $3\times thickness$

Offset Distance = $40\ mm$

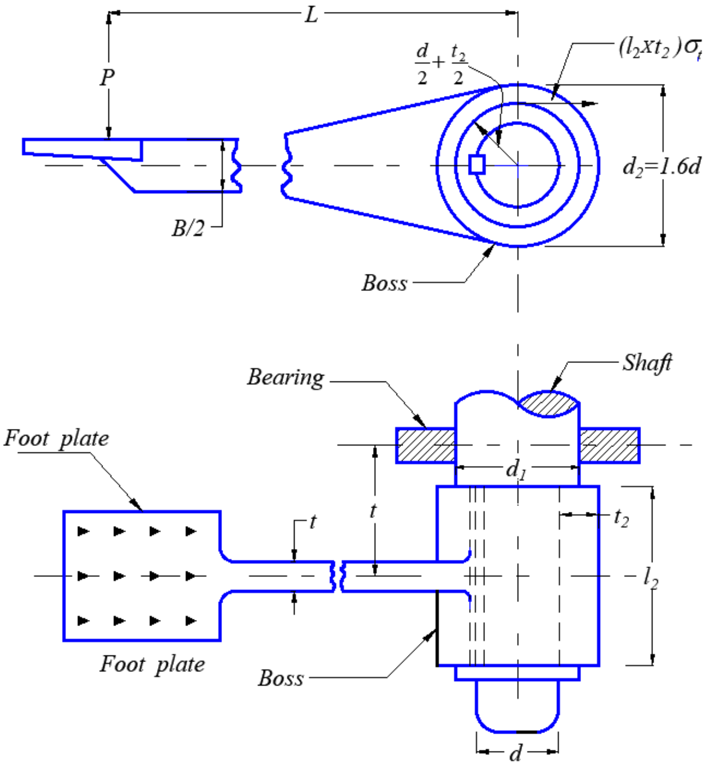


Fig. 2.14: Foot lever

Find:

1. Diameter of shaft
2. Key dimensions
3. Cross-section dimensions of the arm

Solution:

(i) Shaft diameter (d) **:**

Twisting moment, $M_t = P \times L = 500 \times 800 = 400 \times 10^3 \; N - mm$

Twisting moment on the shaft (M_t),

$$400 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 68 \times d^3 = 13.35 d^3$$

$$d^3 = 400 \times 10^3 / 13.35 = 29.9 \times 10^3$$

$$d = 31.05 = 32 \, mm$$

(ii) Boss diameter (d_2) **:**

$$d_2 = 1.6d = 1.6 \times 32 = 52 mm$$

(iii) Boss thickness (t_2) **:**

$$t_2 = 0.3 \times d = 0.3 \times 32 = 10 mm$$

(iv) Boss length (l_2) **:**

$$l_2 = 1.25d = 1.25 \times 32 = 40 \, mm$$

(v) Shaft diameter at bearing centre (d_1) **:**

$$\frac{\pi}{16} \times \tau (d_1)^3 = P \times \sqrt{(l^2 + L^2)}$$

$$\frac{\pi}{16} \times 68 (d_1)^3 = 500 \sqrt{(80)^2 + (800)^2}$$ (Take l = 2 x l_2)

$$13.34 (d_1)^3 = 402 \times 10^3$$

$$(d_1)^3 = 402 \times 10^3 / 13.34 = 30.12 \times 10^3 \quad or \; d_1 = 31.1 = 32 \, mm$$

(vi) Key dimensions:

From Design Data Book, the key dimensions for a shaft of $38 \, mm$ diameter are as follows,

Key width, $w = 12 \, mm$; Key thickness, $t = 8 \, mm$

Twisting moment (M_t) = $400 \times 10^3 = l_1 \times w \times \tau \times \dfrac{d}{2}$

$M_t = l_1 \times 12 \times 68 \times \frac{32}{2} = 13056 \times l_1$

$l_1 = 400 \times 10^3 / 13056 = 30.6\,mm$

$l_1 = l_2 = 31\ mm$

(vii) Dimensions of the arm at a distance of $50\,mm$ *from shaft centre:*

Given that the width of arm is 3 times its thickness i.e., $B = 3t$

Bending moment at the offset distance is

$M_b = 500(800 - 40) = 380 \times 10^3\ Nmm$

Section modulus (Rectangle), $Z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t\left(3t\right)^2 = 1.5t^3\ mm^3$

Tensile bending stress (σ_b) is

$85 = \frac{M_b}{Z} = \frac{380 \times 10^3}{1.5t^3} = \frac{253.3 \times 10^3}{t^3}$

$t^3 = 253.3 \times 10^3 / 73 = 3.47 \times 10^3$

$t = 15.12 = 16\,mm$

$B = 3t = 3 \times 16 = 48\,mm$

Hence, arm width of the foot plate is, $B_1 = B/2 = 48/2 = 24\,mm$

2.12. BELL CRANK LEVER

Bell crank levers have two arms that are at right angles. Typical applications for such levers include signaling systems, Hartnell governors, air pumps for condensers, etc. The design of bell crank lever is similar to that of the others levers as discussed so far. It allows force applied in one direction to be transmitted at an angle, usually 90 degrees. The bell crank lever may have arms that are rectangular, elliptical, or I-shaped.

Bell Crank Lever

The key components of bell crank lever are lever arms (It consists of two arms (input arm and output arm) set at an angle (commonly 90 degrees) to each other), fulcrum (pivot point - point where the two arms meet and pivot) and load and effort points (points where the load and effort forces are applied) as shown in Fig. 2.15.

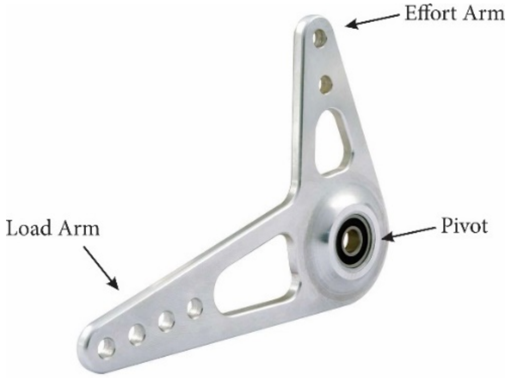


Fig 2.15: Bell crank lever

Example 2.5: A right-angled bell crank lever as shown in Fig. 2.16 with a horizontal arm of $500\ mm$ long and a load of $4.5\,kN$ acts vertically downward through a pin in the forked end of this arm. At the end of the $150\ mm$ long arm which is perpendicular to the $500\ mm$ long arm, a force F act at right angles to the axis of the $150\ mm$ arm through a pin into a forked end. The lever consists of forged steel material and a pin at the fulcrum. Let us take the allowable stress in tension as $75\ MPa$, allowable stress in shear as $60\ MPa$, and allowable bearing pressure is $10\ N/mm^2$ for both the pins and lever.

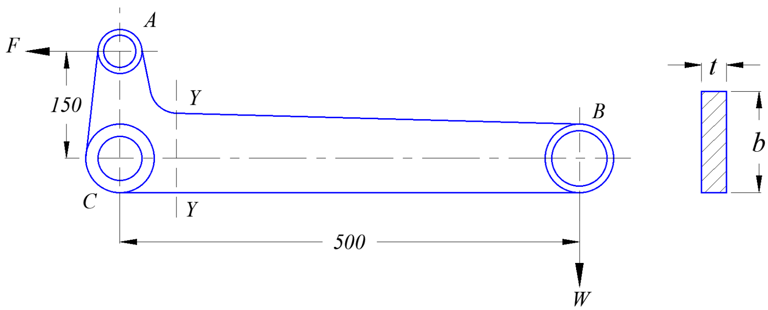


Fig. 2.16: Right-angled bell crank lever

Given data:

Distance $CB = 500\,mm$

Distance $CA = 150\,mm$

Load $W = 4.5kN = 4500N$

Allowable stress in tension $\sigma_t = 75MPa = 75N/mm^2$

Allowable stress in shear $\tau = 60\,MPa = 60N/mm^2$

Allowable bearing pressure $p_b = 10\,N/mm^2$

Find:

1. Design bell crank lever

Solution:

(a) Efforts required to raise the load by bell crank lever:

Moments about the fulcrum C

$$W \times 500 = F \times 150$$

$$F = \frac{W \times 500}{150} = \frac{4500 \times 500}{150} = 15000N$$

Reaction at fulcrum F

$$R_f = \sqrt{W^2 + F^2} = \sqrt{4500^2 + 15000^2} = 15660\ N$$

(b) Design of fulcrum pin

Let us consider the length (l) of the fulcrum pin as $1.25d$, where d is the diameter of fulcrum pin.

Equating the reaction force to the bearing pressure force at fulcrum,

$$15660 = p_b \times l \times d$$

$$15660 = 10 \times 1.25d \times d$$

$$d^2 = 1253$$

$$d = 35.4\ mm = 35\ mm$$

Hence, the length of the fulcrum pin, $l = 1.25d = 1.25 \times 35 = 45mm$

The cross section of fulcrum with $3mm$ bush thickness is as shown Fig. 2.17,

Diameter of hole in the lever = $d + (2 \times 3) = 36 + 6 = 42\,mm$

Diameter of boss at fulcrum = $2d = 2 \times 36 = 72\,mm$

(i) Checking the design against shear stress and bending stress:

Since the pin is in double shear, the shear stress induced in the fulcrum pin is

Reaction force $R_f = 2 \times \frac{\pi}{4} d^2 \times \tau$

$$15660 = 2 \times \frac{\pi}{4} (36)^2 \times \tau$$

$$\tau = 7.7\,N/mm^2$$

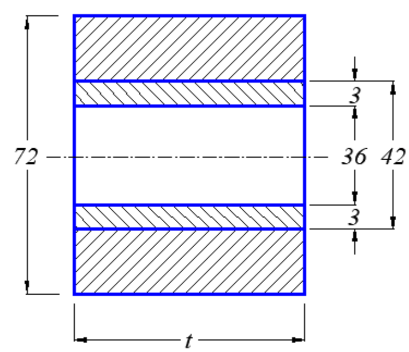


Fig. 2.17: Cross section of fulcrum with bush

Since the calculated value is less than the given limit

$(60N/mm^2)$, the design is safe.

Bending Moment at the fulcrum,

$$M_b = W \times distance(CB)$$

$$M_b = 4500 \times 500 = 2250 \times 10^3\, Nmm$$

Section modulus of the cross section (Rectangle)

$$Z = \frac{I}{y} = \frac{\frac{1}{12} \times 45\left[(72)^3 - (42)^3\right]}{72/2} = 311625\ mm^3$$

Bending stress

$$\sigma_b = \frac{M_b}{Z} = \frac{2250 \times 10^3}{311625} = 7.25\ N/mm^2$$

Since the calculated value is less than the given limit $(75N/mm^2)$, the design is safe.

(c) Design for pins:

Pin A

Since the effort at A $(which\ is\ 15000N)$, is not very much different from the reaction at fulcrum $(which\ is\ 15660N)$, therefore the same dimensions for the pin and boss may be used for the fulcrum pin to reduce spares.

Pin B

Let us consider the length (l) of the fulcrum pin as $1.25d$ where d is the diameter of fulcrum pin.

Equating the reaction force to the bearing pressure force at fulcrum,

$$4500 = p_b \times l_1 \times d_1$$

$$4500 = 10 \times 1.25d_1 \times d_1$$

$$d_1^2 = 360$$

$$d_1 = 18.9 = 20mm$$

Hence, the length of the fulcrum pin,

$$l = 1.25d = 1.25 \times 20 = 25mm$$

Checking the design against shear stress and bending stress

Since the pin is in double shear, the shear stress induced in the fulcrum pin is

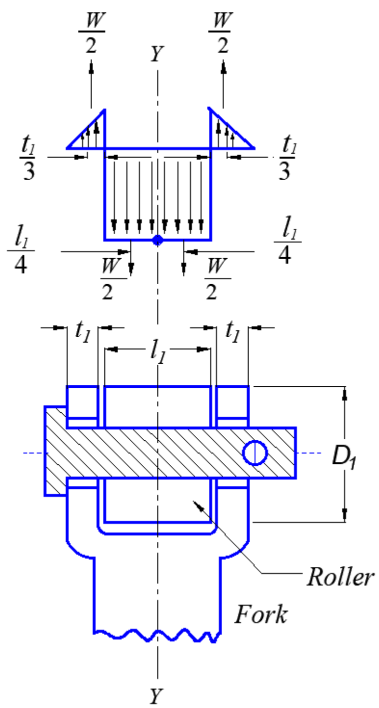


Fig. 2.18: Free body diagram of force acting on fork

Reaction force $R_f = 2\times\frac{\pi}{4}d^2\times\tau$

$$4500 = 2\times\frac{\pi}{4}(20)^2\times\tau$$

$$\tau = 7.12\ N/mm^2$$

Since the calculated value is less than the given limit $(60N/mm^2)$, the design is safe.

Since the end B is a forked end, therefore the thickness of each eye, $t_1 = 1/2 = 25/2 = 12.5mm$

The eye dimensions with $3mm$ bush thickness are,

Inner diameter of each eye = $d+(2\times3) = 20+6 = 26mm$

Outer diameter of eye, $D = 2d_1 = 2\times20 = 40mm$

Bending moment at the fulcrum (Fig. 2.18),

$$M_b = \frac{W}{2}\times\left(\frac{l_1}{2}+\frac{t_1}{3}\right) - \frac{W}{2}\times\frac{l_1}{4}$$

$$M_b = \frac{5}{24}W\times l_1$$

$$M_b = \frac{5}{24}\times4500\times25 = 23438Nmm$$

Section modulus of the cross section (Rectangle)

$$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3$$

$$Z = 786mm^3$$

Bending stress $\sigma_b = \frac{M}{Z} = \frac{23438}{786} = 29.8N/mm^2$

Since the calculated value is less than the given limit $(75N/mm^2)$, the design is safe.

(d) Design for lever:

Let t is thickness of the lever at $Y-Y$ and b is the width or depth of the lever at $Y-Y$

Taking the distance from the center of the fulcrum to $Y-Y$ as $50mm$

Therefore, maximum bending moment at $Y-Y$

$$M_b = 4500(500-50) = 2025\times10^3\ N-mm$$

Section modulus of the cross section (Rectangle) $Z = \frac{1}{6} \times t \times b^2 = \frac{1}{6} \times 3 \times (3t)^2 = 1.5t^3$

Bending stress $\sigma_b = \frac{M}{Z}$ $\Rightarrow 75 = \frac{2025 \times 10^3}{1.5t^3}$

$t^3 = \frac{1350 \times 10^3}{75} = 18 \times 10^3$

$t = 26mm$

Hence, the thickness of the lever at $Y-Y$ is $26m$

The width or depth of the lever at $Y-Y$ is $b = 3t = 3 \times 26 = 78mm$

2.13. C-CLAMP

A C-clamp is a clamping tool widely used in workshops and various industries. It consists of a C-shaped frame, a screw mechanism, and a swivel pad as shown in Fig. 2.19. The screw mechanism applies a clamping force between the swivel pad and the frame, holding objects in place.

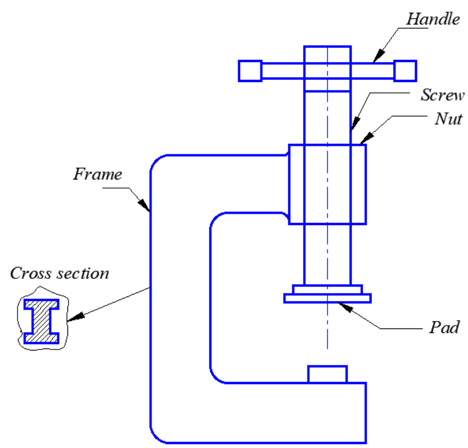


Fig. 2.19: C-clamp

Example 2.6: Design a C-clamp which can exert $4kN$ clamping force considering the distance between the jaws and the screw axis and frame edge as $270\,mm$ and $150\,mm$ respectively. Design screw nut and frame as well.

Given data:

Clamping force, $F = 4\,kN$

Distance between the jaw = $270\,mm$

Screw axis and frame edge distance = $150\,mm$

Find:

1. Design screw
2. Design nut
3. Design frame
4. Design handle

Solution:

1. Design of screw:

(a) Material selection:

Select Low carbon steel $C-30(C-15 \, to \, C-45)$, From Design Data Book,

Yield stress, $\sigma_y = 300 N/mm^2$

Ultimate stress, $\sigma_u = 550 N/mm^2$

(b) Factor of safety:

Let we take factor of safety (FOS) as 4, since the screw is subjected to both compression and torsion.

(c) Permissible stresses:

$$\sigma_c = \frac{\sigma_y}{FOS} = \frac{300}{4} = 75 N/mm^2$$

By maximum shear stress theory, $\tau = \dfrac{\sigma_y \times 0.5}{FOS} = \dfrac{0.5 \times 300}{4} = 37.5 N/mm^2$

$\tau = 38 N/mm^2$

(d) Design:

Considering screw fails under compressive stress, Compressive stress, $\sigma_c = \dfrac{F}{A_c}$

$$A_c = \frac{F}{\sigma_c} = \frac{4 \times 10^3}{75} = 53.33 \ N/mm^2$$

$$\frac{\pi}{4} \times d_c^2 = 53.33$$

$d_c = 8.24 mm$

The diameter of the screw rod must be minimum 30% higher than the calculated value i.e., to account for the torsion

$d_c = 1.3 \times 8.24 = 10.712 mm$

$$A_c = \frac{\pi}{4} \times d_c^2 = \frac{\pi}{4} \times 10.71^2 = 90.13 mm^2$$

From Design Data Book, minimum core area is $227 mm$, hence

Nominal diameter $d_o = 22 mm$

Core diameter $d_c = 17 \ mm$

Pitch of the thread $p = 5 \ mm$

Mean diameter $d_m = \dfrac{d_o + d_c}{2} = 19.5mm$

Helix angle of the thread can be calculated by

$$\tan\alpha = \frac{Pitch}{\pi \times d_m} = \frac{5}{\pi \times 19.5}$$

$$\alpha = 46.6^\circ$$

From Design Data Book, friction angle, $\phi = 6^\circ$

As $\phi > \alpha$ Screw's friction torque, $M_{t1} = p \times \dfrac{d_m}{2}$

$$M_{t1} = P \times \frac{d_m}{2} tan(\phi + \alpha)$$

$$M_{t1} = 4 \times 10^3 \times \frac{19.5}{2} tan(6 + 1.66) = 7.34 \times 10^3 N\text{ - }mm$$

By torsion equation, $\tau = \dfrac{16 M_{t1}}{\pi \times d_c^3} = \dfrac{16 \times 7.34 \times 10^3}{\pi \times 17^3} = 7.6 N/mm^2$

Maximum principal stress in the screw

$$\sigma_{c\max} = \frac{1}{2}\left[\sigma_c + \sqrt{\sigma_c^2 + 4\tau^2}\right]$$

where σ_c is the compressive stress

$$\sigma_c = \frac{F}{A_c} = \frac{4 \times 10^3}{227} = 17.62 N/mm^2$$

$$\sigma_{c\max} = \frac{1}{2}\left[17.62 + \sqrt{17.62^2 + 4(7.60)^2}\right]$$

$$\sigma_{c\max} = 20.44 N/mm^2 < \sigma_c = 75 N/mm^2$$

The selected screw is safe (compression). Otherwise, pick the next standard screw

$$\tau_{\max} = \frac{1}{2}\left[\sqrt{\sigma_c^2 + 4\tau^2}\right]$$

$$\tau_{\max} = \frac{1}{2}\left[\sqrt{17.62^2 + 4(7.60)^2}\right]$$

$$\tau_{\max} = 11.63 N/mm^2 < \tau = 37 N/mm^2$$

The selected screw is safe (Shear)

(i) Buckling of screw

From Design Data Book, the ratio $\frac{H}{d_m}=2$

$$H=2\times d_m=2\times 19.5=39mm$$

Screw length can be taken as

L = Distance between jaws (maximum) + $H/2$

$$L=270+39/2=289.5mm$$

To calculate the buckling load, let us assume the screw as column with the end condition as one end fixed and another end free,

From Design Data Book, use Johnson's parabolic formula

$$F_c=\alpha\times\sigma_y\left(1-\frac{\sigma_y}{4n_1\pi^2Ek^2}\right)$$

From Design Data Book,

n_1 = End condition = 0.25

k = Least radius of gyration = $4.25mm$

E = Young's modulus = $2.1\times 10^5 N/mm^2$

$$F_c=227\times 300\times\left(1-\frac{300}{4\times 0.25\times\pi^2\times 2.1\times 10^5}\left(\frac{289.5}{4.25}\right)^2\right)$$

$$F_c=22.36\times 10^3 N > F=4\times 10^3 N$$

The estimated buckling load is less than the given load, hence the design is safe.

(2) Design of nut

(a) Material selection:

Let us assume nut is made of cast iron,

From Design Data Book, GCI 20 is chosen

Ultimate stress, $\sigma_u=200N/mm^2$

(b) Factor of safety:

Let we take factor of safety (FOS) as 6, since the nut material is brittle.

(c) Permissible stresses in nut:

$$\sigma_t=\frac{\sigma_u}{FOS}=\frac{200}{6}=33.33N/mm^2$$

Consider $\tau = \sigma_t = 33 N/mm^2$

In general, the compressive strength of cast iron is larger than the tensile strength, let us take it as twice of the tensile strength

Hence, $\sigma_c = 2 \times \sigma_t = 2 \times 33 = 66 N/mm^2$

(d) Selection of bearing stress:

From Design Data Book,

Permissible bearing stress $\sigma_b = 80 Kgf/cm^2 = 8 N/mm^2$

Let the no. of threads in contact be assumed as n

$$H = pitch \times n$$

$$n = \frac{H}{pitch} = \frac{39}{5} = 7.8 = 8\ mm$$

The bearing stress in the threads of nut is given by,

$$F = \frac{\pi}{4}\left(d_0^2 - d_c^2\right) \times n \times \sigma_{br}$$

$$4 \times 10^3 = \frac{\pi}{4}\left(22^2 - 17^2\right) \times 8 \times \sigma_{br}$$

$$\sigma_{br} = 3.26 N/mm^2 < \sigma_{br} = 8 N/mm^2$$

Hence, the design is safe.

The shear stress developed in the threads of nuts is given by

$$F = \pi \times d_o \times t_1 \times \tau$$

$$4 \times 10^3 = \pi \times 22 \times 2.5 \times 8 \times \tau_{nut}$$

$$\tau_{nut} = 2.88 N/mm^2 < \tau_{nut} = 33 N/mm^2$$

Hence, the design is safe.

(3) Design of frames

Let we choose I - section as shown in Fig. 2.20 for this condition. It has the minimum material in the middle and maximum at the outer because maximum stress is induced in the outer layer of the section.

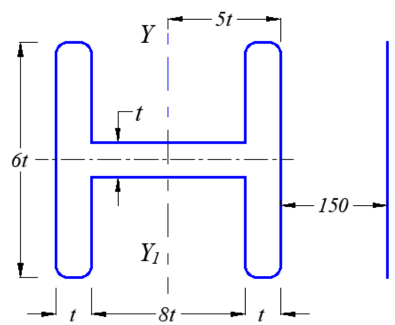


Fig. 2.20: I-Section

(a) Selection of material:

Let us assume nut is made of cast iron, from Design Data Book, GCI 20 is chosen

Ultimate stress, $\sigma_u = 200 N/mm^2$

(b) Factor of safety:

Let us consider an I - section as shown in Fig. 2.20,

Let we take FOS as 6 , since the nut material is brittle.

$$\therefore \sigma_t = \sigma_u / FOS = 50 N / mm^2$$

Area of the I - section is

$$A = (2 \times [\sigma_t \times t]) + (8t \times t) = 20t^2$$

Take moment of inertia about $Y - Y_1$ axis is

$$I_{yy} = \left[\frac{6t \times (10t)^3}{12} \right] \text{-} 2 \left[\frac{(2.5t) \times (8t)^3}{12} \right]$$

$$I_{yy} = 500t^4 \text{ - } 213.33t^4 = 286.67t^4$$

$$Z = \frac{I_{yy}}{y_{\max}} = 57.33t^3$$

$$\sigma_b = \frac{M_b}{Z} = \frac{P \times (150 + 5t)}{57.33t^3} = \frac{4 \times 10^3 \times (150 + 5t)}{57.33t^3} = \frac{69.766(150 + 5t)}{t^3}$$

We know that $\tau_{\max} = \sigma_0 + \sigma_b$

where σ_o is the direct stress and σ_b is the bending stress

$$50 = \frac{F}{A} + \frac{69.766(150 + 5t)}{t^3} = \frac{4 \times 10^3}{20t^2} + \frac{69.766(150 + 5t)}{t^3}$$

$$50t^3 = 200t + 10464.9 + 348.83t$$

$$t = 6.5mm = 7mm$$

Note: Minimum thickness is $5mm$

(4) Design of handle:

(a) Collar friction torque:

Let us assume the condition as uniform pressure,

$$M_{t2} = \frac{2}{3} \mu F \left[\frac{R_1^3 \text{ - } R_2^3}{R_1^2 \text{ - } R_2^2} \right]$$

where R_1 is collar outer radius and R_2 is collar inner radius

Usually, the above-mentioned equation is written as $M_{t2} = \mu P R_M$

M_{t2} is the torque required to overcome collar friction where $R_M = \frac{2}{3}\mu F\left[\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right]$

$R_M = 25mm \quad \mu = 0.25$

$M_{t2} = 0.25\times4\times10^3\times25 = 25\times10^3 N\text{-}mm$

Total Torque, $M_t = M_{t1} + M_{t2} = 7.34\times10^3 + 25\times10^3 = 32.34\times10^3 N\text{-}mm$

(b) Handle length:

Let us take l_b as the handle's effective length, d_h as the diameter of handle

Assuming $300N$ effort applied at end of handle, $M_t = F\times l_h$

$32.34\times10^3 = 300\times l_h$

$l_h = 107.78mm = 110mm$

(c) Handle diameter:

The maximum bending moment can be calculated as

Moment (M_b) = Force $(F)\times$ length (L)

i.e., $M_b = 300\times110 = 33\times10^3 N\text{-}mm$

Let us assume handle is made of C40 material and FOS is taken as 2

$$\sigma_t = \sigma_b = \frac{\sigma_y}{FOS} = \frac{330}{2} = 165N/mm^2$$

$$\sigma_b = \frac{M_b}{Z} = \frac{33\times10^3}{\frac{\pi}{32}d_h^3}$$

$$165 = \frac{33\times10^3}{\frac{\pi}{32}d_h^3}$$

$d_h = 12.67mm = 14mm$

2.14. DESIGN OF OFFSET LINKS

An offset link shown in Fig. 2.21 is a machine element that transmits force between two points with a center distance offset. It's essentially a short beam or lever where the line of action of the applied force doesn't pass through the center of the link. Offset links are commonly found in linkages, mechanisms, and various machines.

Fig. 2.21: Offset link

Example 2.7: Design an offset link to withstand a load of $10\ kN$ at an offset distance of $10\ mm$. Take yield strength of the material as $350\ MPa$ and assume factor of safety as 4 and breadth to thickness ratio as 4.

Given data:

Load = $F = 10\ kN = 10\times10^3\ N$

Offset Distance $= 10\ mm$

Factor of safety FOS = 4

Yield Strength = $350\ MPa$

$b/t = 4$

Find:

1. Design Offset link

Solution:

Yield Stress = direct Stress + bending Stress $\left(\sigma_{yt} = \sigma_o + \sigma_b\right)$

$$\sigma_o = \frac{F}{4t^2}$$

$$\sigma_b = \frac{M_b \times r}{I} \qquad M_b = F(10+2t),\ r = 2t,\ I = \frac{16t^4}{3}$$

$$\sigma_b = \frac{F\left(10+b/2\right)\times 2t}{\frac{16}{3}\times t^4} = \frac{10000\left(10+4t/2\right)\times 2t}{\frac{16}{3}\times t^4} = \frac{37500}{t^3} + \frac{7500}{t^2}$$

$$\sigma_{yt\,(allowable)} = \frac{\sigma_{yt}}{FOS} = \frac{350}{4} = 87.5MPa$$

We know that

$$\sigma_{yt} = \sigma_d + \sigma_b \qquad 87.5 = \frac{F}{4t^2} + \frac{37500}{t^3} + \frac{7500}{t^2}$$

$$87.5t^2 - 10000t - 37500 = 0$$

$$t = 12.22mm$$

$$b = 4t = 48.88 = 50mm$$

2.15. DESIGN OF OVERHANGING CRANK

An overhanging crank, also known as a side crank or cantilever crank, is a crankshaft with a single crank arm projecting from one end. It's commonly used in applications with limited space, such as punching machines, shearing machines, and some single-cylinder engines.

Example 2.8: Calculate the diameter of the crank at $X-X$ as shown in Fig 2.22. which is subjected to $1kN$ load. Take Factor of safety as 2 and tensile strength of material as $400\ MPa$.

Given data:

Load, $F=1\ kN=1\times10^3\ N$

Yield strength = $400\ MPa$

Factor of safety $FOS=2$

Find:

1. Diameter of crank

Solution:

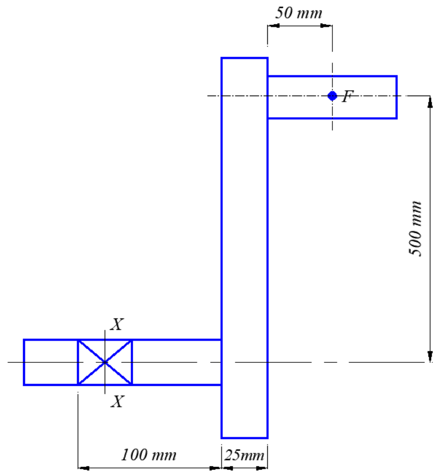


Fig. 2.22: Overhang crank

Bending moment induced at section $X-X$ is given by

$$M_b=F\times(50+25+100)=1000\times175$$

$$M_b=175\times10^3\,Nmm$$

Torque induced at section $X-X$ is given by

$$M_t=F\times500=1000\times500$$

$$M_t=500\times10^3\,Nmm$$

Bending stress $\sigma_b=\dfrac{M_b\times r}{I}=\dfrac{175\times10^3\times(d/2)}{(\pi d^4/64)}=\dfrac{1782.54\times10^3}{d^3}$

Shear stress $\tau=\dfrac{M_t\times r}{J}=\dfrac{500\times10^3\times(d/2)}{(\pi d^4/32)}=\dfrac{2546.48\times10^3}{d^3}$

Maximum shear stress theory is given by $\tau_{\max}=\sqrt{\left(\dfrac{\sigma_b}{2}\right)^2+\tau^2}$

$$\tau_{\max}=\sqrt{\left(\frac{1782.54\times10^3}{2d^3}\right)^2+\left(\frac{2546.48\times10^3}{d^3}\right)^2}=\frac{2697.95\times10^3}{d^3}$$

$$\tau_{yt}=0.5\times400=200\,N/mm^2$$

$$\tau_{yt(allowable)} = \frac{\tau_{yt}}{FOS} = \frac{200}{2} = 100N/mm^2$$

$$100 = \frac{2697.95\times10^3}{d^3}$$

$$d = 29.99 = 30mm$$

Example 2.9: Crank pin in an overhang crank is subjected to $10kN$ tangential load as given in Fig. 2.23. Calculate the maximum principle and shear stress at middle of the crankshaft bearing.

Given data:

Load $F = 10\ kN = 10\times10^3\ N$

Crank shaft diameter = $80\,mm$

Crank pin offset distance $y = 140\,mm$

Distance $x = 120\ mm$

Find:

1. Maximum principal stress
2. Maximum shear stress

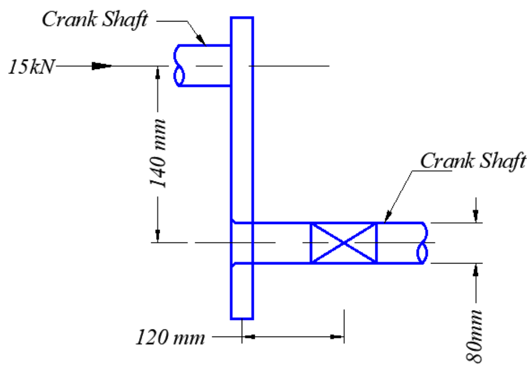


Fig. 2.23: Overhang crank in a crank shaft

Solution: Bending moment

$$M_b = Load \times Distance = F\times x = 10\times10^3\times120 = 1.8\times10^6\,Nmm$$

Torque $M_t = Load\times Distance = F\times y = 10\times10^3\times140 = 2.1\times10^6\,Nmm$

Bending stress $\sigma_b = \frac{M_b}{Z} = \frac{32M}{\pi d^3} = \frac{32\times1.8\times10^6}{\pi\times80^3} = 35.8\,N/mm^2$

Shear stress $\tau = \frac{16M_t}{\pi d^3} = \frac{16\times2.1\times10^6}{\pi\times80^3} = 20.9\,N/mm^2$

Principal stress $\sigma = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$

$$\sigma = \frac{35.8}{2} + \frac{1}{2}\sqrt{(35.8)^2 + 4\times(20.9)^2} = 45.4N/mm^2 = 45.4MPa$$

Maximum shear stress $\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$

$$\tau_{max} = \frac{1}{2}\sqrt{(35.8)^2 + 4\times(20.9)^2} = 27.5N/mm^2 = 27.5MPa$$

2.16. ARM OF PULLEY

Pulley arms, also known as spokes or webs, connect the central hub of a pulley to its rim as shown in Fig. 2.24. They play a crucial role in transmitting power and withstanding the forces acting on the pulley. As compared to rectangular cross-sections, elliptical cross-sections reduce aerodynamic losses during pulley rotation. Therefore, the arms will always have an elliptical cross-section.

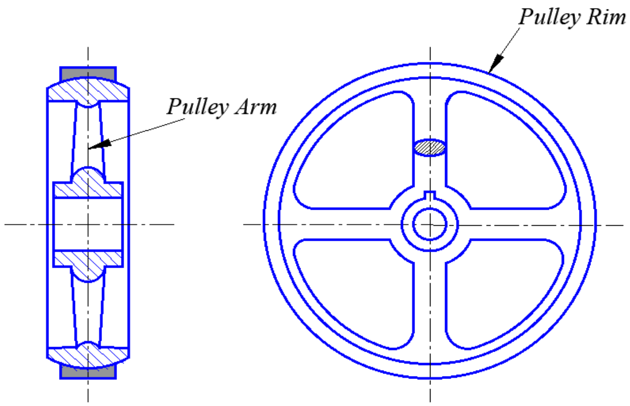


Fig. 2.24: Arm of pulley

These pulley arms are designed by estimating the bending stress using simple formula. During any given time, half of the arms carry the load as the belt surrounds the pulley's rim for an angle of 180° as noticed in Fig. 2.25.

The total amount of torque transmitted by the pulley is given by,

$$M_t = F \times R\left(\frac{n}{2}\right) \tag{2.50}$$

$$F \times R = 2\left(\frac{M_t}{n}\right) \tag{2.51}$$

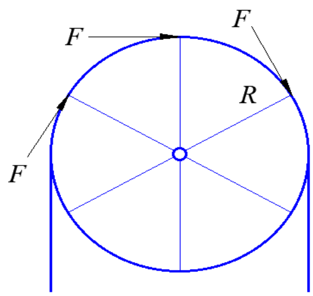


Fig.2.25: Load distribution in pulley

where M_t is torque transmitted in $N-mm$, F is force (Tangential) in the arm end in N, R is rim radius in mm and n is number of arms.

Fig. 2.26 represents the bending moment experienced on the pulley arm and it is given by

$$M_b = F \times R$$

$$M_b = 2\left(\frac{M_t}{n}\right) \tag{2.52}$$

where M_b is the maximum bending moment in $N-mm.$

In an elliptical cross-section, the major axis is located in the plane of rotation. Here, a and b denote the minor and major axes respectively,

Moment of Inertia about x axis, $I = \dfrac{\pi a b^3}{64}$

We know that the major axis in an elliptical cross-section is usually two times the minor axis 'a' i.e., $b = 2a$

$$I = \frac{\pi a^4}{8} \; ; \qquad y = \frac{b}{2} = a$$

(2.53)

The bending stress induced in the arm can be calculated as

$$\sigma_b = \frac{M_b y}{I} = \frac{\left(2M_t / N\right)a}{\left(\pi a^4 / 8\right)}$$

$$a^3 = \frac{16 M_t}{\pi N \sigma_b}$$

$$a = 1.72 \times \sqrt[3]{\frac{M_t}{N \sigma_b}} \qquad (2.54)$$

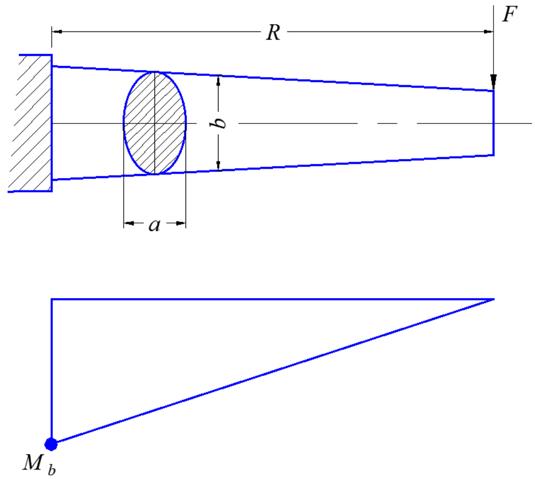


Fig.2.26: Bending moment on pulley arm

where σ_b is the permissible bending stress.

The arms of the pulley are tapered from hub to rim. The taper is usually 1/48 to 1/32.

Example 2.10: A 450 mm diameter pulley made of FG 150 (Cast Iron) has to transmit $8\,kW$ power at a speed of $600\,rpm$. The number of arms in the pulley is 4 and take factor of safety as 5. Calculate the dimensions of the arm.

Given data:

Power $P = 8kW$

Speed $N = 600\ rpm$

Diameter of pulley = $450\,mm$

Number of arms $n = 4$

Factor of safety FOS = 5

Find:

1. Dimensions of arm

Solution:

(i) Permissible stress:

$$\sigma_t = \frac{\sigma_{ut}}{FOS} = \frac{150}{5} = 30N / mm^2$$

Bending moment on arm

Torque and power relationship is given by, $P = \dfrac{2\pi N \times M_t}{60}$

From this, torque transmitted can be calculated, $M_t = \dfrac{60 \times P}{2\pi N}$

$$M_t = \frac{60 \times 8 \times 10^6}{2\pi \times 600} = 127388.5 Nmm$$

The tangential force in the arm can be calculated from Eq. (2.51)

$$F = \frac{M_t}{R\left(\frac{n}{2}\right)} = \frac{127388.5}{225\left(\frac{4}{2}\right)} = 283.08N$$

$$M_b = F \times R = 283.08 \times 225 = 63694.26 Nmm$$

(ii) Cross-section of arm:

The cross section of the arm can be calculated from torque and permissible stress using the following equation

$$a = 1.72 \times \sqrt[3]{\frac{M_t}{n\sigma_b}} = 1.72 \times \sqrt[3]{\frac{127388.5}{4 \times 30}} = 17.54mm = 20mm$$

$$b = 2a = 40mm$$

2.17. BEARINGS

A bearing is a machine element that facilitates smooth and efficient movement between two parts, typically supporting rotational or linear motion. Bearings reduce friction between moving parts, ensuring that machinery operates smoothly and efficiently. Bearings support various types of loads, including radial loads, which are forces acting perpendicular to the shaft, and axial loads, which are forces acting parallel to the shaft as shown in Fig 2.27. Additionally, bearings can support combined loads, which are a combination of radial and axial forces.

Bearings

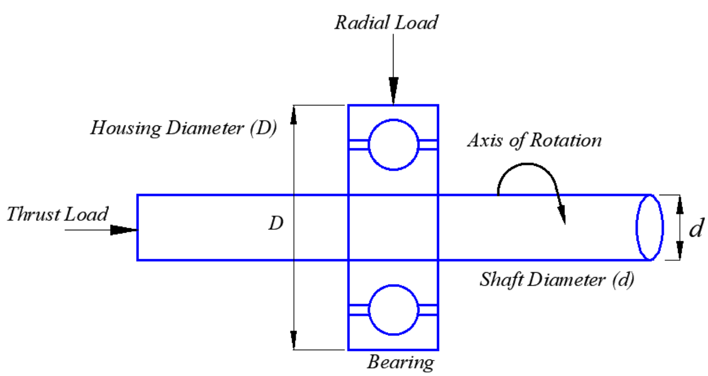


Fig. 2.27: Load acting on bearing

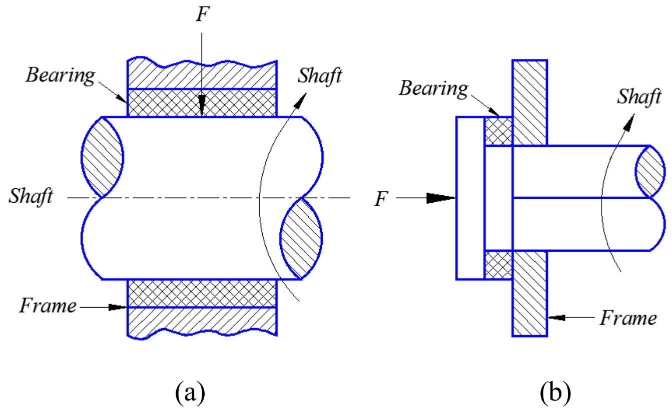


Fig. 2.28: (a) Radial bearing and (b) Thrust bearing

2.17.1 Classification of bearings

Bearings are categorized based on the type of force they endure and the kind of friction involved. They are classified into radial and thrust bearings according to the direction of the force applied, as illustrated in Fig. 2.28. Radial bearings support loads perpendicular to the shaft's axis, while thrust bearings support loads along the shaft's axis. Additionally, bearings are broadly classified into sliding contact bearings, also known as plain bearings or bushings, and rolling contact bearings, which include ball bearings and roller bearings, depending on the type of friction between the shaft and the bearing surface.

(i) Sliding contact bearing:

Sliding contact bearings operate on the principle of sliding friction. In these bearings, the shaft or axle slides over a surface (bearing surface), which is often lubricated to reduce friction and wear as shown in Fig. 2.29. The bearing material can be metallic or non-metallic and must have good frictional properties and wear resistance.

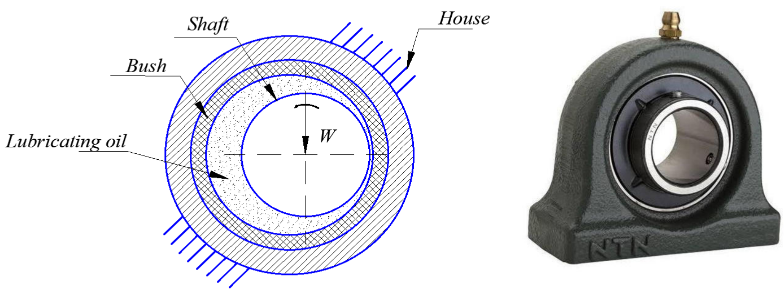


Fig. 2.29: Sliding contact bearing

Advantages:

- Simple design with fewer parts, making them easy to manufacture and maintain
- Capable of handling heavy and shock loads
- Operates with less noise and vibration

- Performs well at low speeds and in applications with oscillating movements
- Generally cheaper to produce

Disadvantages:

- Higher friction compared to rolling contact bearings
- More heat generation
- More susceptible to wear

Applications:

Sliding contact bearings are commonly used in automotive engines, in industrial machinery for crankshafts, camshafts, and turbines, and in household appliances for door hinges.

(ii) Rolling contact bearings:

Rolling contact bearings reduce friction by replacing sliding motion with rolling motion using rolling elements such as balls or rollers. These elements are housed between inner and outer races as shown in Fig. 2.30, and the rolling motion significantly reduces friction compared to sliding contact bearings. Rolling contact bearings include ball bearings that uses balls as the rolling elements meanwhile cylindrical, tapered, or spherical roller elements are used in the roller bearings.

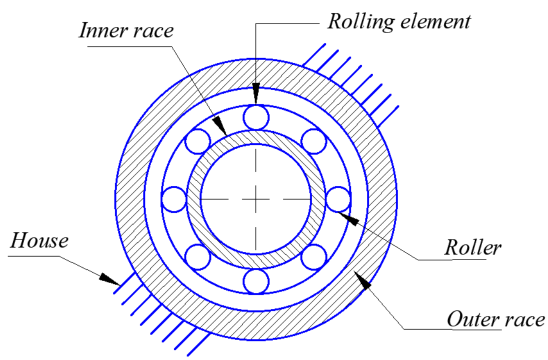


Fig. 2.30: Rolling contact bearing

Advantages:

- Lower friction compared to sliding bearings
- Support both radial and axial loads
- Long service life
- Less maintenance
- Capable of operating at higher speeds

Disadvantages:

- Generate more noise during operation
- Sensitive to vibrations and shocks
- Design and manufacturing are more complex, leading to higher production costs

Applications:

Ball bearings are suitable for light to moderate loads and are used in electric motors, bicycles, and computer fans. Roller bearings are suitable for heavier loads and are used in conveyor belt rollers, heavy machinery, and automotive applications.

2.17.2. Bearing materials

Bearing materials are pivotal in determining the overall performance and lifespan of bearings, and their selection is based on the specific operational demands of the application. The most commonly used material is high-carbon chromium steel (often referred to as bearing steel, such as AISI 52100), prized for its high hardness, fatigue resistance, and ability to maintain precision under high loads and speeds. This steel is typically through-hardened and provides excellent wear resistance and dimensional stability.

For applications requiring corrosion resistance, stainless steels such as AISI 440C are preferred due to their ability to withstand harsh environments while maintaining mechanical properties. Additionally, advanced ceramic materials like silicon nitride (Si_3N_4) offer exceptional hardness, low density, and high-temperature stability, making them ideal for high-speed and high-precision applications where reduced friction and wear are critical.

Polymers and composite materials are also utilized in bearing applications, especially where weight reduction, corrosion resistance, and cost-effectiveness are important. Polytetrafluoroethylene (PTFE) and other engineered plastics can provide self-lubricating properties, reducing the need for maintenance and lubrication.

2.18. TERMINOLOGY OF BALL BEARING

Ball bearings are essential to deliver smooth and efficient motion by reducing friction between moving parts. Understanding the terminology associated with ball bearings as shown in Fig. 2.31 is crucial for students studying the design of machine elements.

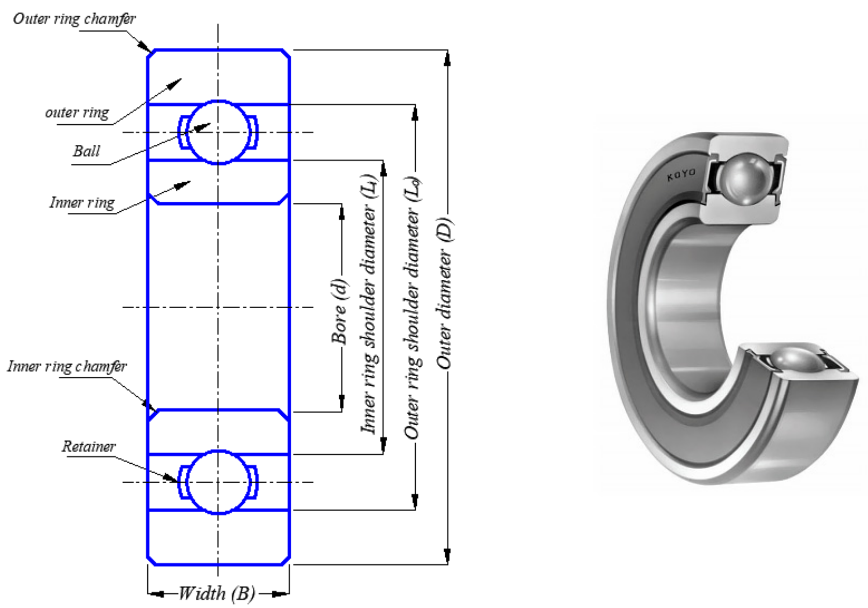


Fig. 2.31: Ball bearing terminology

(i) Inner race: Also known as the inner ring or inner ring raceway, this is the inner part of the bearing that is usually attached to the rotating shaft. The balls roll along a groove on the inner race.

(ii) Outer race: Also referred to as the outer ring or outer ring raceway, this is the outer part of the bearing that remains stationary or is attached to the bearing housing. The balls roll along a groove on the outer race.

(iii) Rolling elements (Balls): These are the spherical components that roll between the inner and outer races, reducing friction and allowing smooth motion. The number, size, and material of the balls can vary based on the bearing design and application.

(iv) Cage (or Retainer): The cage is a structure that holds the balls in place, evenly spacing them around the bearing. It prevents the balls from coming into contact with each other and guides their motion.

(v) Seal/Shields: Seals and shields are components that protect the bearing from contaminants such as dust, dirt, and moisture. Seals usually provide a tight seal to prevent ingress of contaminants and retain lubrication, while shields offer less protection but reduce friction.

(vi) Lubrication: This is a critical aspect of ball bearings, involving the application of grease or oil to reduce friction and wear between the rolling elements and raceways. Proper lubrication extends the lifespan of the bearing and ensures smooth operation.

(vii) Clearance (Internal clearance): Clearance refers to the small gap between the rolling elements and the raceways. It allows for thermal expansion and helps accommodate slight misalignments. Clearance can be radial (perpendicular to the shaft) or axial (along the shaft).

(viii) Preload: Preload is the application of an axial load to the bearing during assembly to eliminate internal clearance and provide rigidity. Preloading enhances the performance of the bearing by reducing noise and improving stiffness.

(ix) Load rating: This term describes the maximum load a bearing can support. Load ratings include the dynamic load rating, which is the maximum load a bearing can endure while rotating, and the static load rating, which is the maximum load a bearing can support when stationary.

(x) Bearing number: This is a standardized code used to identify the bearing type, size, and other specific features. Bearing numbers are essential for selecting bearing for a specific application.

(xi) Fatigue life: The fatigue life of a ball bearing refers to the number of revolutions or the time period a bearing can operate under a given load before showing signs of fatigue, such as spalling or pitting. It is a critical factor in determining the bearing's lifespan.

2.19. TYPES OF ROLLING CONTACT BEARINGS

Rolling contact bearings, also known as antifriction bearings, utilize rolling elements like balls or rollers placed between inner and outer rings to support rotating shafts and reduce friction. Different types of rolling contact bearing are shown in Fig. 2.32.

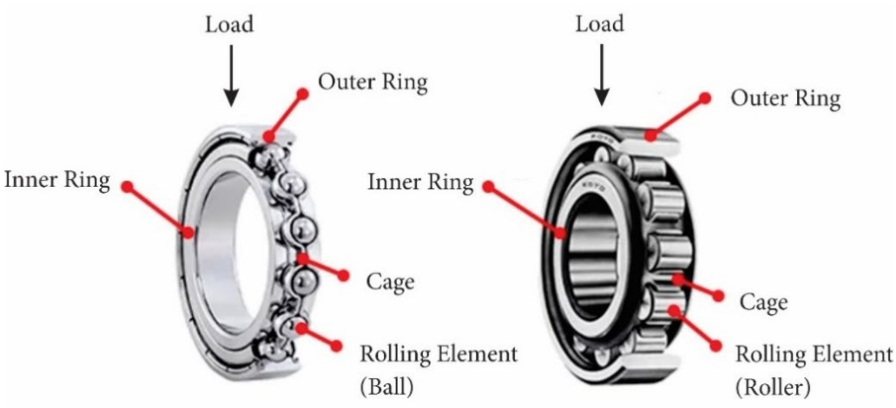


Fig. 2.32: Ball and roller bearing

2.19.1. Types of ball bearings

(a) Deep groove ball bearings:

Deep groove ball bearings (Fig. 2.33) are the most common type of ball bearings. They consist of an inner ring, an outer ring, a cage, and a set of balls. The grooves on the inner and outer rings are deep, allowing the bearing to support radial and axial loads in both directions. They are suitable for high-speed applications and provide low friction.

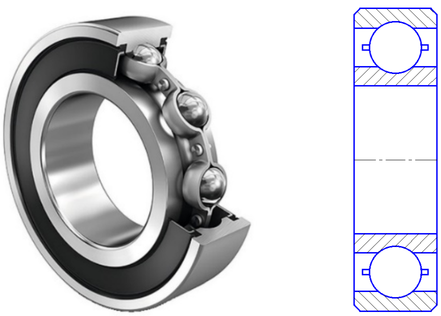


Fig. 2.33: Deep groove ball bearings

Applications: Electric motors, household appliances, automobiles, and industrial machinery.

(b) Angular contact ball bearings:

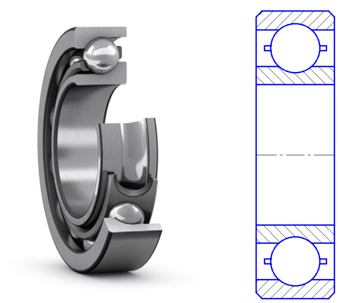


Fig. 2.34: Angular contact ball bearings

Angular contact ball bearings (Fig. 2.34) are designed to handle both radial and axial loads simultaneously. The contact angle between the ball and the raceway (inner and outer rings) allows the bearing to support higher axial loads. They are often used in pairs or sets to accommodate thrust loads in multiple directions.

Applications: Machine tool spindles, pumps, and gearboxes.

(c) Self-aligning ball bearings:

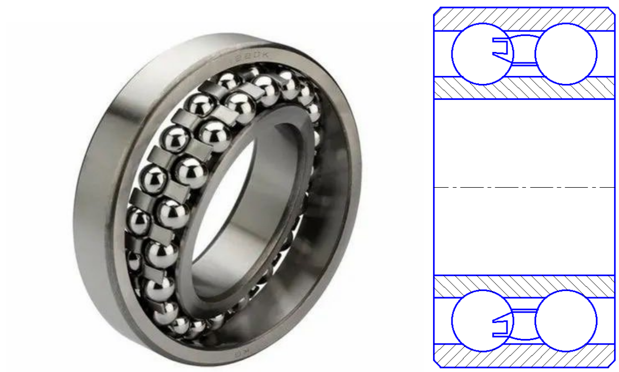


Fig. 2.35: Self-aligning ball bearings

Self-aligning ball bearings (Fig. 2.35) have two rows of balls and a common spherical raceway in the outer ring. This design allows the bearing to self-align and accommodate misalignment of the shaft relative to the housing. They are particularly useful in applications where shaft deflection or misalignment is expected.

Applications: Agricultural machinery, textile machinery, and conveyors.

(d) Thrust ball bearings:

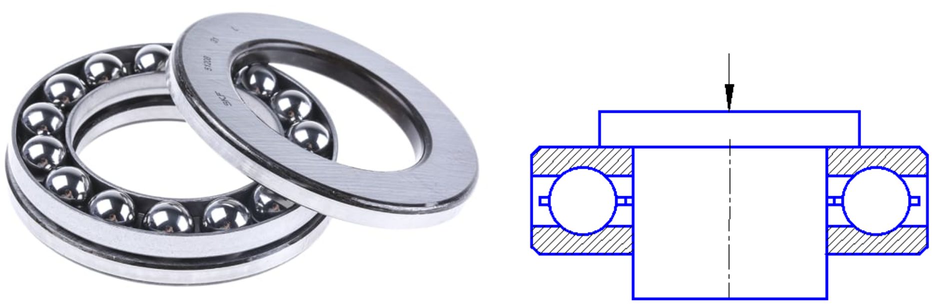


Fig. 2.36: Thrust ball bearings

Thrust ball bearings (Fig. 2.36) are designed to support axial loads only. They consist of two rings (shaft and housing washers) with ball and cage assemblies in between. These bearings are not suitable for radial loads.

Applications: Automotive clutches, low-speed reducers, and vertical shafts.

2.19.2. Types of roller bearings

(a) Cylindrical roller bearings:

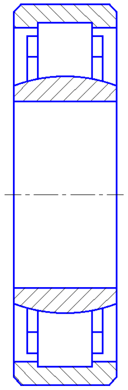



Fig. 2.37: Cylindrical roller bearings

Cylindrical roller bearings (Fig. 2.37) use cylindrical rollers as rolling elements, which are in linear contact with the raceways. This design allows for high radial load capacity and low friction. They are suitable for high-speed applications but cannot accommodate significant axial loads.

Applications: Electric motors, generators, and machine tool spindles.

(b) Spherical roller bearings:

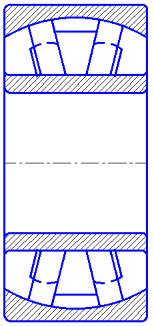



Fig. 2.38: Spherical roller bearings

Spherical roller bearings (Fig. 2.38) have barrel-shaped rollers and can accommodate both radial and axial loads. They are self-aligning and can handle shaft misalignment. These bearings are robust and suitable for heavy-duty applications.

Applications: Mining equipment, paper mills, and construction machinery.

(c) Tapered roller bearings:

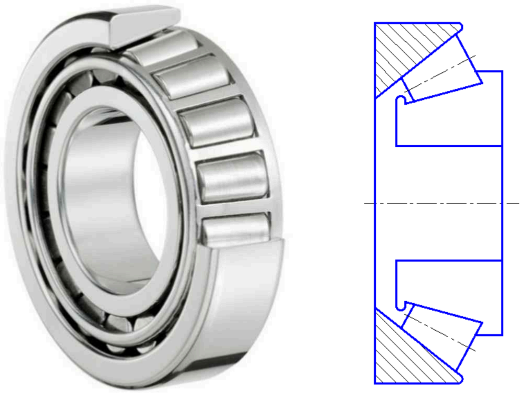


Fig. 2.39: Tapered roller bearings

Tapered roller bearings (Fig. 2.39) have conical rollers and raceways, which allows them to support combined radial and axial loads. The angle of the taper determines the load capacity and the direction of the load.

Applications: Automotive wheel hubs, gearboxes, and heavy machinery.

(d) Needle roller bearings:

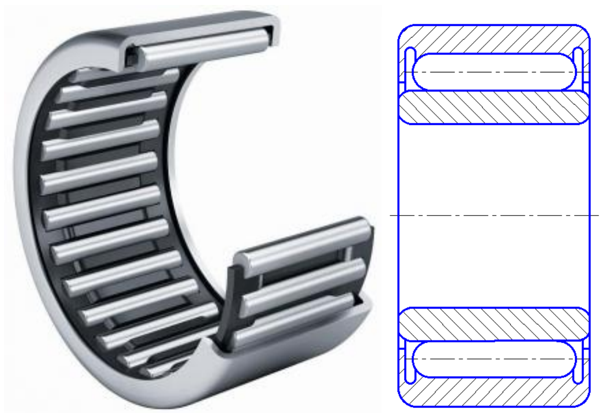


Fig. 2.40: Needle roller bearings

Needle roller bearings (Fig. 2.40) have long, thin rollers that resemble needles. Despite their small cross-section, they have a high load-carrying capacity and are suitable for applications with limited radial space.

Applications: Transmissions, compressors, and automotive components.

(e) Thrust roller bearings:

Fig. 2.41: Thrust roller bearings

Thrust roller bearings (Fig. 2.41) are designed to support high axial loads. They can come in various designs, such as cylindrical, spherical, and tapered thrust roller bearings. These bearings provide high load capacity and rigidity.

Applications: Heavy machinery, marine applications, and industrial presses.

2.20. PRINCIPLE OF SELF-ALIGNING BEARING

Self-aligning bearings are designed to accommodate misalignments between the shaft and the bearing housing. The principle behind their operation relies on their unique construction, which includes an inner ring with two raceways, an outer ring with a spherical raceway, and the rolling elements (either balls or rollers).

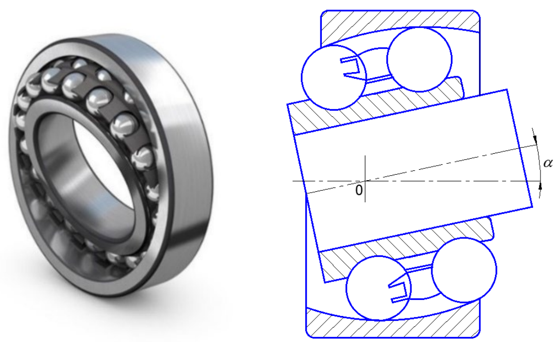


Fig. 2.42: Self aligning bearing principle

When the shaft is perfectly aligned with the bearing, the inner ring and the rolling elements are aligned concentrically with the outer ring as shown in Fig. 2.42. The self-aligning bearing is designed to mitigate these issues. The outer ring of the self-aligning bearing has a spherical raceway, whose center of curvature coincides with the bearing's center. This design allows the inner ring, which houses the rolling elements, to pivot freely around the spherical surface of the outer ring. As a result, when the shaft is misaligned, the inner ring and the rolling elements adjust their position relative to the outer ring, aligning themselves with the shaft.

2.21. SELECTION OF BEARING TYPE

Selecting the right type of bearing for various applications requires considering specific operating conditions and requirements.

- For applications involving low loads, ball bearings are generally preferred due to their ability to handle light loads efficiently. They offer low friction and smooth operation, making them ideal for use in small electric motors, household appliances, and light-duty machinery
- When there is a possibility of misalignment, self-aligning ball bearings and spherical roller bearings, can adjust to shaft deflections and mounting inaccuracies
- For medium load applications, cylindrical roller bearings and tapered roller bearings are often selected. These bearings can handle higher radial loads compared to ball bearings and offer increased load-carrying capacity
- In situations where both radial and thrust loads are present, angular contact ball bearings and tapered roller bearings are commonly used. Angular contact ball bearings can support combined loads due to their angled raceways, while tapered roller bearings are designed to handle significant axial and radial loads simultaneously
- For applications requiring maximum speed, deep groove ball bearings are typically the best choice. These bearings have a simple design with low frictional resistance, allowing them to operate at high speeds efficiently
- Deep groove ball bearings and hybrid ceramic bearings operate quietly, with minimal vibration, ensuring a smoother and quieter operation. It can be used where noise reduction is crucial, such as in household appliances, electric motors, and office machinery
- For machine tools requiring high rigidity and precision, angular contact ball bearings and tapered roller bearings are typically used. These bearings provide high stiffness and support precise movements, essential for maintaining accuracy in machining operations

2.22. STATIC LOAD CARRYING CAPACITY

The static load carrying capacity of a ball bearing, denoted by C_o, is a critical parameter defining the maximum load the bearing can endure without experiencing permanent plastic deformation. C_o is established by determining the load that induces a specific level of permanent deformation, typically expressed as a percentage of the ball diameter (e.g., 0.0001%). Exceeding this load threshold leads to permanent indentation on the balls and races, exceeding their elastic limit. This compromises the bearing's internal geometry, elevates friction, and ultimately accelerates wear and potential failure. By selecting a ball bearing with a C_o value exceeding the anticipated load in an application, engineers ensure the bearing operates within its elastic regime, guaranteeing smooth movement, extended lifespan, and reliable performance.

2.22.1. STRIBECK'S equation

A bearing's static load capacity is determined by Stribeck's equation.

Assumptions are as follows:

1. The races maintain their circular shape despite being rigid.
2. The balls are evenly spaced.
3. The ball on the upper half does not support any load.

$$C_o = \frac{kd^2q}{5} \tag{2.55}$$

where d is ball diameter in mm,

k factor (Depends on radii of curvature & modulus of material) and q is number of balls.

2.23. DYNAMIC LOAD CARRYING CAPACITY

The dynamic load carrying capacity of a bearing is a vital parameter that defines the bearing's ability to withstand dynamic loads during operation. This capacity is crucial for predicting the bearing's lifespan and ensuring the reliability and efficiency of the machinery it supports. The dynamic load carrying capacity (C) refers to the constant load that a bearing can theoretically endure for a rating life of one million revolutions.

In bearing terminology, the average life refers to the number of revolutions or hours that a group of identical bearings will operate before showing signs of fatigue. It is a statistical measure indicating the central tendency of bearing life distribution. The L_{10} life, also known as the B_{10} life, is a widely used concept in bearing engineering. It denotes the number of revolutions at which 90% of a large batch of identical bearings are expected to still be operational without showing signs of fatigue. Conversely, it implies that 10% of the bearings will have failed by this point. The L_{10} life is calculated based on standardized testing and is often used as a benchmark for bearing reliability.

Several factors affect the dynamic load carrying capacity of a bearing. These include the load magnitude and direction, where the actual load on the bearing, including radial and axial components, influences the dynamic load capacity. Material properties, particularly the material composition of the bearing races and rolling elements, affect their strength and fatigue resistance. Proper lubrication is critical as it reduces friction and wear, significantly impacting the bearing's lifespan and dynamic load capacity. Operating conditions, such as temperature, contamination, and speed, also play a role in affecting the bearing's performance and durability.

2.24. EQUIVALENT BEARING LOAD

In bearing design, understanding the concepts of equivalent bearing load and equivalent dynamic load is essential for selecting the appropriate bearing that can endure the actual operational conditions. The equivalent bearing load is a hypothetical load that represents the combined effect of radial and axial loads acting on a bearing. It simplifies the complex loading conditions into a single load value that can be used for design calculations and comparison purposes. This equivalent load is crucial because it allows engineers to assess the bearing's capacity to handle the applied loads effectively.

The equivalent dynamic load is a related concept that specifically pertains to the dynamic conditions of the bearing's operation. It is the constant stationary load that, if applied to the bearing, would have the same effect on the bearing's lifespan as the actual varying loads encountered during operation. This load helps in determining the bearing's rating life, making it a vital parameter in bearing selection and design.

Equivalent dynamic load can be expressed as follows:

$$W_d = XVF_r + \Upsilon F_a \tag{2.56}$$

where W_d is equivalent dynamic load (N), F_r is radial load (N), F_a is axial or thrust load (N), V is race-rotation factor, X is radial load factor and Υ is axial load factor

The values of X and Υ depend on the bearing type and the ratio of the axial load to the radial load.

Another important consideration in bearing load calculations is the race-rotation factor. The race-rotation factor accounts for the fact that the relative motion between the bearing races and the rolling elements affects the load distribution within the bearing. When the inner race rotates while the outer race is stationary (common in many applications), the load is evenly distributed among the rolling elements, there the value of V is 1. However, if the outer race rotates while the inner race is stationary, the load distribution can be different V equals to 1.2. This factor must be considered to ensure accurate load calculations and bearing selection. However, in majority of applications only the inner case rotates, hence

$$W_d = XF_r + \Upsilon F_a \tag{2.57}$$

Under condition when bearing is subjected to radial loads F_r alone,

$$W_d = F_r \tag{2.58}$$

Under condition when bearing is subjected to thrust loads F_a alone,

$$W_d = F_a \tag{2.59}$$

2.25. LOAD-LIFE RELATIONSHIP

The load-life relationship of ball helps to predict the lifespan of a bearing based on the loads it experiences during operation. This relationship is typically expressed through the bearing life equation, which connects the applied load to the bearing's expected operational life.

The relationship between the bearing life, the equivalent dynamic load and dynamic load carrying capacity is described by the following equation:

$$L_{10} = \left(\frac{C}{W}\right)^p \tag{2.60}$$

where L_{10} is rated bearing life (in million revolutions), C is dynamic load capacity (N), and $p=3$ (for ball bearings) & 1/3 (for roller bearings). Rearranging Eq. (2.60),

$$C = W\left(L_{10}\right)^{1/p}$$

For all types of ball bearings,

$$C = W\left(L_{10}\right)^{1/3} \tag{2.61}$$

This cubic relationship indicates that even small increases in the applied load W can significantly decrease the bearing life L_{10}. For instance, if the load is doubled, the bearing life is reduced by a factor of eight. Conversely, reducing the load can greatly extend the bearing's lifespan.

For all types of roller bearings,

$$C = W\ \left(L_{10}\right)^{0.3} \tag{2.62}$$

The relationship between the number of millions of revolutions in a year and the number of working hours is

$$L_{10} = \frac{60NL_{10h}}{10^6} \tag{2.63}$$

where L_{10h} is rated bearing life (hours) and N *is* speed of rotation (rpm)

Ball bearing life is often quantified using the L_{10} life or B_{10} life, which represents the number of revolutions at which 90% of a group of identical bearings are expected to still be operational without showing signs of fatigue. This is a statistical measure that provides a reliable estimate of bearing longevity under specified load conditions.

Example 2.11: Estimate the radial load carrying capacity of a taper roller bearing subjected to dynamic load carrying limit of $20kN$ operating at a speed of $450\ rpm$. Take the desired life as $10000h$ for 90% of the bearing.

Given data: ***Load*** $C = 20\ kN = 20\times10^3\ N$

Life $L_{10h} = 10000h$

Speed $N = 450\ rpm$

Find:

1. Radial load

Solution:

(i) Calculation of bearing life (L_{10}):

$$L_{10} = \frac{60NL_{10h}}{10^6} = \frac{60(450)(10000)}{10^6}$$

$$L_{10} = 270 \text{ million revolution}$$

(ii) Calculation of equivalent radial load:

$$C = W(L_{10})^{0.3}$$

$$W = \frac{C}{(L_{10})^{0.3}} = \frac{20\times10^3}{(270)^{0.3}} = 3729.25\ N$$

It was given that the roller bearing is under pure radial load, hence

$$F_r = W = 3729.25N$$

2.26. SELECTION OF BEARING LIFE

The life of ball bearings in different applications is a crucial aspect of design and maintenance, directly impacting the performance, reliability, and longevity of the equipment. Bearings are subjected to a variety of load conditions, environmental factors, and operational demands that influence their lifespan.

2.26.1. Load factor

The load factor is used to modify the basic dynamic load rating and bearing life equations to reflect actual operating conditions more precisely. The load factor helps to bridge the gap between the theoretical load capacity of a bearing and the actual load it experiences during operation. For example, manufacturing imperfections, misalignments, shock loads, and uneven load distribution can all affect the loads acting on the bearing. The load factor compensates for these additional stresses, ensuring that the bearing's calculated life more closely matches its real-world performance.

The load factor is determined based on the specific application and operational environment. For instance, in applications subject to high shock loads or vibrations, a higher load factor is used to reflect the increased stresses. Conversely, for applications with stable, well-aligned loads and minimal shocks, a lower load factor may be appropriate.

2.27. SELECTION OF BEARING FROM MANUFACTURER’S CATALOGUE

Selecting the right ball bearing for your application requires careful consideration of various factors. Here's a step-by-step guide to choose the most suitable bearing from manufacturer's catalog:

1. Determine the required shaft diameter based on the given radial and axial loads that the bearing will support.
2. Choose the appropriate bearing type specific to the application' requirements. Consider factors such as load type (radial, axial, or combined), operating speed, temperature range, and environmental conditions.

Bearings may include deep groove ball bearings, angular contact ball bearings, cylindrical roller bearings, etc.:

- Deep groove ball bearings: For radial and moderate axial loads
- Angular contact ball bearings: For combined load
- Thrust ball bearings: For pure axial loads
- Self-aligning ball bearings: For applications with misalignment
- Other types: Specific types depending on special requirements

3. Refer to the manufacturer's catalog to identify the thrust factors (X and Y) specific to the selected bearing type. These factors account for the radial and axial load distributions within the bearing and are used in subsequent calculations.
4. Calculate the equivalent dynamic load (C) acting on the bearing using the given radial and axial loads, along with the thrust factors (X and Υ).

   $C = XF_r + \Upsilon F_a$
5. Estimate the expected life of the bearing in millions of revolutions (L_{10}) using the calculated equivalent dynamic load.

   $$L_{10} = \frac{60NL_{10h}}{10^6}$$
6. Determine the dynamic load carrying capacity of the selected bearing series.

   $$C = W\ (L_{10})^{1/3}$$
7. Check whether the selected bearing series has the required dynamic load carrying capacity to support the calculated equivalent dynamic load, else choose the next series and move to step 3.

The selection of ball bearing is thus done by trial-and-error method. The same procedure may be adopted to other types of bearing as well.

Let us consider a numerical example to understand the method of selection of bearing from the manufacturer catalogue.

Example 2.12: Design a suitable bearing which is subjected to a radial load of $4000\,N$, an axial load of $5000\,N$ and a speed of $1600\,rpm$. The bearing should have a life expectancy of 15,000 hours and fit with $60mm$ bore diameter.

Step 1: Given that the shaft diameter is $60mm$

Step 2: As the bearing is subjected to radial and moderate axial loads, deep groove ball bearing is selected. Let us consider SKF 6312 bearing for this application. This bearing is selected based on the shaft diameter.

Step 3: For the selected bearing C_{10} is $7800\ N$ and f_0 is 13, Hence

$$\frac{f_a f_0}{C_o} = \frac{1200 \times 13}{52000} = 0.3$$

The corresponding thrust factors X and Υ are 0.56 and 2.071 respectively. The value of Y is calculated using interpolation.

Step 4: Equivalent dynamic load, $W_d = XF_r + \Upsilon F_a = 0.56(1200) + 2.071(4000) = 8956\,N$

Step 5: Life of bearing, $L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60 \times 360 \times 20000}{10^6} = 432$ million revolution

Step 6: Dynamic load carrying capacity, $C = P(L_{10})^{1/3} = 67{,}703\,N$

Step 7: Since the dynamic load carrying capacity of SKF 6312 bearing is $85{,}200\,N$, the selected bearing is suitable for the selected application.

2.27.1. Bearing series

Selecting the appropriate ball bearing series from a manufacturer's catalog involves a detailed understanding of the load capacities, dimensions, and suitability for specific applications. In the catalogue, the 'series' of the bearing is often mentioned i.e., light, medium and heavy.

- Light series ball bearings are designed for applications with relatively low loads and speeds. They are characterized by smaller dimensions and lower load-carrying capacities compared to medium and heavy series bearings. Light series bearings are suitable for applications where space constraints or weight considerations are critical.
- Medium series ball bearings strike a balance between load capacity and size. They are suitable for a wide range of general-purpose applications with moderate loads and speeds. Medium series bearings offer versatility and are commonly used in various industries, including automotive, industrial machinery, and appliances.
- Heavy series ball bearings are designed for applications with high loads and demanding operating conditions. They feature larger dimensions and higher load-carrying capacities compared to light and medium series bearings. Heavy series bearings are suitable for heavy-duty applications such as mining equipment, construction machinery, and large industrial machinery.

  In addition to the bearing series classification, each bearing series is typically denoted by 3 or 4 digits (Ex. 6207) that provide further information about its design and characteristics:

  - The bore diameter of the bearing is usually designated in the last 2 letters of the series. i.e., Multiply last 2 digits by 5. For bore diameters below $20\,mm$, specific codes are used (e.g., 00 for $10\,mm$, 01 $for\,12\,mm$, 02 $for\,15\,mm$ and 3 $for\,17\,mm$).

  Example: xx07 represents that diameter of bearing is $35\,mm$

  - Bearing series are denoted in the next digit i.e., 3rd from the right. i.e., '1' means the extra light series; '2' means light series; '3' means medium series and '4' means heavy series

Example: X207 represents light series bearing with $35mm$ bearing.

- The fourth digit (occasionally fifth digit) is used to represent the bearing series.

  Example: 6 refers to deep groove ball bearing.

Suffixes and prefixes:

These indicate specific features such as internal design, clearance, cage type, and seals.

- Cages: Often specified by a letter (e.g., " J " for a pressed steel cage).
- Seals and Shields: Indicated by letters such as " Z " for a shield on one side, or " ZZ " for shields on both sides.
- Clearance: Indicated by a letter and number combination (e.g., "C3" for greater than normal clearance).

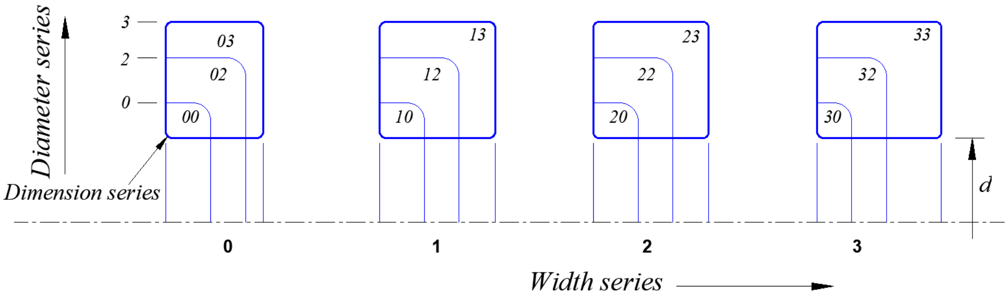


Fig.2.43: Bearing dimension series

For a selected bore diameter, light series bearings offer the compact dimensions. However, their load-carrying capacity is the lowest among the three series. Compared to light series bearings of the same bore diameter, medium series bearings exhibit a dynamic load carrying capacity that is 30 to 40 percent higher and note that the space occupied for the medium series bearing is little large. Finally, the heavy series bearings can attribute to a dynamic load carrying capacity that is 20 to 30 percent higher for the bearing with same bore diameter.

According to ISO, the bearing dimension series is illustrated in Fig. 2.43. It uses a two-digit code (middle two numbers in the bearing series) to categorize bearings based on their width and diameter series.

The width series is designated by the first digit of the two-digit code and determines the width of the bearing in relation to its diameter. The series are as follows:

- 8: Extra narrow
- 0: Very narrow
- 1: Narrow
- 2: Medium
- 3: Medium
- 4: Wide

- 5: Very wide
- 6: Extra wide

For instance, a bearing with a width series of "1" will be narrower than a bearing with a width series of "4".

The diameter series is represented by the second digit and organizes the bearings in ascending order based on their outer diameter relative to the bore diameter i.e., thickness between the bore and outside diameters. The series are as follows:

7, 8, 9 – represents smaller outer diameters relative to the bore diameter

0, 1, 2, 3, 4 - represents larger outer diameters relative to the bore diameter.

In this system, a bearing with a diameter series of "1" will have a smaller outer diameter relative to the bore diameter than a bearing with a diameter series of "3".

Example of Bearing Dimension Series Code

Let's us consider the series code 23:

- First Digit "2" indicates a medium width.
- Second Digit "3" suggests a medium-to-large outer diameter in relation to the bore diameter.

Example 2.13: A shaft with a diameter of $75\ mm$ and a rotation speed of $125\ rpm$ requires a single-row deep groove ball bearing. Design the bearing which is subjected a radial load of $21kN$, and there is no thrust load applied. The bearing is expected to last for $10000\ hours$.

Given data:

Radial load, $F_r = 21000N$; Axial load, $F_a = 0$

Diameter of the shaft, $d = 75mm$

Life hours = $10000\ h$

Speed = $125\ rpm$

Find:

1. Design single-row deep groove ball bearing

Solution:

(i) Bearing type:

Single-row deep groove ball bearing

(ii) Load:

No axial Load, $F = F_a - 0N$

Radial Load, $F = F_r - 2000N$

(iii) Bearing life:

$$L_{10} = \frac{60n\,L_{10h}}{10^6}$$

$$L_{10} = \frac{60(125)(10000)}{10^6}$$

$$L_{10} = 75 \text{ million revolutions}$$

(iv) Dynamic load capacity:

$$C = W\,(L_{10})^{1/3} = 21000\,(75)^{1/3}$$

$$C = 88560.43\ N$$

(v) Selection of bearing series:

From Design Data Book, for the given shaft diameter of $75mm$, bearing No. 6315 is selected.

Example 2.14: Select a single-row deep groove ball bearing subjected to radial load and thrust load of $6kN$ and $2kN$ respectively which is expected to have $25000\,hours$ of life $L10h$. The shaft attached to the bearing runs at 1400 rpm .

Given data:

Radial load $F_r = 6\ kN = 6\times10^3\ N$

Thrust load $F_a = 2\ kN = 2\times10^3\ N$

Life hours = $25000\,hr$

Speed $N = 1400\ rpm$

Diameter of the shaft = $65mm$

Find:

1. Design single-row deep groove ball bearing

Solution:

(i) X and Υ factors:

The X and Υ factors has to be obtained from Design Data Book, through trial and error process. It is understood that the bearing should undergo both radial and thrust loads. For the deep groove ball bearing,

The value of X is 0 and Υ is 1 when $\left(\frac{F_a}{F_r}\right) \le e$

The value of X remains 0.56 and Y changes between 1 and 2 when $\left(\frac{F_a}{F_r}\right) > e$

Let we take the average value, $\Upsilon = 1.5$ and proceed,

Therefore, $X = 0.56 \quad \Upsilon = 1.5 \qquad F_r = 6000\ N \quad F_a = 2000\ N$

(ii) Equivalent bearing load:

$$W = XF_r + \Upsilon F_a = 0.56(6000) + 1.5(2000) = 6360\ N$$

(iii) Life of bearing:

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(1400)(25000)}{10^6}$$

$$L_{10} = 2100\ million\ revolutions$$

$$C = W(L_{10})^{1/3} = (6360)\times(2100)^{1/3} = 81444.89\ N$$

From Design Data Book, it is observed that for the shaft of $75mm$ diameter, Bearing No. 6315 (C = 90000N) is suitable for the above data.

For this bearing, $C_o = 72000\ N$, Therefore,

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{2000}{6000}\right) = 0.333$$

$$\left(\frac{F_a}{C_o}\right) = \left(\frac{2000}{72000}\right) = 0.0277$$

From Design Data Book, Let we take e as 0.23 (approximately)

Using linear interpolation, Υ can be calculated

$$\Upsilon = 2 - \frac{(2-1.8)}{(0.04-0.025)}\times(0.0277-0.025) = 1.8$$

and X = 0.56

(iv) Dynamic load capacity:

$$W = XF_r + \Upsilon F_a = 0.56(6000) + 1.8(2000) = 6960\ N$$

$$C = W(L_{10})^{1/3} = 6960\ (2100)^{1/3} = 89128\ N$$

(v) Selection of bearing:

From Design Data Book, Bearing No. 6315 ($C = 90000N$) is suitable for the above application.

Example 2.15: Design two single row ball bearing to use in shaft with $12mm$ and $20\,mm$ diameter respectively for transmitting power from a pulley to spur gear. The shaft rotates at 650 *rpm* and the pulley weight is $120N$. The forces in the transmission systems are $F_1 = 475N$; $F_2 = 175N$; $F_t = 480N$; $F_r = 170N$. Consider the expected life of bearing as $6000h$ and load factor as 2.5.

Given data:

Speed $N = 650\ rpm$

Diameter of shaft 1 = $12mm$

Diameter of shaft 2 = $20mm$

Load, $F_1 = 475N$; Load, $F_2 = 175N$

Pulley weight $W = 120\ N$

Load factor = 2.5

Tangential load, $F_t = 480N$; Radial load, $F_r = 170N$

Life hours = $6000h$

Find:

1. Design two single row ball bearing

Solution:

(i) Radial and axial forces:

The shaft is subjected to different forces as shown in Fig. 2.44.

Let us consider the forces acting in vertical direction and take moment about B_1

$$F_r(100) + W(400) - R_{V2}(250) = 0$$

$$170(100) + 120(400) - R_{V2}(250) = 0$$

$$R_{V2} = 260\ N$$

$$R_{V1} + R_{V2} = F_r + W$$

$$R_{V1} + 260 = 170 + 120$$

$$R_{V1} + 260 = 290$$

$$R_{V1} = 30\ N$$

Let us consider the forces acting in vertical direction and take moment about B_1

$$F_t(100) + (F_1 + F_2)(400) - R_{H2}(250) = 0$$

$$480(100) + (475 + 175)(400) - R_{H2}(250) = 0$$

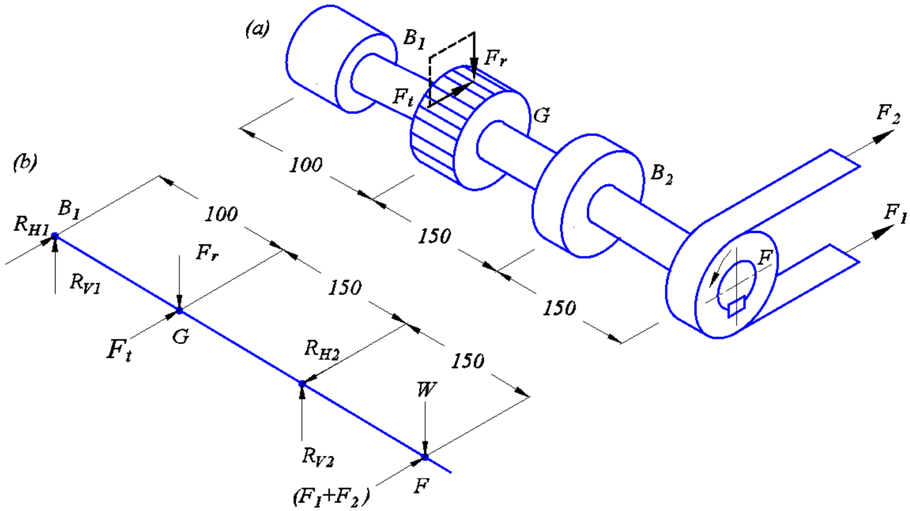


Fig. 2.44: (a) Power transmission system (b) Forces acting on the system

$R_{H2} = 1232\ N$

$R_{H2} = R_{H1} + F_t + (F_1 + F_2)$

$1232 = R_{H1} + 480 + (475 + 175)$

$R_{H1} = 102\ N$

From the reaction forces, net reactions of the bearings can be calculated as

$R_1 = \sqrt{(R_{V1})^2 + (R_{H1})^2}$

$R_1 = \sqrt{(30)^2 + (102)^2} = 106.32\,N$

$R_2 = \sqrt{(R_{V2})^2 + (R_{H2})^2}$

$R_2 = \sqrt{(260)^2 + (1232)^2} = 1259.1\,N$

As the reactions created in the bearings are in radial direction, we can consider those reactions as the radial load,

$R_1 = F_{r1} = 106.32\ N$; $\quad R_2 = F_{r2} = 1259.1\ N$

It can be noticed that the bearing has no axial thrust, therefore

$F_{a1} = F_{a2} = 0$

(ii) Dynamic load capacities:

From the Eq. (2.58),

$F_1 = F_{r1} = 106.32\ N$

$F_2 = F_{r2} = 1259.1\ N$

From the Eq. (2.63), $L_{10} = \dfrac{60nL_{10h}}{10^6} = \dfrac{60(650)(6000)}{10^6}$

L_{10} = 234 million rev.

The dynamic load carrying capacity including load factor. Given that load factor is 2.5

$$C_1 = F_1 (L_{10})^{1/3} (Load\ factor)$$

$$C_1 = (106.32)\ (234)^{1/3} (2.5) = 1638\ N$$

$$C_2 = F_2 (L_{10})^{1/3} (Load\ factor)$$

$$C_2 = (1259.1)\ (234)^{1/3} (2.5) = 19424.7\ N$$

(iii) Selection of bearings:

From Design Data Book, the appropriate deep groove ball bearing for $12\,mm$ is Bearing 6001 (Suitable for B_1) and ball bearing for $20\,mm$ is Bearing 6404 (Suitable for B_2).

Example 2.16: $1100\,N$ of axial load and $2300\,N$ of radial load is applied in deep groove ball bearing series 6002 . Calculate 60% of the expected life of bearing that can operate under the given load.

Given data:

Axial load = $F_a = 1100\,N$

Radial load = $F_r = 2300\,N$

Bearing series = 6002

Find:

1. 60% of the expected bearing life

Solution:

(i) X and Y factors:

From Design Data Book, for bearing series 6002 ,

$C_o = 2550\ N\ and\ C = 4400\ N$

Given that

$F_a = 1100\ N\ and\ F_r = 2300\ N$

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{1200}{2300}\right) = 0.5217$$

$$\left(\frac{F_a}{C_o}\right)=\left(\frac{1100}{2550}\right)=0.43$$

From Design Data Book, $\left(\frac{F_a}{F_r}\right)>e$

Calculating Υ from the table using linear interpolation.

$$\Upsilon=1.2-\frac{(1.2-1.0)}{0.5-0.25}\times(0.43-0.25)=1.056 \text{ and } X=0.56$$

(ii) Bearing life (L_{10})**:**

Equivalent bearing load must be calculated to find the bearing life,

$$W = XF_r + \Upsilon F_a = 0.56(2300)+1.056(1100)$$

$$=1288+1161=2449\ N$$

From the Eq. (2.61), bearing life is $C = W(L_{10})^{1/3}$

$$4400 = 2449\ (L_{10})^{1/3}$$

$$L_{10} = 10.42\ million\ revolutions$$

(iii) Bearing life (L_{60})**:**

60% life of the bearings is around 6 times of the bearing (L_{10}) life. Therefore,

$$L_{60} = 6\times L_{10} = 6(10.42) = 62.52\ million\ revolutions$$

2.28. LIMITING SPEED

The limiting speed of a ball bearing is a crucial parameter that determines the maximum operational speed at which the bearing can function reliably without excessive heat generation, lubrication failure, or mechanical damage. Understanding and managing the limiting speed is essential for ensuring the performance and longevity of bearings in high-speed applications.

2.28.1. Factors affecting limiting speed

Several factors influence the limiting speed of ball bearings, including bearing type and design, materials, lubrication, heat dissipation, precision and tolerance, load conditions, and operating environment. Different bearing types, such as deep groove and angular contact, have varying limiting speeds due to their design characteristics. High-quality materials with good thermal properties and strength, such as high-carbon chromium steel or ceramics, enhance the limiting speed. Proper lubrication reduces friction and heat generation, thus increasing the limiting speed.

The choice of lubricant, whether oil or grease, and its viscosity play a significant role in determining the bearing's speed limit. Efficient heat dissipation mechanisms, such as cooling systems or specially designed bearing housings, prevent overheating and enable higher speeds. High-precision bearings with tight tolerances run more smoothly and can achieve higher speeds.

The magnitude and direction of applied loads impact the limiting speed, with radial and axial loads needing careful consideration. External factors such as ambient temperature, contamination, and humidity can influence the bearing's limiting speed, with clean and controlled environments supporting higher speed operation.

2.29. BEARING FAILURE

Bearing failure is a critical concern in mechanical systems, as it can lead to unexpected downtime, costly repairs, and even catastrophic equipment failure. Understanding the various types of bearing failure is essential for diagnosing issues, improving design, and implementing effective maintenance strategies.

2.29.1. Types of bearing failure

(i) Fatigue failure: Fatigue failure, also known as spalling, occurs when the bearing material fails due to repeated stress cycles over time. This type of failure typically manifests as flaking or pitting on the surface of the raceways or rolling elements. Fatigue failure is influenced by factors such as load magnitude, load cycles, material properties, and lubrication quality. Bearings designed with appropriate load ratings and high-quality materials, along with proper lubrication, can mitigate fatigue failure.

(ii) Wear: Wear is a common failure mode resulting from the gradual removal of material from the bearing surfaces due to friction. This can be caused by inadequate lubrication, contamination, misalignment, or excessive loads. There are several types of wear, including abrasive wear, adhesive wear, and fretting. Preventing wear involves ensuring proper lubrication, maintaining clean operating environments, and avoiding misalignment and overloading.

(iii) Lubrication failure: Lubrication failure occurs when the lubricant fails to adequately protect the bearing surfaces, leading to increased friction, heat generation, and wear. This can be due to insufficient lubricant quantity, incorrect lubricant type, contamination, or degradation over time. Symptoms of lubrication failure include discoloration of bearing components, increased operating temperature, and noise. Regular maintenance, proper lubricant selection, and contamination control are key to preventing lubrication failure.

(iv) Contamination: Contamination refers to the presence of foreign particles, such as dirt, dust, metal shavings, or moisture, within the bearing. Contaminants can enter through damaged seals, improper handling, or a contaminated operating environment. Contamination leads to abrasive wear, increased friction, and potential bearing seizure. Preventive measures include using high-quality seals, maintaining clean assembly areas, and implementing effective filtration systems.

(v) Electrical erosion: Electrical erosion, also known as fluting, occurs when electric currents pass through the bearing, causing localized melting and material removal. This is common in electric motors and

generators where stray electrical currents can flow through the bearing. Electrical erosion results in distinctive patterns on the bearing surfaces and can lead to premature failure. Solutions include proper grounding, using insulated bearings, and employing current bypass techniques.

(vi) Corrosion: Corrosion is the chemical reaction between the bearing material and the environment, leading to the formation of rust or other corrosive products. It is often caused by exposure to moisture, acids, or other corrosive agents. Corrosion weakens the bearing material, increases friction, and can result in catastrophic failure. Preventive measures include using corrosion-resistant materials, applying protective coatings, and controlling environmental exposure.

(vii) Misalignment: Misalignment occurs when the bearing and its associated components are not aligned properly, resulting in uneven load distribution and excessive stress on certain areas. This can cause premature wear, overheating, and eventual bearing failure. Misalignment can be due to improper installation, shaft deflection, or structural issues. Ensuring precise alignment during installation and using flexible couplings can help prevent misalignment-related failures.

(viii) Overloading: Overloading happens when the bearing is subjected to loads exceeding its designed capacity. This can be due to incorrect bearing selection, sudden impact loads, or operational changes. Overloading accelerates wear, increases heat generation, and can cause immediate or premature failure. Proper load analysis, selecting bearings with appropriate load ratings, and avoiding shock loads are essential to prevent overloading.

2.30. DIAGNOSIS AND PREVENTION

To effectively diagnose and prevent bearing failures, several strategies can be implemented:

Regular inspection: Routine inspection of bearings for signs of wear, contamination, lubrication issues, and alignment can help identify potential problems early.

Condition monitoring: Using techniques such as vibration analysis, temperature monitoring, and acoustic emission can provide early warning signs of bearing issues.

Proper lubrication: Ensuring the correct type and amount of lubricant is used and regularly maintained can significantly extend bearing life.

Cleanliness: Maintaining a clean operating environment and protecting bearings from contaminants through proper sealing and filtration systems.

Correct installation: Following manufacturer guidelines for bearing installation to ensure proper alignment and fit.

Load management: Ensuring bearings are not subjected to loads beyond their capacity through proper design and operational controls.

2.31. LUBRICATION OF ROLLING CONTACT BEARINGS

Antifriction bearings require lubrication in order to reduce friction between the balls and the races. Other objectives of the bearing include dissipating frictional heat, preventing corrosion, and protecting it from dirt. Oil and grease are both types of lubricants. Compared with grease, oil offers the following advantages:

- The frictional heat is carried more effectively by it
- Under load, it feeds more easily into contact areas of the bearing
- The bearing will be more effectively cleaned by flushing out dirt, corrosion, and foreign particles

Grease-lubricated bearings offer advantages such as simple housing design, minimal maintenance costs, better sealing against rust, and reduced leakage potential. For the selection of the lubricant, the following guidelines should be followed:

Grease is suitable for applications where the temperature is below 100°C, whereas lubricating oils are preferable for applications where the temperature exceeds 100°C.

- Grease is suitable when the product of bore (in mm) $\times$ speed (in rpm) is less than 200 000. Lubricating oils are recommended for higher values.
- Low and medium loads are best handled by grease, while high loads are best handled by lubricating oils.
- Bearings, such as gearboxes, are lubricated using the same lubricating oil used for lubricating other parts of a system.

High speed, heavy load applications require lubricating oils, while grease provides the most cost-effective and easiest form of lubrication for the remaining majority of applications.

UNIT SUMMARY

- Cotter joint connects two coaxial rods, providing a reliable way to transmit axial loads. The cotter is a flat wedge-shaped piece inserted into slots cut into the rods and locked in place.
- There is a slight taper in the width dimension b of the cotter; usually a taper of 1 in 24.
- Between the slots and the cotter, there is a clearance of 2 to 3 mm.
- Turnbuckle is used for adjusting the length or tension in cables, rods, or tie rods, a turnbuckle consists of a pair of eye bolts connected by a central nut
- Bell crank lever is used to change the direction of force, typically at a 90-degree angle, these levers are crucial in various mechanical applications
- Based on the direction of force, bearings are classified into radial and thrust bearings
- A radial bearing supports the load, which is perpendicular to the axis of the shaft. A thrust bearing supports the load, which acts along the axis of the shaft
- Depending upon the type of friction between the shaft and the bearing surface, bearings are broadly classified into sliding contact bearings, also known as plain bearings or bushings, and rolling contact bearings, called ball bearings and roller bearings
- Self-aligning ball bearings use two rows of balls and a common raceway, allowing for misalignment of the shaft and housing
- The life of a bearing is related to the load it carries, with higher loads reducing bearing life
- The maximum load a bearing can withstand without permanent deformation is basic static load rating
- The relationship between the bearing life and load is described by the following equation:

$$L_{10} = \left(\frac{C}{W}\right)^{p}$$

  For all types of ball bearings, $C = W(L_{10})^{3}$

  For all types of roller bearings, $C = W(L_{10})^{0.3}$

- The equivalent dynamic load (W) acting on the bearing using the given radial and axial loads, along with the thrust factors (X and Υ).

  $W = XF_r + \Upsilon F_a$

- Each bearing series is typically denoted by 3 or 4 digits (Ex. 6207) that provide further information about its design and characteristics
- The bore diameter of the bearing is usually designated in the last 2 letters of the series. i.e., Multiply last 2 digits by 5

- Bearing series are denoted in the next digit i.e., 3rd from the right
- The fourth digit (occasionally fifth digit) is used to represent the bearing series
- The maximum operating speed of a bearing without excessive heat generation or wear is called limiting speed

EXERCISES

Multiple Choice Questions

1. A cotter joint is primarily used for
   (a) axial tensile loads (b) axial compressive loads
   (c) shear loads (d) torsional loads
2. Upset cotters provide a tighter fit compared to regular cotters because
   (a) they are made of a stronger material
   (b) their head creates a wedging action when bent
   (c) They have a longer shank
   (d) they require a smaller slot
3. A disadvantage of cotter joints is
   (a) high load capacity
   (b) simple design and assembly
   (c) susceptible to becoming loose due to vibration
   (d) suitable for high-precision applications
4. Knuckle joints are suitable for
   (a) high-speed applications (b) high compressive loads
   (c) flexible connections (d) precise positioning
5. In a knuckle joint, the pin connecting the two links experiences
   (a) pure bending moment (b) pure shear force
   (c) combined bending and shear force (d) pure tensile force
6. The knuckle pin in a knuckle joint is typically made of
   (a) wood (b) plastic
   (c) high-strength steel (d) rubber
7. The main function of a turnbuckle is
   (a) to create a fixed joint (b) to adjust the tension in a cable or rod
   (c) to transmit torque (d) to limit movement in one direction
8. A turnbuckle consists of
   (a) two threaded rods connected by a pin
   (b) a welded eye at each end and a central body

(c) two threaded rods with a coupling mechanism
(d) a solid bar with a tapered end on each side

9. Turnbuckles are commonly used in applications like
   (a) bearings (b) gears
   (c) cables and tie rods (d) electrical connections

10. When designing a hand lever, an important consideration is
    (a) material strength only (b) strength, ergonomics, and ease of operation
    (c) lever length alone (d) manufacturing cost only

11. To increase the mechanical advantage (ratio of output force to input force) of a lever, you should
    (a) decrease the effort arm length
    (b) increase the load arm length
    (c) use a thicker material for the lever
    (d) decrease the distance between the fulcrum and the applied force

12. The distance from the fulcrum to the point of application of force on a lever is called the
    (a) moment arm (b) effort arm
    (c) load arm (d) mechanical advantage

13. Sliding contact bearings experience
    (a) high friction and wear (b) smooth and low-friction operation
    (c) limited load capacity (d) all of the above

14. Rolling contact bearings, like ball bearings, offer
    (a) high starting torque (b) low friction and high efficiency
    (c) ability to handle misalignment (d) all of the above

15. In a bearing, the inner ring that fits on the shaft is called the
    (a) outer race (b) inner race
    (c) ball cage (d) separator

16. The radial clearance in a bearing refers to the
    (a) tightness of the fit between the balls and races
    (b) play between the inner and outer rings
    (c) material of the balls
    (d) lubrication method used

17. The life of a ball bearing (how long it lasts) is inversely proportional to the
    (a) shaft diameter (b) applied load
    (c) bearing type (d) lubrication quality

18. Examples of sliding contact bearings include
    (a) ball bearings and roller bearings
    (b) bushings and plain bearings
    (c) needle bearings and thrust bearings
    (d) all of the above
19. Rolling contact bearings offer several advantages over sliding contact bearings. Which is NOT an advantage?
    (a) lower friction and wear
    (b) higher efficiency and longer life
    (c) ability to handle high shock loads
    (d) reduced maintenance requirements
20. The basic static load rating (C_0) of a bearing represents
    (a) maximum speed at which the bearing can operate
    (b) maximum static load the bearing can withstand
    (c) fatigue life of the bearing
    (d) recommended lubrication method
21. The basic dynamic load rating (C) of a bearing indicates
    (a) maximum allowable misalignment
    (b) load capacity for a specified fatigue life
    (c) material of the bearing components
    (d) recommended maintenance schedule
22. Limiting speed for a ball bearing refers to the
    (a) maximum permissible rotational speed
    (b) minimum lubrication requirement
    (c) recommended replacement interval
    (d) type of sealing used
23. When selecting a ball bearing from a manufacturer's catalogue, the following factors are considered
    (a) price only
    (b) bearing type, size, load capacity, and speed
    (c) brand reputation only
    (d) availability of a local supplier
24. Manufacturer's catalogues provide information on
    (a) Installation instructions only
    (b) Bearing dimensions, capacities, and limitations
    (c) Troubleshooting tips for bearing failures
    (d) Recommended lubricants for specific applications
25. In addition to bearing type, size, and load capacity, another important factor to consider when selecting a ball bearing is
    (a) color of the bearing housing
    (b) lubrication method and sealing requirements
    (c) manufacturer's logo design
    (d) the date the bearing was manufactured

Answers to Multiple Choice Questions

1. (a) 2. (b) 3. (c) 4. (b) 5. (c) 6. (c) 7. (b) 8. (c) 9. (c) 10. (b) 11. (b) 12. (c) 13. (a) 14. (b) 15. (b) 16. (b) 17. (b) 18. (b) 19. (c) 20. (b) 21. (b) 22. (a) 23. (b) 24. (b) 25. (b)

Short and Long Answer Type Questions

1. Describe the primary function of a cotter joint.
2. Explain the working principle of a cotter joint. Discuss the advantages and disadvantages of using cotter joints compared to other fastening methods. Include real-world examples of applications where cotter joints are preferred.
3. How does an upset cotter provide a tighter fit compared to a regular cotter?
4. Compare and contrast the design and functionality of regular cotters and upset cotters. Explain how the wedging action of an upset cotter creates a stronger and more secure joint.
5. What type of loads are knuckle joints best suited for?
6. Describe the construction and key features of a knuckle joint. Explain how the design of a knuckle joint allows it to handle high compressive loads. Provide a comparison between knuckle joints and cotter joints in terms of load capacity and applications.
7. Briefly describe two applications where knuckle joints are commonly used.
8. Discuss two specific applications where knuckle joints are crucial components. Explain the forces acting on the knuckle joint in each application and how the joint design facilitates efficient load transfer.
9. What is the main purpose of a turnbuckle?
10. Explain the mechanism of a turnbuckle and how it allows for tension adjustment. Discuss different turnbuckle designs and their suitability for various applications. Include real-world examples of where turnbuckles are used.
11. Provide an example of a situation where a turnbuckle would be a critical component.
12. Select a specific application (e.g., rigging, musical instrument tuning) where turnbuckles play a vital role. Explain how turnbuckles ensure proper tensioning and how failure of a turnbuckle could impact the system's performance or safety.
13. List three important factors to consider when designing a hand or foot lever.
14. Discuss the design considerations for hand and foot levers. Explain how factors like lever material, strength, ergonomics, and placement of the fulcrum, effort arm, and load arm influence the lever's functionality and user comfort. Provide examples of good and bad lever design practices.
15. How does the length of the effort arm and load arm on a lever affect the mechanical advantage?

16. Explain the concept of mechanical advantage in levers. Discuss the relationship between the lengths of the effort arm and load arm and how it affects the force required to operate the lever. Include calculations or diagrams to illustrate your explanation.
17. What is the main drawback of using sliding contact bearings?
18. Describe the principle of operation of sliding contact bearings. Explain the limitations of sliding contact bearings, including factors that contribute to friction and wear. Discuss different types of sliding contact bearings and their applications.
19. How do rolling contact bearings achieve lower friction compared to sliding contact bearings?
20. Explain the working principle of rolling contact bearings, such as ball bearings, and how they achieve lower friction compared to sliding contact bearings. Discuss the advantages and disadvantages of rolling contact bearings and provide examples of their applications in various machinery.
21. Give an example of an application where each type of bearing (sliding and rolling contact) might be preferable.
22. Define the term "clearance" in relation to bearings.
23. Explain the concept of clearance in bearings. Discuss the different types of clearance (e.g., radial clearance, axial clearance) and their impact on bearing performance, lubrication requirements, and lifespan.
24. Differentiate between the inner and outer race of a bearing.
25. Describe the components of a rolling contact bearing, such as the inner race, outer race, balls, and cage. Explain the function of each component and how they work together to provide smooth and efficient operation.

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NPTEL VIDEOS

1. Lecture - 14 Design of Fasteners - II - Design of Machine Elements - I by Prof.B.Maiti, Department of Mechanical Engineering, IIT Kharagpur.

Design of Fasteners - II

2. Lecture - 13 Design of Fasteners – I - Design of Machine Elements - I by Prof.B.Maiti, Department of Mechanical Engineering, IIT Kharagpur.

Design of Fasteners – I

3. Lecture - 13 Design of Power Screw - Design of Machine Elements - I by Prof.B.Maiti, Department of Mechanical Engineering, IIT Kharagpur.

Design of power screws

4. Lecture 28 Rolling Element Bearings - Tribology by Dr. Harish Hirani, Department of Mechanical Engineering, IIT Delhi.

Rolling Element Bearings 1 — Lecture 28

Rolling Element Bearings 2 — Lecture 29

Rolling Element Bearings 3 — Lecture 30

5. Lecture 31 Selection of Rolling Element Bearings - Tribology by Dr. Harish Hirani, Department of Mechanical Engineering, IIT Delhi.

Selec. Roll. Elem. Bea.

6. Lecture 31 Friction of Rolling Element Bearing - Tribology by Dr. Harish Hirani, Department of Mechanical Engineering, IIT Delhi.

Fric. Roll. Elem. Bea.

3 Design of Shafts, Keys, Couplings and Spur Gears

UNIT SPECIFICS

Through this unit, the following aspects are discussed:

- Shafts – types, materials, sizes
- Design of shafts – strength and rigidity criteria
- American Society of Mechanical Engineers (ASME) code of design for line shafts
- Keys – design, effect of key ways on strength of the shaft
- Design of couplings – muff, protected type flange, bush-pin type flexible coupling
- Spur gear – design, Lewis equation, power transmission capacity

RATIONALE

The third unit of this book helps the students to have good understanding of the type of shafts used in the industrial applications. After completing this course, a student will be able to choose a suitable material and size to design a shaft using strength and rigidity criteria according to its properties and applications. Keys are essential components in power transmission systems, serving to securely connect rotating elements. This unit also presents the design of keys and effect of keyways on strength of the shaft for effective power transmission between shafts. By learning this unit, students will be able to design the various couplings and spur gears used for power transmission. Through this unit, the students will be able to use the Lewis equation for evaluating the beam strength and also determine power transmission capacity of spur gear in bending.

PRE-REQUISITES

Strength of materials, Engineering mechanics

UNIT OUTCOMES

List of outcomes of this unit is as follows:

On the successful completion of the unit, students will be able to

U3-O1: Analyze and select the types of shafts, shaft materials and size of the shaft for various machine and industrial applications.

U3-O2: Design the solid and hollow shafts using the strength and rigidity criteria.

U3-O3: Design and analyze the keys for power transmission between the shafts and effect of keyways on strength of the shaft.

U3-O4: Design the various couplings used to connect rotating shafts and transmit the power.

U3-O5: Design the spur gear and analyze the power transmission capacity of the spur gears in bending

Unit 3 Outcomes	**Mapping with Course Outcomes** (1 – weak correlation, 2 – medium correlation, 3 – strong correlation)				
	CO-1	**CO-2**	**CO-3**	**CO-4**	**CO-5**
U3-O1	**3**	**3**	**1**	**3**	**3**
U3-O2	**3**	**3**	**1**	**3**	**3**
U3-O3	**3**	**3**	**2**	**3**	**3**
U3-O4	**3**	**3**	**1**	**3**	**3**
U3-O5	**3**	**3**	**1**	**3**	**3**

3.1. INTRODUCTION

A Shaft is a rotating element, circular in cross section, which is used in rotating machinery to transmit rotary motion and torque from one location to another location. A shaft with keyway is shown in Fig. 3.1.

Fig. 3.1: Shaft with keyway

The shaft receives power through a tangential force, and the resulting torque or twisting moment generated within the shaft enables the transfer of power to different machines connected to the shaft. The various members such as pulleys, gears etc., are mounted in the shaft to transfer the power from one shaft to another. The presence of these elements, along with the forces applied to them, induces bending in the shaft. The shaft supports different components, which are attached to shaft through keys or splines.

Shafts

3.2. TYPES OF SHAFTS

Mechanical shafts are generally classified into four broad types as shown in Fig. 3.2:

(a) (b) (c) (d)

Fig. 3.2: Types of shafts (a) Transmission shaft (b) Axle shaft (c) Spindle shaft (d) Machine shaft

(i) Transmission shafts: A transmission shaft stands as a crucial component in machinery which is used to transmit power between the power source and the machines to be operated.

(ii) Axle shafts: An axle is a stationary form of a shaft that provides support for rotating pulleys and wheels without transmitting any torque itself. Axle shaft acts as a support for rotating body.

(iii) Spindle shafts: A spindle is a rotating shaft equipped with a fixture designed to hold a tool (or work piece in the context of milling, grinding, or drilling spindles) or, in the case of a turning spindle, the work piece itself. The spindle shaft serves as a support, positioner, and rotational drive for the tool or work piece.

(iv) Machine shafts: These shafts constitute an integral part of the machine, situated within the assembly. An illustration of a machine shaft is the crankshaft in an automobile engine. Here, we considered two categories for calculating the torque required to overcome the friction viz. (1) torque required to raise the load and (2) torque required to lower the load.

3.3. SHAFT MATERIALS

The material properties of mechanical shafts play a crucial role in determining their performance and suitability for specific applications. The shaft material should have the following properties:

- High strength
- Good machinability
- Low notch sensitivity factor
- Good heat treatment properties
- High wear resistance properties

The essential strength required to withstand loading stresses significantly influences the selection of materials and their treatments. Numerous shafts are crafted from low carbon, either cold-drawn or hot-rolled steel.

The material employed for shafts consists of carbon steel grades, including $40C_8$, $45C_8$, $50C_4$, and $50C_{12}$. Standard transmission shafts are composed of medium carbon steels, like $30C_8$ or $40C_8$, which have a carbon content ranging from 0.15 to 0.40 percent. It's usual to refer to these steels as machinery steels. Alloy steels or high carbon steels like 45C8 or 50C8 are used when increased strength is needed. Nickel,

nickel-chromium, and molybdenum steels are examples of alloy steels. Transmission shafts are commonly made from the alloy steel grades $16Mn_5Cr_4$, $40Cr_4Mo_2$, $16Ni_3Cr^2$, $35Ni_5Cr^2$, $40Ni_6Cr^4Mo_2$, and $40Ni_{10}Cr^3Mo_6$. The cost of alloy steels is higher than that of regular carbon steels. Alloy steels, however, are stronger, harder, and more resilient. Components with large section diameters can also attain high values of strength and hardness. When it comes to corrosion resistance, alloy steels outperform regular carbon steels. Thus, in certain situations, these benefits outweigh the alloy steel's greater price.

Low carbon steels are used to manufacture commercial shafts, which are produced through hot rolling and then sized either by cold drawing or by turning and grinding. Cold drawing yields a stronger shaft compared to hot rolling. However, cold drawn shafts have some drawbacks; their diameter and straightness tolerances are less precise than those of shafts finished through turning and grinding. Additionally, residual tensions are created at and around the shaft's surface by cold drawing. The residual stresses partially release during machining operations such as milling and slotting, which are necessary to form the key slot. This results in the distortion of the shaft. An intricate and costly procedure involves straightening a twisted and deformed shaft. As a result, the majority of transmission shafts undergo turning and grinding following hot rolling. To get the needed strength and hardness, they are further toughened by oil-quenching. Commercially, steel bars with a diameter of up to 200 mm are available. Billets are forged into bars for very large sizes, and standard turning and grinding procedures are used to finish them. Commercial shafts available in standard sizes and are used for general engineering and structural applications. The Standard diameters (in mm) of steel bars used for structural and general engineering purposes are 5, 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 90, 100, 110, 120, 140, 160, 180, 200. The standard length of the shafts are 5 m, 6 m and 7 m.

3.4. DESIGN OF SOLID SHAFTS USING STRENGTH CRITERIA

Axial tensile force, bending moment, torsional moment, or their combinations can be applied to a transmission shaft. Majority of transmission shafts experience both bending and torsional moments at the same time. The process of designing a transmission shaft involves figuring out the proper shaft diameter based on stiffness and strength factors. The tensile stress of a shaft under axial tensile force is determined by

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$$\sigma_t = \frac{F}{\left(\frac{\pi d^2}{4}\right)} \quad \Rightarrow \quad \sigma_t = \frac{4F}{\pi d^2} \tag{3.1}$$

When the shaft is subjected to pure bending moment, the bending stresses are given by,

$$\sigma_b = \frac{M_b r}{I} = \frac{M_b\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{64}\right)} \quad \Rightarrow \quad \sigma_b = \frac{32 M_b}{\pi d^3} \tag{3.2}$$

When the shaft experiences a purely torsional moment, the torsional shear stress can be expressed as:

$$\tau = \frac{M_t r}{J} = \frac{M_t\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{32}\right)} \quad \Rightarrow \quad \tau = \frac{16 M_t}{\pi d^3} \tag{3.3}$$

Building Mohr's circle yields the principal stress σ_1, and principal shear stress $\tau_{\max}$, when the shaft is subjected to a combination of stresses. The normal and shear stresses are denoted as σ_x and τ respectively. To calculate the normal stress σ_x, two cases are considered.

Case I: The shaft is subjected to a combination of axial force, bending moment and torsional moment, the normal stress σ_x is as follows,

$$\sigma_x = \sigma_t + \sigma_b \tag{3.4}$$

Case II: The shaft is subjected to a combination of bending and torsional moments without any axial force, the normal stress σ_x is as follows,

$$\sigma_x = \sigma_b \tag{3.5}$$

The values of σ_t and σ_b are calculated from Eqs. (3.1) and (3.2) respectively.

The following expressions are derived to calculate the principal stress σ_1, and principal shear stress $\tau_{\max}$, using Mohr's Circle,

$$\text{Principal stress } \sigma_1 = \left(\frac{\sigma_x}{2}\right) + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{3.6}$$

$$\text{Principal shear stress } \tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{3.7}$$

Fundamental formulae for shaft design are used from Eqs. (3.1) to (3.7). It is not required to employ all of these equations every time, though. Combining the aforementioned equations can result in the development of simple formulae for shaft design. Either maximum principal stress theory or maximum shear stress theory can be used to design the shaft. These theories will be utilized for transmission shafts that experience both torsional and bending moments.

1. ***Maximum principal stress theory:*** σ_1 is denoted as the maximum principal stress. Since, the shaft is subjected to bending and torsional moments without any axial force, the normal stress σ_x is as follows,

$$\sigma_x = \sigma_b = \frac{32M_b}{\pi d^3} \tag{3.8}$$

Also,

$$\tau = \frac{16M_t}{\pi d^3} \tag{3.9}$$

Substituting Eqs. (3.8) and (3.9) in Eq. (3.6),

$$\sigma_1 = \frac{16M_b}{\pi d^3} + \sqrt{\left(\frac{16M_b}{\pi d^3}\right)^2 + \left(\frac{16M_t}{\pi d^3}\right)^2}$$

$$\sigma_1 = \frac{16}{\pi d^3} + \left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right] \tag{3.10}$$

The permissible value of maximum principal stress is given by,

$$\sigma_1 = \frac{S_{yt}}{FOS} \tag{3.11}$$

Eqs. (3.10) and (3.11) are used to determine shaft diameter on the basis of principal stress theory. According to experimental research, the maximum principal stress theory provides accurate predictions for brittle materials. Since ductile materials like steel are used to make shafts, this theory does not apply to shaft design.

2. ***Maximum shear stress theory:*** $\tau_{\max}$ is denoted as principal shear stress. Substituting Eqs. (3.8) and (3.9) in Eq. (3.7),

$$\tau_{\max} = \sqrt{\left(\frac{16M_b}{\pi d^3}\right)^2 + \left(\frac{16M_t}{\pi d^3}\right)^2}$$

$$\tau_{\max} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2} \tag{3.12}$$

Based on the maximum shear stress theory,

$$S_{sy} = 0.5S_{yt} \tag{3.13}$$

The permissible value of maximum shear stress is given by,

$$\tau_{\max} = \frac{S_{sy}}{FOS} = \frac{0.5S_{yt}}{FOS} \tag{3.14}$$

Using the maximum shear stress theory, the shaft diameter is calculated using Eqs. (3.12) and (3.14).

For ductile materials, the maximum shear stress theory is appropriate. Applying this theory to shaft design makes more sense than basing the shaft design on primary stress theory, as the shafts are composed of ductile materials. In a summary, rewrite Eqs. (3.12) and (3.10),

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2} \tag{3.15}$$

$$\sigma_1 = \frac{16}{\pi d^3} + \left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right] \tag{3.16}$$

Where $\sqrt{(M_b)^2 + (M_t)^2}$ is equivalent torsional moment, $\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right]$ is equivalent bending moment. When designing shafts, the equivalent torsional moment is applied based on the maximum shear stress theory of failure. Based on the maximum principal stress theory of failure, the equivalent bending moment is applied in the design of shafts.

Example 3.1: A solid shaft is subjected to a bending moment of $3460\ kN-mm$ and a torque of $1150\ kN-mm$. Determine the diameter of the shaft given the factor of safety as 6 and assuming ultimate bending stress as $690\ N/mm^2$ and ultimate shear stress as $516\ N/mm^2$.

Given Data:

Bending moment $M_b = 3460\ kN-mm = 3460\times10^3\ N-mm$

Torque $M_t = 1150\ kN-mm = 1150\times10^3\ N-mm$

Ultimate bending stress $\sigma_{tu} = 690\ N/mm^2$

Ultimate shear stress $\tau_u = 516\ N/mm^2$

Factor of safety $FOS = 6$

Find:

1. Diameter of the shaft d

Solution:

We know that the allowable bending stress,

$$\sigma_b = \frac{\sigma_{tu}}{FOS} = \frac{690}{6} = 115\,N/mm^2$$

and allowable shear stress, $\tau = \frac{\tau_u}{FOS} = \frac{516}{6} = 86\,N/mm^2$

Based on the maximum shear stress theory, equivalent twisting moment,

$$M_{te} = \sqrt{M_b^2 + M_t^2} = \sqrt{\left(3460\times10^3\right)^2 + \left(1150\times10^3\right)^2} = 3.646\times10^6\,N-mm$$

Also, equivalent twisting moment $M_{te} = \frac{\pi}{16}\tau d^3$

$$3.646\times10^6 = \frac{\pi}{16}\times 86\times d^3$$

$$d^3 = \frac{16\times3.646\times10^6}{86\pi}$$

$$d = \sqrt[3]{\frac{16\times3.646\times10^6}{86\pi}} == 60\ mm$$

Based on the maximum normal stress theory, equivalent bending moment,

$$M_{be} = \frac{1}{2}\left(M_b + \sqrt{M_b^2 + M_t^2}\right) = \frac{1}{2}\left(M_b + M_{te}\right)$$

$$= \frac{1}{2}\left(3460\times10^3 + 3.646\times10^6\right)$$

$$= 3.553\times10^6\ N-mm$$

Also, equivalent bending moment $M_{be} = \dfrac{\pi}{32}\sigma_b d^3$

$$3.553\times10^6 = \frac{\pi}{32}\times115\times d^3$$

$$d^3 = \frac{32\times3.553\times10^6}{115\pi}$$

$$d = \sqrt[3]{\frac{32\times3.553\times10^6}{115\pi}} = 68\ mm$$

Take the highest value of the two values, we have

$d = 68\ mm$ say $= 70\ mm$

Example 3.2: A mild steel shaft transmits $40\ kW$ power at $280\ rpm$. Maximum torque transmitted exceeds mean torque by 20%. Maximum shear stress is $60\,N/mm^2$. Compute the diameter of the shaft.

Given Data:

Power transmitted $P = 40\ kW\ = 4000\ W$

Speed of the shaft $N = 280\ rpm$

Maximum torque $M_t = 20\%$ excess of mean torque

Maximum shear stress $\tau_{\max} = 60\,N/mm^2$

Find:

1. Diameter of the shaft d

Solution:

We know that the maximum shear stress, $\tau_{\max} = \dfrac{16(M_t)_{mean}}{\pi d^3}\ N/mm^2$

The torque transmitted by the solid shaft is,

$$(M_t)_{mean} = \frac{60P}{2\pi N}\times 10^3\, N-mm = \frac{60\times 4000}{2\pi\times 280}\times 10^3\, N-mm$$

$$(M_t)_{mean} = 1.36\times 10^5\ N-mm$$

Maximum Torque $M_t = 1.2\times (M_t)_{mean} = 1.2\times 1.36\times 10^5 = 1.63\times 10^5\ N-mm$

Then, the diameter of the shaft,

$$d^3 = \frac{16M_t}{\pi\tau_{\max}} = \frac{16\times 1.63\times 10^5}{\pi\times 60} = 13{,}852.846$$

$$d = \sqrt[3]{13{,}852.846} = 24\ mm$$

The diameter of the shaft $d = 24\ mm$ say 25 mm.

3.5. DESIGN OF HOLLOW SHAFTS USING STRENGTH CRITERIA

The following formula provides the shaft's torsional shear stress and bending stresses:

Torsional shear stress $\tau = \dfrac{M_t r}{J}$ (3.17)

Torsional bending stress $\sigma_b = \dfrac{M_b r}{I}$ (3.18)

Fig. 3.3 displays the distribution of torsional shear stress and bending stresses. At the center of the shaft, it is noted that there are no bending or torsional shear stresses ($r = 0$ and $y = 0$) and negligibly small nearer to the center of the shaft, where the radius is small. The resisting stresses due to external bending and torsional moments increase, when the radius increases. As a result, outer fibers are better at withstanding applied moments.

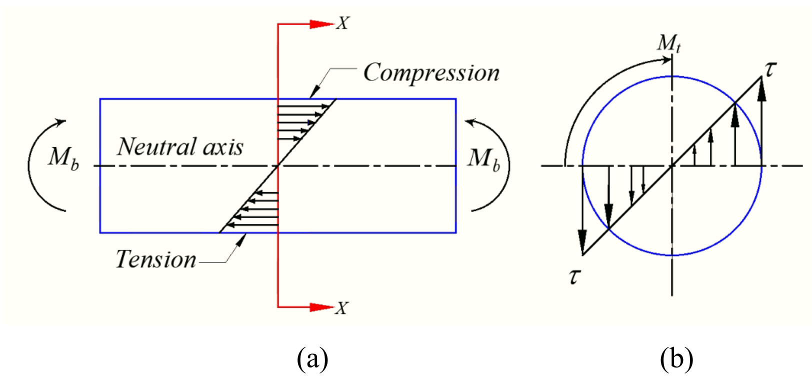


Fig. 3.3: (a) Distribution of bending stresses (b) Distribution of torsional shear stresses

The material from the center of hollow shafts is removed and redistributed to a larger radius, which enhances their strength compared to solid shafts of the same weight. Here are the benefits of hollow shafts over solid shafts.

1. Hollow shafts display greater stiffness than solid shafts of equal weight.
2. The natural frequency of a hollow shaft is higher than that of a solid shaft of the same weight.

The disadvantages of hollow shaft is,

3. It is costlier than solid shaft
4. It requires more space, since the diameter of the hollow shaft is more

Hollow shafts are used in bore well and deep hole drilling to allow coolants and control cables to pass through. They are also used for railway wagon axles and car propeller shafts. Epicyclic gearboxes, in which one shaft spins inside another, is one of its applications. The procedure of extrusion is typically used to make hollow shafts. Hollow shafts are subjected to axial force, bending moment, torsional moment or combination of these loads. Therefore, it is crucial to determine the appropriate inner and outer diameters from both strength and rigidity perspectives when designing a hollow shaft.

Assume, $C = \frac{d_i}{d_o}$ where d_i is inside diameter of the hollow shaft (mm), d_o is outside diameter of the hollow shaft (mm), C is ratio of inside diameter to outside diameter.

The tensile stress of a shaft under axial tensile force is determined by

$$\sigma_t = \frac{F}{\frac{\pi}{4}\left(d_o^2 - d_i^2\right)} = \frac{F}{\frac{\pi}{4}\left(d_o^2 - C^2 d_o^2\right)} \Rightarrow \sigma_t = \frac{4F}{\pi d_o^2\left(1 - C^2\right)} \tag{3.19}$$

The bending stress of a shaft under bending moment is determined by

$$\sigma_b = \frac{M_b y}{I} \tag{3.20}$$

For hollow shaft,

$$I = \frac{\pi\left(d_o^4 - d_i^4\right)}{64} = \frac{\pi\left(d_o^4 - C^4 d_o^4\right)}{64}$$

$$I = \frac{\pi d_o^4\left(1 - C^4\right)}{64} \tag{3.21}$$

and $y = \frac{d_o}{2}$ (3.22)

Substituting Eqs. (3.21) and (3.22) in Eq. (3.20)

$$\sigma_b = \frac{32 M_b}{\pi d_o^3\left(1 - C^4\right)} \tag{3.23}$$

The torsional shear stress of a shaft under a pure torsional moment is determined by,

$$\tau = \frac{M_t r}{J} \tag{3.24}$$

For hollow shaft,

$$J = \frac{\pi\left(d_o^4 - d_i^4\right)}{32} = \frac{\pi\left(d_o^4 - C^4 d_o^4\right)}{32}$$

$$J = \frac{\pi d_o^4\left(1 - C^4\right)}{32} \tag{3.25}$$

and $\quad r = \dfrac{d_o}{2}$ (3.26)

Substituting Eqs. (3.25) and (3.26) in Eq. (3.24)

$$\tau = \frac{16 M_t}{\pi d_o^3\left(1 - C^4\right)} \tag{3.27}$$

Building Mohr's circle diagram for hollow shaft, similar to solid shaft, the normal and shear stresses are denoted as σ_x and τ respectively. To calculate the normal stress σ_x, two cases are considered.

Case I: The shaft is subjected to a combination of axial force, bending moment and torsional moment, the normal stress σ_x is as follows,

$$\sigma_x = \sigma_t + \sigma_b \tag{3.28}$$

Case II: The shaft is subjected to a combination of bending and torsional moments without any axial force, the normal stress σ_x is as follows,

$$\sigma_x = \sigma_b \tag{3.29}$$

The following expressions are derived to calculate the principal stress σ_1, and principal shear stress $\tau_{\max}$, using Mohr's Circle,

Principal stress $\sigma_1 = \left(\dfrac{\sigma_x}{2}\right) + \sqrt{\left(\dfrac{\sigma_x}{2}\right)^2 + (\tau)^2}$ (3.30)

Principal shear stress $\tau_{\max} = \sqrt{\left(\dfrac{\sigma_x}{2}\right)^2 + (\tau)^2}$ (3.31)

Either the maximum shear stress theory or the maximum principal stress theory can be used to design the hollow shaft. Applying these failure theories, let us assume that the hollow shaft experiences combined bending and torsional moments in the absence of any axial force.

(i) Maximum principal stress theory:

Substituting Eqs. (3.23) and (3.27) in Eq. (3.30),

$$\sigma_1 = \frac{16M_b}{\pi d_o^3\left(1-C^4\right)} + \sqrt{\left(\frac{16M_b}{\pi d_o^3\left(1-C^4\right)}\right)^2 + \left(\frac{16M_t}{\pi d_o^3\left(1-C^4\right)}\right)^2}$$

$$\sigma_1 = \frac{16}{\pi d_o^3\left(1-C^4\right)} + \left[M_b + \sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}\right] \tag{3.32}$$

The permissible value of maximum principal stress is given by,

$$\sigma_1 = \frac{S_{yt}}{FOS} = \frac{16}{\pi d_o^3\left(1-C^4\right)} + \left[M_b + \sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}\right] \tag{3.33}$$

Based on the maximum principal stress theory, the hollow shaft's outer diameter can be found using Eq. (3.33).

(ii) Maximum shear stress theory:

$\tau_{\max}$ is denoted as principal shear stress.

Substituting Eqs. (3.23) and (3.27) in Eq. (3.31),

$$\tau_{\max} = \sqrt{\left(\frac{16M_b}{\pi d_o^3\left(1-C^4\right)}\right)^2 + \left(\frac{16M_t}{\pi d_o^3\left(1-C^4\right)}\right)^2}$$

$$\tau_{\max} = \frac{16}{\pi d_o^3\left(1-C^4\right)}\sqrt{\left(M_b\right)^2 + \left(M_t\right)^2} \tag{3.34}$$

Based on the maximum shear stress theory,

$S_{sy} = 0.5S_{yt}$

The permissible value of maximum shear stress is given by,

$$\tau_{\max} = \frac{S_{sy}}{FOS} = \frac{0.5S_{yt}}{FOS} = \frac{16}{\pi d_o^3\left(1-C^4\right)}\sqrt{\left(M_b\right)^2 + \left(M_t\right)^2} \tag{3.35}$$

Eqs. (3.10) and (3.32) show that, with the exception of the term $\left(1-C^4\right)$, the formulas for σ_1 for solid and hollow shafts are equivalent. With the exception of the term $\left(1-C^4\right)$, the formulas for $\tau_{\max}$ for solid and hollow shafts are likewise comparable.

Example 3.3: A hollow plain carbon steel shaft is transmitting $50\,kW$ power at 500 rpm. The inside diameter of the hollow shaft is 0.6 times of outside diameter. The permissible shear stress is $84\ N/mm^2$. Calculate the inner and outside diameters of the shaft.

Given Data:

Power transmitted $P = 50\ kW\ = 50000\ W$

Speed of the shaft $N = 500\ rpm$

Inner diameter of the shaft $d_i = 0.6d_o$

Permissible shear stress $\tau = 84\,N/mm^2$

Find:

1. Inner diameter of the shaft d_i
2. Outer diameter of the shaft d_o

Solution:

The torque transmitted by the hollow shaft is,

$$M_t = \frac{60P}{2\pi N}\times 10^3\,N-mm = \frac{60\times 50000}{2\pi\times 500}\times 10^3\,N-mm$$

$$M_t = 9.549\times 10^5\ N-mm$$

Inner and outer diameter of the shaft

$$C = \frac{d_i}{d_o} = 0.6$$

From Eq. (3.27), $\tau = \dfrac{16M_t}{\pi d_o^3\left(1-C^4\right)}$

$$\therefore \quad 84 = \frac{16\times 9.549\times 10^5}{\pi d_o^3\left(1-(0.6)^4\right)}$$

$$d_o = 40.52\ mm$$

Inner diameter $d_i = 0.6d_o = 0.6(40.52) = 24.31\ mm$

3.6. DESIGN OF SOLID SHAFTS USING TORSIONAL RIGIDITY CRITERIA

The shafts are designed based on either lateral rigidity or torsional rigidity in certain applications. If a transmission shaft does not twist excessively in response to an external torque, it is considered to have torsional stiffness. Similar to this, if the transmission shaft does not bend excessively when subjected to external forces and bending moments, it is considered rigid based on its lateral rigidity. It is required to design the shaft based on torsional rigidity, or the allowable angle of twist per meter of shaft length, in specific applications, such as machine tool spindles. The following gives the twist angle θ_r (in radians):

$$\theta_r = \frac{M_t l}{JG} \text{ (in radians)}$$

$$\theta = \frac{180}{\pi} \times \frac{M_t l}{JG} \quad \text{(in degrees)} \tag{3.36}$$

Where

$$J = \frac{\pi d^4}{32} \text{ for solid circular shaft} \tag{3.37}$$

Combining Eqs. (3.36) and (3.37)

$$\theta = \frac{584 M_t l}{G d^4} \tag{3.38}$$

Where θ is angle of twist (in deg.), l is length of shaft subjected to twisting moment (mm), M_t is torsional moment ($N-mm$), G is modulus of rigidity (N/mm^2). Eq. (3.38) is used to design the shaft on the basis of torsional rigidity. The permissible angle of twist for machine tool applications is 0.25° per metre length. For line shafts, 3° per metre length is the limiting value.

Example 3.4: Determine the diameter of the solid steel shaft to transmit 90 kW at 290 rpm, if the angle of twist per meter length is not exceed 0.1^o. The modulus of rigidity of the shaft material is $0.84 \times 10^5 \ N/mm^2$.

Given Data:

Power transmission $P = 90 \ kW$

Speed $N = 290 \ rpm$

Angle of twist $\theta = 0.1^o$ per meter = $0.1 \times \frac{\pi}{180} = 0.00174$ radians

Length of the shaft $l = 1m = 1000$ mm

Modulus of rigidity $G = 0.84 \times 10^5 \ N/mm^2$

Find:

1. The diameter of the solid shaft d.

Solution:

From torsion equation,

$$\frac{M_t}{J} = \frac{G\theta}{l}$$

$$\therefore \quad J = \frac{M_t l}{G\theta} \tag{i}$$

Power transmitted $P = \frac{2\pi N M_t}{60}$

Torque on the shaft $M_t = \frac{P \times 60}{2\pi N} = \frac{90 \times 10^3 \times 60}{2\pi \times 290} = 2965\, N-m$

$M_t = 2965 \times 10^3\, N-mm$

For solid shaft, polar moment of inertia $J = \frac{\pi}{32} d^4$

Substitute the value of J and M_t in Eq. (i),

$$\frac{\pi}{32} d^4 = \frac{M_t l}{G\theta}$$

$$\frac{\pi}{32} d^4 = \frac{2965 \times 10^3 \times 1000}{0.84 \times 10^5 \times 0.00174}$$

Diameter of the solid shaft $d = 120\ mm$

3.7. DESIGN OF HOLLOW SHAFTS USING TORSIONAL RIGIDITY CRITERIA

The allowable twist angle per shaft meter determines the design of hollow shafts based on torsional rigidity. The following gives the angle of twist θ_r (in radians):

From Eq. (3.36)

$$\theta_r = \frac{M_t l}{JG} \text{ (in radians)}$$

$$\theta = \frac{180}{\pi} \times \frac{M_t l}{JG} \quad \text{(in degrees)}$$

From Eq. (3.37), for hollow circular shaft $J = \frac{\pi d^4}{32} = \frac{\pi\left(d_o^4 - d_i^4\right)}{32} = \frac{\pi d_o^4\left(1 - C^4\right)}{32}$ (3.39)

Combining Eqs. (3.36) and (3.39)

$$\theta = \frac{584 M_t l}{G d_o^4 \left(1 - C^4\right)} \tag{3.40}$$

Eq. (3.40) is used to design the hollow shaft on the basis of torsional rigidity.

Example 3.5: A hollow shaft transmit power $500\ kW$ at 100 rpm. The maximum torque is 25% higher than the mean torque. The allowable shear stress is 60 MPa. The shaft is not to twist more than 1^o in a length of 3m. Find the diameter of the shaft if the external to internal diameter is 8:3. Assume modulus of rigidity as $85\ GPa$.

Given Data:

Power transmitted $P = 500\ kW = 500x10^3\ W$

Speed of the shaft $N = 100\ rpm$

Maximum torque $M_t = 25\%$ higher than mean torque

Inner diameter of the shaft $d_i = \frac{3}{8} d_o$

Length of the shaft $l = 3\ m$

Allowable shear stress $\tau_{\max} = 60\ MPa = 60\, N / mm^2$

Angle of twist $\theta = 1^o / 3m$ length

Modulus of rigidity $G = 85 GPa = 85 \times 10^3\ N / mm^2$

Find:

1. Inner diameter of the shaft d_i
2. Outer diameter of the shaft d_o

Solution:

Power transmitted $P = \dfrac{2\pi N \left(M_t\right)_{mean}}{60}$

$$\left(M_t\right)_{mean} = \frac{60P}{2\pi N} = \frac{60 \times 500 \times 10^3}{2 \times \pi \times 100} = 47 \times 10^3\ \ N - m$$

$$\left(M_t\right)_{mean} = 47 \times 10^6\ \ N - mm$$

Maximum torque transmitted $M_t = 1.2 \times \left(M_t\right)_{mean} = 1.2 \times 47 \times 10^6\ \ N - mm$

We know that,

$$\tau_{\max} = \frac{16 M_t}{\pi d_o^3 \left(1 - C^4\right)}$$

$$d_o^3 = \frac{16M_t}{\pi\tau_{\max}\left(1-C^4\right)} = \frac{16\times 56.4\times 10^6}{\pi\times 60\left(1-0.375^4\right)}$$

$$d_o = \sqrt[3]{4.8\times 10^6} = 158.7 \text{ mm}$$

We know that, the polar moment of Inertia for hollow shaft

$$J = \frac{\pi d_o^4\left(1-C^4\right)}{32} = \frac{3.14\times d_o^4\left(1-0.375^4\right)}{32} = 0.0962\times\left(d_o\right)^4$$

The angle of twist $\theta = \frac{180}{\pi}\times\frac{M_t l}{JG} == \frac{180}{3.14}\times\frac{56.4\times 10^6\times 3000}{0.0962\left(d_o\right)^4\times 85\times 10^3}$

$$\left(d_o\right)^4 = \frac{180}{3.14}\times\frac{56.4\times 10^6\times 3000}{0.0962\times 1\times 85\times 10^3} = 12\times 10^8$$

$$d_o = \sqrt[4]{12\times 10^8} = 186.12\ mm$$

Taking larger of two values, the outer diameter of the hollow shaft is 186.12 mm say 190 mm

The inner diameter of the hollow shaft $d_i = \frac{3}{8}d_o = \frac{3}{8}\times 190 = 71.25\ mm$.

3.8. ASME CODE FOR THE DESIGN OF TRANSMISSION SHAFTS

The design of transmission shafts in accordance with the ASME Code follows the maximum shear stress theory. This approach considers the impact of fatigue and shock by incorporating two constants such as K_m and K_t into the maximum shear stress equation. Based on the ASME code, the bending moment M_b and twisting moment M_t are multiplied by factors K_m and K_t respectively to account for shock and fatigue in operating condition. Therefore, Eq. (3.35) can be rearranged as,

$$d_o^3 = \frac{16}{\pi\tau_{\max}\left(1-C^4\right)}\sqrt{\left(K_m M_b + \frac{\alpha F d_o\left(1+C^2\right)}{8}\right)^2 + \left(K_t M_t\right)^2} \tag{3.41}$$

Where $\alpha = \frac{1}{1-0.0044\left(\frac{L}{k}\right)}$ for $\frac{L}{k} < 115$

L is length of the shaft, k is radius of gyration of the shaft, $k = \frac{d_o^2}{16}\left(1+C^2\right)$

$$\alpha = \frac{\sigma_{oc}\left(\frac{L}{k}\right)^2}{\pi^2 nE} \quad \text{for } \frac{L}{k} > 115$$

σ_{oc} is compression yield stress, $n=1$ for hinged ends; $n=2.25$ for fixed ends; $n=1.6$ for both ends pinned. The recommended values of K_m and K_t are listed in Table 3.1 for various load conditions.

Table 3.1: Shock and fatigue factors for the ASME code

Nature of Load	K_m	K_t
Stationary Shafts:		
Gradually applied load	1.0	1.0
Suddenly applied load	1.5-2.0	1.5-2.0
Rotating Shafts:		
Gradually applied or steady load	1.5	1.0
Suddenly applied loads with minor shocks only	1.5-2.0	1.0-1.5
Suddenly applied loads with heavy shocks	2.0-3.0	1.5-3.0

When designing transmission shafts, ASME regulation states that the maximum permissible shear stress may be taken as

(a) 55 N/mm^2 for ordinary steel shaft

(b) 42 N/mm^2 for shafts with keyway

3.9. KEYS

A key is a machine element which is used to connect transmission shafts with rotating elements such as gears, flywheels and pulleys. A key assembly consists of shaft, hub and key, is shown in Fig. 3.4.

The key is accommodated by grooves, or keyways, in the rotating shaft, hub, or both. The basic functions of the key are:

1. Transmit the power from shaft to the hub of the mating element and vice versa
2. Prevent the slip between shaft and mating element like gears, pulley etc.

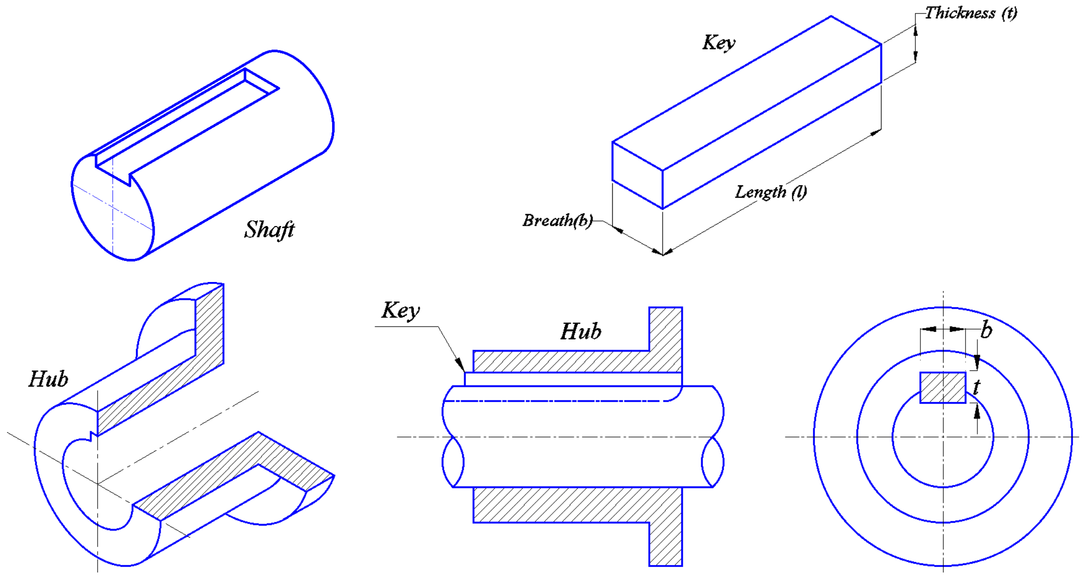


Fig. 3.4: Key joint

A keyway is a slot or recess that has been cut into the hub or shaft to hold the key. The keyway is machined using vertical or horizontal milling machine. Because of the keyway, the shaft experiences stress concentration and the part becomes weak. Generally, keys are manufactured from mild steel or plain carbon steel like 45C8 or 50C8 in order to withstand shear and compressive stresses resulting from transmission of torque**.**

The keys are classified as,

1. *Sunk key:* Sunk key is a key in which the half of the thickness covered in the keyway of the shaft and remaining half in keyway of the hub or boss of the pulley. Sunk key transmits the power due to shear resistance of the key and there is no slip around the shaft. Hence, it is used for heavy duty applications.
2. *Saddle key:* Saddle key is a key which covers keyway of the hub only. It is likely to slip around the shaft. Hence, it is used for comparatively light loads.
3. *Tangent key:* Tangent keys are fitted in pair at right angles where each key withstands torsion in one direction only. Tangent keys are used in high-torque applications.
4. *Round key:* A round key fits into holes drilled partially through the shaft and partially through the hub. For low power drives, round keys are considered to be the most suitable.
5. *Splines:* Keys are integral parts of the shaft that fits into the keyways drilled into the hub. Such a kind of shaft is called as splined shaft. Compared to shafts with a single keyway, splined shafts are comparatively stronger. When the force to be transmitted is large relative to the shaft's size, like in sliding gear and car gearboxes, splined shafts are employed.

3.9.1. Design of sunk keys

A sunk key is a key in which half of the key fits into the keyway of the pulley and the remaining half resides in the keyway of the shaft.

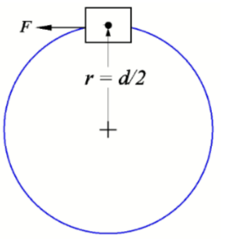


Fig. 3.5: Radial distance of shaft with key

Let M_t = torque transmitted by the shaft

d = shaft diameter

F = tangential force at shaft radius

τ, σ_c = permissible stresses in shear and crushing respectively

Then $M_t = F \times \frac{d}{2}$ (Fig. 3.5)

$$\therefore \quad F = \frac{2M_t}{d} \tag{3.42}$$

(i) Considering shear strength of key:

Due to power transmission, shearing and crushing failures of the key may take place.

Let l = length of the key; b = width of the key; t = thickness of the key

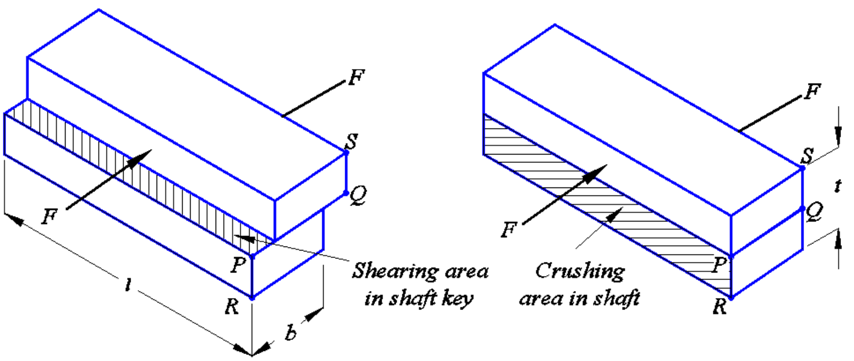
Area in shear = lb (Refer Fig. 3.6 (a))

$$\text{Shear stress induced in the key} = \frac{F}{lb} = \frac{2M_t}{lbd} \leq \tau \tag{3.43}$$

(ii) Considering crushing strength of key:

The key may crush either in the shaft or in the hub.

Area in crushing = $\frac{lt}{2}$ (Refer Fig. 3.6 (b))



(a) (b)

Fig. 3.6: (a) Shearing area of the key (b) Crushing area of the key

$$\text{Crushing stress induced in key} = \frac{2F}{lt} = \frac{4M_t}{ltd} \leq \sigma_c \tag{3.44}$$

From Eq. (3.43), Shear stress induced in the key $\tau = \frac{F}{lb}$ $\quad\therefore\; F = \tau \times l \times b$

From Eq. (3.44), Crushing stress induced in key $\sigma_c = \frac{2F}{lt}$ $\quad\therefore\; F = \sigma_c \times \frac{l \times t}{2}$

If the key is to be equally strong in shear and crushing, then

$$\tau \times l \times b = \sigma_c \times \frac{l \times t}{2}$$

$$\Rightarrow \frac{2b}{t} = \frac{\sigma_c}{\tau} \tag{3.45}$$

Generally $\sigma_c = 2\tau$. Hence $b = t$, *i.e.* a square key is the best size.

From Eq. (3.43), Shear stress induced in the key $\tau = \frac{2M_t}{lbd}$ $\quad\therefore\; M_t = \tau \times \frac{l \times b \times d}{2}$

Torsional shear strength of the shaft $M_t = \frac{\pi}{16} \times d^3 \times \tau_s$

If the key is to be equally strong in shear as the shaft, then

$$\frac{l \times b \times d}{2} \times \tau = \frac{\pi}{16} \times d^3 \times \tau_s$$

$$l = \frac{2\pi}{16} \times \frac{d^3}{b \times d} \times \frac{\tau_s}{\tau} = \frac{\pi \times d^2 \times \tau_s}{8 \times b \times \tau}$$

The usual proportion for width (b) of the rectangular key is $\frac{d}{4}$

$$l = \frac{4\pi \times d^2 \times \tau_s}{8 \times d \times \tau} = 1.571 \times d \times \frac{\tau_s}{\tau} \tag{3.46}$$

If the material of the key is same as that of shaft, then $\tau = \tau_s$

$$l = 1.571d \tag{3.47}$$

Example 3.6: Design a sunk key to transmit 20 kW power at 600 rpm through 65 mm shaft diameter. The permissible shear and crushing stresses for key material are $60\ N/mm^2$ and $160\ N/mm^2$.

Given Data:

Power transmitted $P = 20\ kW = 20 \times 10^3\ W$

Speed of the shaft $N = 600\ rpm$

Diameter of the shaft $d = 65\ mm$

Permissible shear stress in key $\tau_s = 60\ N/mm^2$

Permissible crushing stress in key $\sigma_c = 160\ N/mm^2$

Find:

1. Design a sunk key

Solution:

Refer Design Data Book, for shaft diameter $d = 65\ mm$

Width of the key $b = 20\ mm$

Thickness of the key $t = 12\ mm$

Power transmitted by the shaft $P = \dfrac{2\pi N M_t}{60}$

Therefore, Torque transmitted $M_t = \dfrac{60P}{2\pi N} = \dfrac{60 \times 20 \times 10^3}{2 \times \pi \times 600}$

$$M_t = \frac{60P}{2\pi N} = \frac{60 \times 20 \times 10^3}{2 \times \pi \times 600} = 318.47\ N-m$$

$$M_t = 318471\ N-mm$$

Considering shearing failure of key,

From Eq. (3.43), Length of key $l > \dfrac{2M_t}{bd\tau_s} = \dfrac{2 \times 318471}{bd\tau_s} = \dfrac{2 \times 318471}{20 \times 65 \times 60} = 8.165\ mm$

Considering crushing failure of key,

From Eq. (3.44), Length of key $l > \dfrac{4M_t}{td\sigma_c} = \dfrac{4 \times 318471}{12 \times 65 \times 160} = 10.2\ mm \simeq 11\ mm$

By comparing two values, length of key $l = 11\ mm$. This value is very low while comparing the width and thickness of key. Normally, the length of the key is same as the length of the hub. The length of the hub is not given. Hence,

Length of the key $l = 4 \times b = 4 \times 20 = 80\ mm$

Width of the key $b = 20\ mm$

Thickness of the key $t = 12\ mm$

Example 3.7: A shaft 38 mm diameter is connected with a pulley by means of key. The maximum shear stress of the shaft is $80\ MPa$. Find the length of the key so that the stress in the key is not to exceed $50\ MPa$ and length of the key is 4 times the width.

Given Data:

Diameter of the shaft $d = 38\ mm$

Maximum shear stress of the shaft $\tau = 80\ MPa = 80\ N/mm^2$

Permissible shearing stress in key $\tau_s = 50\ MPa = 50\ N/mm^2$

Length of the key $l = 4 \times b$

Find:

1. Length of the key l

Solution:

Torque transmitted $M_t = \frac{\pi}{16} d^3 \tau = \frac{\pi}{16}(38)^3 \times 50 = 538432\ N-mm$

Considering shearing failure of key $\tau_s = \frac{2M_t}{lbd} \Rightarrow 50 = \frac{2\times 538432}{4\times b\times b\times 38}$

$$b = \sqrt[2]{\frac{2\times 538432}{4\times 50\times 38}} = 11.9\ mm \simeq 12\ mm$$

Length of the key $l = 4b = 4\times 12 = 48\ mm$

3.10. EFFECT OF KEYWAYS ON STRENGTH OF SHAFT

The keyway cut into the shaft or hub diminishes the shaft's load-carrying capacity due to stress concentration near the keyway corners and a reduction in the shaft's cross-sectional area. Additionally, it decreases the shaft's torsional strength. The weakening effect of the keyway is quantified by the following relation, based on experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) \tag{3.48}$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = $Thickness\ of\ key(t)/2$

It is generally assumed that the strength of a keyed shaft is 75% of that of a solid shaft, which is somewhat higher than the value derived from the aforementioned relation. If the keyway is excessively long and the key is of the sliding type, the angle of twist increases according to a ratio k_θ as described by the following relation:

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right) \tag{3.49}$$

Where k_θ is reduction factor for angular twist.

3.11. DESIGN OF COUPLINGS

A coupling is a machine element that connects two rotating shafts for the purpose of transmitting power. The coupling is an essential element due to the following reasons:

a. To provide a connection between two shafts to transmit energy from the drive side to driven side

b. To provide for easy disassembly during maintenance of mechanical systems

c. To provide a joint between two shafts to facilitate easy transportation

d. To accommodate misalignment between two shafts

e. To absorb and dampen shock loads generated during operation

The lateral or angular misalignment may exist, if the center lines of the respective shafts are not parallel. Based on the condition, either rigid or flexible coupling is used to connect the shafts. The couplings can be classified as shown in Fig. 3.7.

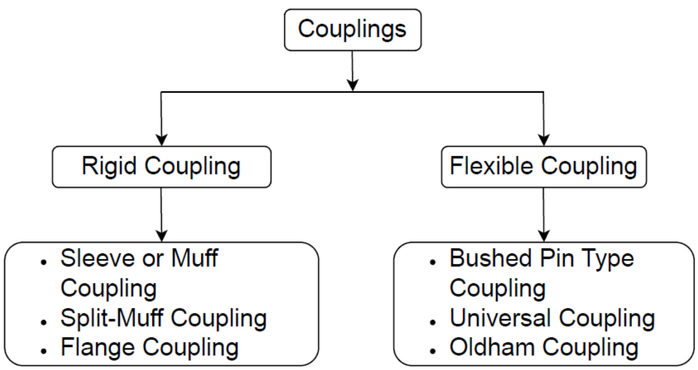


Fig. 3.7: Types of couplings

3.11.1. Design of muff coupling

A muff coupling, also known as a sleeve coupling or box coupling, is a type of rigid coupling. Its construction is illustrated in Fig. 3.8.

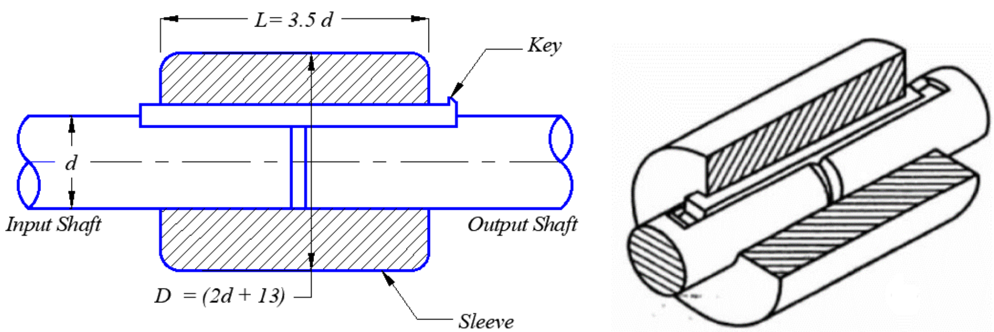


Fig. 3.8: Muff Coupling

The coupling consists of a hollow cylinder that is fitted over the ends of two shafts. Torque is transmitted from the input shaft to the sleeve and then to the output shaft, with the key being an integral part of both the shafts and the sleeve. The muff coupling offers several advantages:

- It is easy to design and manufacture because it consists of just two components: the sleeve and the key.
- There are no projecting parts and the external surface of the sleeve is smooth
- It is compact in size
- Cost is less than other types of coupling

The disadvantages of the muff coupling are:

- It is difficult to assemble or dismantle
- It is rigid coupling and cannot accommodate the misalignment of shafts
- It cannot absorb shocks and vibrations, since there is no flexible element
- It is difficult to install where shaft lengths are very small

Compared to flexible couplings, muff couplings are less common because of these drawbacks. The muff couplings are used on shafts with a diameter up to 70 mm. Steel or cast iron can be used to make the sleeve.

(i) Diameter of the shaft d:

The diameter of the shaft may be calculated from the Eq. (3.50),

$$M_t = \frac{60P}{2\pi N} \text{ and } \tau = \frac{16M_t}{\pi d^3} \tag{3.50}$$

Where M_t is torque transmitted by the shaft, d is diameter of the shaft, τ is permissible shear stress for sleeve material.

(ii) *Dimensions of the sleeve:*

The dimensions of the sleeve are calculated using the empirical relation from Eq. (3.51),

$$D = 2d + 13\ mm;\ L = 3.5d \tag{3.51}$$

Where D represents outer diameter of the sleeve, L represents the length of the sleeve.

The torsional shear stress induced in the sleeve is verified using Eq. (3.52),

$$\tau = \frac{M_t r}{J} \text{ where } J = \frac{\pi\left(D^4 - d^4\right)}{32} \qquad \text{and } r = \frac{D}{2} \tag{3.52}$$

(iii) *Dimensions of the key :*

The width b, and thickness t, of the key are taken from Design Data Book for shaft diameter d. The length of the key $l =$ total length of the sleeve .

$$l = L = 3.5d \tag{3.53}$$

Based on the dimensions of the key, check the shear and compressive stress induced in the key from Eq. (3.43) and (3.44) respectively.

$$\tau = \frac{2M_t}{lbd} \quad \text{and} \quad \sigma_c = \frac{4M_t}{ltd}$$

Example 3.8: Design a muff coupling to connect the two shafts in order to transmit 80 kW power at 400 rpm. The permissible shearing and crushing stresses for shaft and key material are $60\ N/mm^2$ and $100\ N/mm^2$ respectively. The permissible shear stress for muff material is $15\ N/mm^2$. Assume that the maximum torque transmitted is equal to mean torque.

Given Data:

Power transmitted $P = 80\,kW = 80\times 10^3\ Watts$

Speed $N = 400\,rpm$

Permissible shearing stress for the shaft and key $[\tau]_{shaft} = [\tau]_{key} = 60\ N/mm^2$

Permissible crushing stress for the shaft and key $[\sigma_c]_{shaft} = [\sigma_c]_{key} = 100\ N/mm^2$

Permissible shear stress for muff material $[\tau]_{muff} = 15\ N/mm^2$

Maximum torque transmitted $M_t = (M_t)_{mean}$

Find:

1. Design a muff coupling

Solution:

(i) *Diameter of the shaft* d:

Power transmitted $P = \dfrac{2\pi N (M_t)_{mean}}{60}$

$$(M_t)_{mean} = \frac{60P}{2\pi N} = \frac{60\times 80\times 10^3}{2\times 3.14\times 400} = \frac{60\times 80\times 10^3}{2\times 3.14\times 400} = 1911\,N-m$$

$$M_t = (M_t)_{mean} = 1911\times 10^3\ N-mm$$

Where $M_t = \dfrac{\pi[\tau]_{shaft}\,d^3}{16}$; $\quad \Rightarrow d^3 = \dfrac{16M_t}{\pi[\tau]_{shaft}} = \dfrac{16\times 1911\times 10^3}{3.14\times 60} = 54.55\ mm$

Diameter of the shaft $d = 55\ mm$

(ii) *Design of sleeve*:

Outer diameter of the sleeve $D = 2d + 13$

Inner diameter of the sleeve $d =$ diameter of the shaft $= 55\ mm$

$D = 2 \times 55 + 13 = 123\ mm$

Length of the sleeve $L = 3.5d = 3.5 \times 55 = 192.5 \simeq 193\ mm$

(iii) Design of key:

From Design Data Book, for shaft $d = 55\ mm$ we have

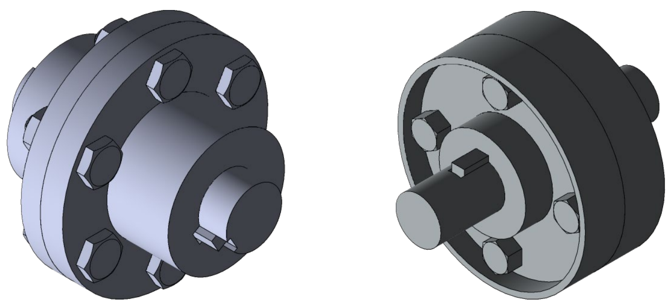
Width of the key $b = 16\ mm$; Thickness of the key $t = 10\ mm$

Length of the key $l = L = 3.5d = 3.5 \times 55 = 192.5\ mm$

3.11.2. Design of flange coupling

A flange coupling as shown in Fig. 3.9 is a mechanical device used to connect two shafts together at their ends to transmit power. It consists of two separate flanges that are bolted together. The power is transmitted from driving shaft with left side flange assembly to the driven shaft with right side flange assembly through bolts. The flange coupling is used to transmit heavy loads and large shafting.

Fig. 3.9: Rigid flange coupling



(a) (b)

Fig. 3.10: Types of rigid flange coupling (a) Unprotected type (b) Protected type

The flange coupling is a rigid type of coupling. There are two types of rigid flange coupling, unprotected and protected as shown in Fig. 3.10. The bolts and nuts are projected from the flange in unprotected rigid coupling, which is dangerous to the operator may lead to an accident. Whereas, the bolts and nuts are covered by protecting circumferential rims in the protected type flange coupling. The fractured bolts will hit against this rim and eventually fall down, if a bolt breaks while the machine is operating. This protects against harm to the operator. The advantages of the rigid couplings are: simple in construction, high torque transmission capacity, easy to assemble and dismantle. The disadvantages are: requires more radial space, cannot accommodate misalignment between the axes of the two shafts, used only where the motion is free from shocks and vibrations.

3.11.2.1. Protected type flange coupling

Transmission of torque from the shaft to the central flange and vice versa is the function of the hub in the protected type coupling. There are holes in the central flange to accommodate the bolts. These bolts are used to transmit torque from one flange to the other. The outer circumferential rim covers the projecting bolt heads and nuts for safety to the operator. The flanges are generally made from cast iron by casting or steel by a forging process. The various parts of the protected type flange coupling are shown in Fig. 3.11. A common method for calculating the dimensions of the flanges are using the standard proportions in terms of the shaft diameter (d). Because no stress analysis is involved, designing the coupling using these standard proportions is easy. The dimensions of the flanges are:

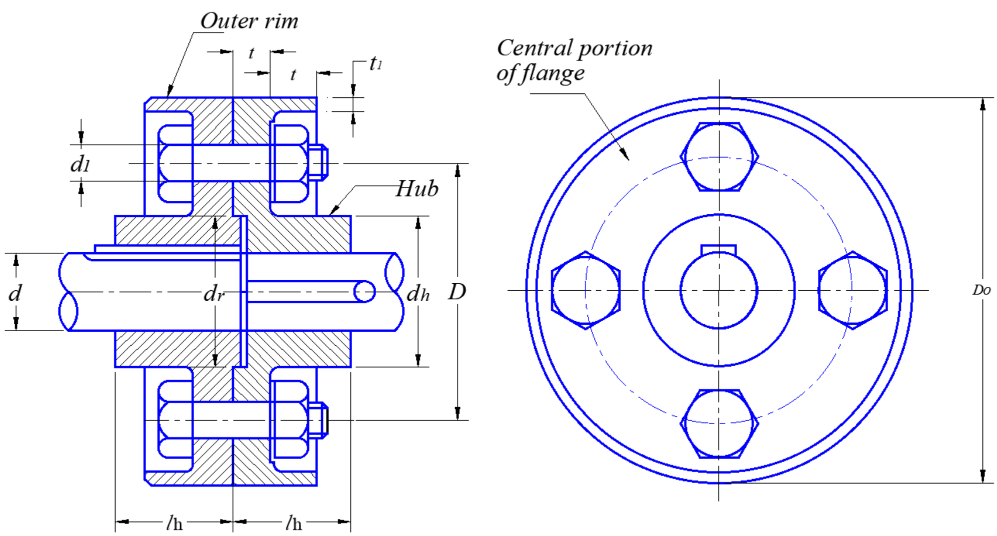


Fig. 3.11: Protected type flange coupling

(i) Outside diameter of hub $d_h = 2d$

(ii) length of hub or effective length of key $l_h = 1.5d$

(iii) pitch circle diameter of bolts $D = 3d$

(iv) thickness of flanges $t = 0.5d$

(v) thickness of protecting rim $t_1 = 0.25d$

(vi) diameter of spigot and recess $d_r = 1.5d$

(vii) outside diameter of flange $D_o = \left(4d + 2t_1\right)$

3.11.3. Design Procedure for protected type flange coupling

The basic procedure for finding out the dimensions of the protected type rigid flange coupling consists of following steps:

(i) Shaft diameter:

Calculate the shaft diameter by using following two equations:

$$M_t = \frac{60\times10^6\left(kW\right)}{2\pi N} \quad \text{and} \quad \tau = \frac{16M_t}{\pi d^3} \tag{3.54}$$

(ii) Dimensions of flanges:

Calculate the dimensions of the flanges by following empirical equations:

$$d_h = 2d;\ \ l_h = 1.5d;\ \ D = 3d;$$

$$t = 0.5d;\ \ t_1 = 0.25d$$

$$d_r = 1.5d;\ \ D_O = \left(4d + 2t_1\right) \tag{3.55}$$

The torsional shear stress in the hub can be calculated by considering it as a hollow shaft subjected to torsional moment M_t.

The inner and outer diameters of hub are d and d_h respectively. The torsional shear stress in the hub is given by,

$$\text{Torsional shear stress } \tau = \frac{M_t r}{J} \tag{3.56}$$

$$\text{Where } J = \frac{\pi\left(d_h^4 - d^4\right)}{32};\ r = \frac{d_h}{2}$$

The flange at the junction of the hub is under shear while transmitting the torsional moment M_t. From Fig. 3.11,

Area under shear $= \left(\pi d_h\right)\times t$;

Shear force $=$ area $\times$ shear stress $= \pi d_h t\tau$

Resisting torque $=$ shear force $\times \left(\dfrac{d_h}{2}\right)$

$$\therefore \quad M_t = (\pi d_h t\tau)\times\frac{d_h}{2}$$

$$M_t = \frac{1}{2}\pi d_h^2 t\tau \tag{3.57}$$

(iii) Diameter of bolts:

The forces acting on individual bolts due to the transmission of torque are shown in Fig. 3.12.

Equating the applied torque to the resisting torque,

$$M_t = F\times\frac{D}{2}\times n \tag{3.58}$$

Where M_t represents torque transmitted by the coupling, F represents force acting on each bolt, D represents pitch circle diameter of the bolts, n represents number of bolts. It is important to note that the bolts experience direct shear stress due to the force F, rather than torsional shear stress, as no torque acts about the bolt's axis. Thus, the force F only causes direct shear stress.

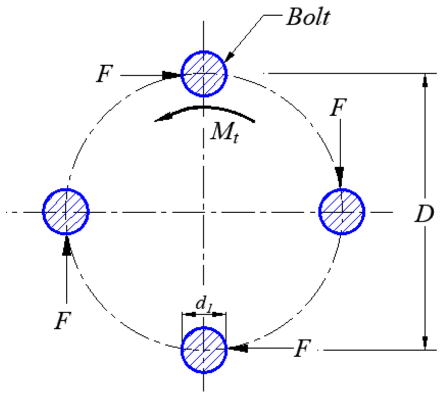


Fig. 3.12: Forces acting on individual bolts due to the transmission of torque

The direct shear stress in the bolt is calculated by,

$$\tau = \frac{F}{\left(\frac{\pi}{4}d_1^2\right)} \tag{3.59}$$

From Eqs. (3.58) and (3.59),

$$\tau = \frac{8M_t}{\pi D n d_1^2} \tag{3.60}$$

Decide the number of bolts using following guidelines:

$n = 3$ for $d < 40$ mm

$n = 4$ for $40 \le d < 100$ mm

$n = 6$ for $100 \le d < 180$ mm

Determine the diameter of bolt by Eq. (3.60). Rearranging the equation,

$$d_1^2 = \frac{8M_t}{\pi D n \tau} \tag{3.61}$$

Where τ is permissible shear stress for the bolt material.

The compressive stress in the bolt can be determined by referring to Fig. 3.11,

Crushing area of each bolt $= d_1 t$

Crushing area of all bolts $= n d_1 t$

Comprehensive force $= n d_1 t \sigma_c$

Torque $= M_t = (N d_1 t \sigma_c) \times \frac{D}{2}$

or

$$\sigma_c = \frac{2M_t}{N d_1 t D} \tag{3.62}$$

Eq. (3.62) is used to check the compressive stress in the bolt.

(iv) Dimensions of keys:

From Design Data Book, determine the standard cross-section of flat key. The length of the key in each shaft is l_h. Therefore,

$$l = l_h \tag{3.63}$$

With these dimensions of key, check the shear and compressive stresses in the key by Eqs. (3.43) and (3.44) respectively.

$$\tau = \frac{2M_t}{dbl} \text{ and } \sigma_c = \frac{4M_t}{dtl} \tag{3.64}$$

Example 3.9: Design a protected type of CI flange coupling to connect two shafts in order to transmit $7.5\ kW$ at $720\ rpm$. The following permissible stresses may be assumed; Permissible shear stress for shaft, bolt and key material $= 33\ N/mm^2$, Permissible crushing stress for bolt and key material $= 60\ N/mm^2$, Permissible shear stress for CI flange $= 15\ N/mm^2$.

Given Data:

Power transmitted $P = 7.5\ kW$

Speed $N = 720\ rpm$

Permissible shearing stress for the shaft, bolt and key

$[\tau]_{shaft} = [\tau]_{bolt} = [\tau]_{key} = 33\ N/mm^2$

Permissible crushing stress for the bolt and key $\left[\sigma_c\right]_{bolt} = \left[\sigma_c\right]_{key} = 60\ N/mm^2$

Permissible shear stress for CI Flange material $\left[\tau\right]_{CI} = 15\ N/mm^2$

Find:

1. Design a protected type of CI flange coupling

Solution:

Power transmitted $P = \dfrac{2\pi N M_t}{60}$

$$M_t = \frac{60P}{2\pi N} = \frac{60\times7.5\times10^3}{2\times3.14\times720} = 99.5\,N-m$$

$$M_t = 99.5\times10^3\ N-mm$$

Where $M_t = \dfrac{\pi\left[\tau\right]_{shaft} d^3}{16}$; $\quad \Rightarrow d^3 = \dfrac{16M_t}{\pi\left[\tau\right]_{shaft}} = \dfrac{16\times99.5\times10^3}{3.14\times33}$

$d = 24.9\ mm$

Diameter of the shaft $d = 25\ mm$

Empirical relations:

a. outside diameter of hub $d_h = 2d = 2\times25 = 50\ mm$

b. length of hub or effective length of key $l_h = 1.5d = 1.5\times25 = 37.5\ mm$

c. pitch circle diameter of bolts $D = 3d = 3\times25 = 75\ mm$

d. thickness of flanges $t = 0.5d = 0.5\times25 = 12.5\ mm$

e. thickness of protecting rim $t_1 = 0.25d = 0.25\times25 = 6.25\ mm$

f. diameter of spigot and recess $d_r = 1.5d = 1.5\times25 = 37.5\ mm$

g. outside diameter of flange $D_O = \left(4d + 2t_1\right) = \left(4\times25 + 2\times6.25\right) = 112.5\ mm$

(i) Design of Key:

From Design Data Book, for shaft $d = 25\ mm$ we have width of the key $b = 8\ mm$, thickness of the key $t = 7\ mm$, length of the key = length of the hub $l = 37.5\ mm$.

Check for shear and compressive stress for key:

Shear stress in the key $\tau = \frac{2M_t}{dbl} = \frac{2\times 99.5\times 10^3}{25\times 8\times 37.5} = 26.5\ N/mm^2$

Compressive stress in the key $\sigma_c = \frac{4M_t}{dtl} = \frac{4\times 99.5\times 10^3}{25\times 7\times 37.5} = 60\ N/mm^2$

This induced shear and compressive stress in the key is $26.5\ N/mm^2$ and $60.64\ N/mm^2$ respectively. These stresses are less than allowable stress, so key is safe against the shearing and crushing.

(ii) No. of Bolts:

For $d = 25\ mm$, No. of bolts $n = 3$

(iii) Diameter of the bolt:

$$d_1^2 = \frac{8M_t}{\pi D n \tau} = \frac{8\times 99.5\times 10^3}{\pi\times 75\times 3\times 33}$$

$$d_1 = 5.84\ mm$$

Check for bolt, $\sigma_c = \frac{2M_t}{n d_1 t D} = \frac{2\times 99.5\times 10^3}{3\times 5.84\times 12.5\times 75} = 12.12\ N/mm^2$

This induced stress $12.12\ N/mm^2$ is less than allowable stress $60\ N/mm^2$, so bolt is safe.

Example 3.10: Design cast iron flange coupling for a steel shaft transmitting $15kW$ at 200 rpm having an allowable shear stress of $40\,N/mm^2$. The working stress in the bolt should not exceed 30 N/mm^2 Assume that the same material is used for shaft and key and that the crushing stress is twice the value of its shear stress. The maximum torque is $24\,\%$ greater than the full load torque. The shear stress for cast iron is 14 N/mm^2.

Given Data:

Power $P = 15kW = 15\times 10^3 W$

Speed $N = 200\ rpm$

Allowable shear stress of the steel shaft $(\tau)_s = (\tau)_k = 40\ N/mm^2$

Working stress of the bolt $(\tau)_b = 30\ N/mm^2$

Crushing stress $\tau_c = 2\times(\tau)_s = 2\times 40 = 80\ N/mm^2$

Maximum torque $M_t = 1.24\times(M_t)_{mean}$

Shear stress of the cast iron $(\tau)_c = 14\ N/mm^2$

Find:

1. Design cast iron flange coupling

Solution:

(i) Design of Hub:

$$\left(M_t\right)_{mean} = \frac{P\times 60}{2\pi N} = \frac{15\times 10^3\times 60}{2\times\pi\times 200} = 716.19\times 10^3\ N-mm$$

$$M_t = 1.24\times\left(M_t\right)_{mean} = 1.24\times 716.19\times 10^3 = 888.07\times 10^3\ N-mm$$

Also, $M_t = \frac{\pi}{16}\times(\tau)_s\times d^3 \qquad \therefore \quad 888.07\times 10^3 = \frac{\pi}{16}\times 40\times d^3$

$$d^3 = \frac{888.07\times 10^3\times 16}{\pi\times 40}$$

$d = 48.35 \simeq 50mm$

Outer diameter $D = 2d = 2\times 50 = 100mm$

Length of hub $h = 1.5d = 1.5\times 50 = 75mm$

By considering it as hollow Shaft,

$$M_t = \frac{\pi}{16}\times(\tau)_c\times\left[\frac{D^4-d^4}{D}\right]$$

$$\frac{888.07\times 10^3\times 16}{\pi} = (\tau)_c\times 683593.75$$

$$(\tau)_c = \frac{888.07\times 10^3\times 16}{\pi\times 683593.75}$$

$(\tau)_c = 6.61N/mm^2 < 14N/mm^2$

$\therefore$ Design is Safe.

(ii) Design of key:

Width = thickness

$$w = t = \frac{d}{4} = \frac{50}{4} = 12.5(or)14mm$$

Length of key equals to length of hub $l = L = 75mm$

$$M_t = l\times w\times(\tau)_k\times\frac{d}{2}$$

$$888.07\times 10^3 = 75\times 14\times(\tau)_k\times\frac{50}{2}$$

$$(\tau)_k = 33.83N/mm^2$$

$$(\sigma)_c = 2(\tau)_k = 2\times 33.83 = 67.66N/mm^2$$

(iii) Design of flange:

$$t = 0.5\times d = 0.5\times 50 = 25mm$$

$$T = \pi\left(\frac{D^2}{2}\right)\times(\tau)_c\times t_f$$

$$888.07\times 10^3 = \pi\left(\frac{100^2}{2}\right)\times(\tau)_c\times 25$$

$$(\tau)_c = 2.26N/mm^2$$

(iv) Design for bolts:

$n = 4,$

Pitch circle diameter, $D_1 = 3d = 3\times 50 = 150\ mm$

$$T = \frac{\pi}{4}(d_1)^2\times(\tau)_b\times n\times\frac{D_1}{2}$$

$$888.07\times 10^3 = \frac{\pi}{4}(d_1)^2\times 30\times 4\times\frac{150}{2}$$

$$d_1^{\ 2} = \frac{888.07\times 10^3\times 4\times 2}{600\times 30\times\pi}$$

$$d_1 = 11.20mm$$

Outer diameter $D_2 = 4d = 4\times 50 = 200\ mm$

Thickness $t_f = 0.25\times d = 0.25\times 50\ = 12.5mm$

3.11.4 Bushed – pin type flexible coupling

Use of a rigid coupling is limited to situations when there is no vibration or shock and the axes of the two shafts are perfectly aligned. Shaft axes cannot be perfectly aligned in practice. When rigid coupling is employed under such conditions, the misalignment results in excessive bearing reactions, which wear and generate vibrations.

A flexible element, such as a rubber bush, is used in a flexible coupling to connect the driven and driving flanges. This flexible rubber bush absorbs vibrations and shocks in addition to adjustment for the misalignment. Fig. 3.13 depicts the construction of the flexible coupling. A rubber bushing and pins in place of bolts distinguish it from the rigid type of flexible coupling. The are two flanges in bushed pin type flexible coupling, one is connected to the input shaft and other is connected to output shaft. The diameter of the pin is enlarged in the input flange where a rubber bush is mounted over the pin. There is a brass lining on the inside of the rubber bush, in order to reduce the wear. Power is transmitted from the input shaft to output shaft through the input flange, rubber bush, pin, output flange. A common method for calculating the dimensions of the flanges of bushed pin type flexible coupling are using the standard proportions in terms of the shaft diameter. Rigid and flexible connections differ fundamentally from one another.

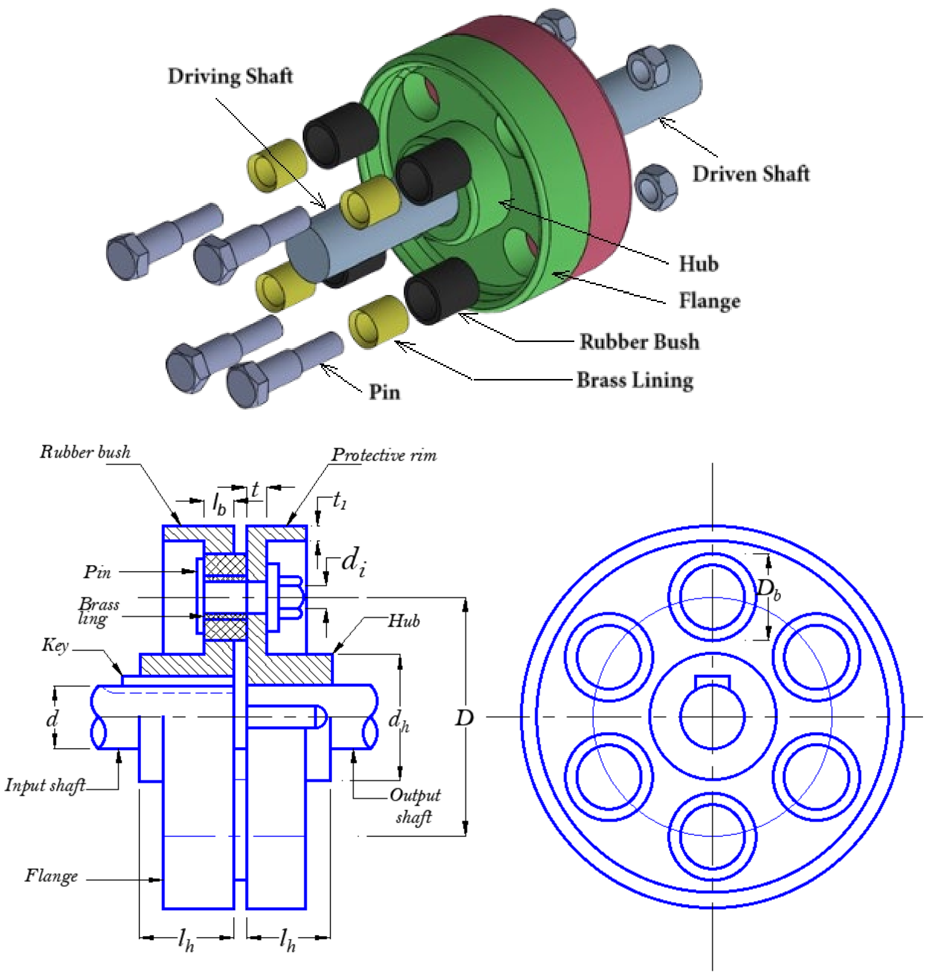


Fig. 3.13: Bushed – pin type flexible coupling

Two flanges are identical in rigid coupling, whereas the diameter of the pins are different in flexible coupling. The rubber bush is accommodated in the input flange which is having larger diameter than the diameter of the pins accommodated in output flange and it is shown in Fig. 3.13. Hence, the two flanges have varying thicknesses and hole diameters.

3.11.4.1. Design procedure for bushed – pin type flange coupling

The basic procedure for finding out the dimensions of bushed – pin type flexible coupling consists of following steps:

(i) Shaft diameter:

Calculate the shaft diameter using Eq. (3.65):

$$M_t = \frac{60\times10^6 (kW)}{2\pi N} \text{ and } \tau = \frac{16M_t}{\pi d^3} \tag{3.65}$$

(ii) Flange dimensions:

Determine the dimensions of the flanges using empirical formulas:

$d_h = 2d;\quad l_h = 1.5d;\quad D = 3d$ to $4d;$

$$t = 0.5d;\quad t_1 = 0.25d \tag{3.66}$$

The torsional shear stress in the hub can be determined by treating it as a hollow shaft exposed to a torsional moment M_t. The hub's inner and outer diameters are d and d_h respectively. The formula for the torsional shear stress in the hub is given by,

$$\tau = \frac{M_t r}{J} \tag{3.67}$$

Where $J = \frac{\pi\left(d_h^4 - d^4\right)}{32};\quad r = \frac{d_h}{2}$

The shear stress in the flange at the hub junction is determined using Eq. (3.57) and is expressed as follows,

$$M_t = \frac{1}{2}\pi d_h^2 t\tau \tag{3.68}$$

(iii) Pin diameter:

The number of pins is typically either 4 or 6, and the diameter of the pins is determined using the following empirical equation:

$$d_1 = \frac{0.5d}{\sqrt{n}} \tag{3.69}$$

Calculate the shear stress in the pins by

$$\tau = \frac{8M_t}{\pi d_1^2 Dn} \tag{3.70}$$

The shear stress calculated by the Eq. (3.70) should be less than $35\,N/mm^2$. Determine the bending stresses in the pins and verify that they are within the acceptable limits.

(iv) Bush dimensions:

Determine the outer diameter of the rubber bush using the following formula:

Torque transmitted by the coupling $M_t = \frac{1}{2} p_b D_b l_b D n$ (3.71)

Where M_t is torque transmitted by the coupling, p_b is permissible intensity of pressure or bearing pressure $\left(N/mm^2\right)$ between the flange and the rubber bush, D_b is outer diameter of the bush, D is diameter of the pitch circle for the pins, l_b length of the bush that effectively engages with the input flange (mm), n is number of bushes or pins.

Assume, the length to the outer diameter for the rubber bush is 1. Therefore, $\frac{l_b}{D_b} = 1 \Rightarrow l_b = D_b$.

Torque transmitted by the coupling $M_t = \frac{1}{2} p_b D_b^2 D n$ (3.72)

If we assume the allowable bearing pressure between the rubber bushing and the cast iron flange is $1\ N/mm^2$, then the Eq. (3.72) can be written as,

$$M_t = \frac{1}{2} D_b^2 D n \tag{3.73}$$

Use the following relationship to determine the rubber bush's effective length:

$$l_b = D_b \tag{3.74}$$

(v) Dimensions of keys:

From Design Data Book, determine the standard cross-section of flat. Each shaft's key has a length of l_h. Consequently,

$l = l_h$ (3.75)

With the above dimensions of the key, check the shear and compressive stresses in the key by Eqs. (3.43) and (3.44) respectively.

$$\tau = \frac{2M_t}{dbl}; \text{ and } \sigma_c = \frac{4M_t}{dhl} \tag{3.76}$$

Example 3.11: Design a bush-and-pin type flexible coupling to transfer $35\ kW$ of power at $960\ rpm$ from a motor shaft to pump shaft. Assume the maximum torque is 25% higher than the average torque. The permissible shear and crushing stresses for the shaft and key materials are $35\ N/mm^2$ and $70\ N/mm^2$

respectively. The permissible shear stress for cast iron is $15\ N/mm^2$ and the bearing pressure for rubber bush is $0.9\ N/mm^2$. Assume that the pin material is same as that of shaft and key.

Given Data:

$P = 35\ kW$; $N = 960\ rpm$; $M_t = 1.25(M_t)_{mean}$; $[\tau]_{shaft} = [\tau]_{key} = 35\ N/mm^2$

$[\sigma_c]_{shaft} = [\sigma_c]_{key} = 70\ N/mm^2$; $[\tau]_{CI} = 15\ N/mm^2$; $p_b = 0.9\ N/mm^2$

Find:

1. Design a bushed – pin type flexible coupling

Solution:

(i) Shaft Diameter:

$$(M_t)_{mean} = \frac{60\times10^6(kW)}{2\pi N} = \frac{60\times10^6\times35}{2\times3.14\times960} = 348.32\times10^3\ N-mm$$

Maximum torque transmitted $M_t = 1.25(M_t)_{mean}$

$$M_t = 1.25\times348.32\times10^3 = 435\times10^3\ N-mm$$

(ii) Diameter of the shaft:

We know, Maximum torque transmitted by the shaft $M_t = \frac{\pi}{16}[\tau]_{shaft}\, d^3$

$$d^3 = \frac{16M_t}{\pi[\tau]_{shaft}} = \frac{16\times435\times10^3}{3.14\times35}$$

Diameter of the shaft $d = 39.85\ mm \simeq 40\ mm$

(iii) Dimensions of flanges:

The dimensions of the flange are

Outside diameter of the hub $d_h = 2d = 2\times40 = 80\ mm$

Length of hub or effective length of key $l_h = 1.5d = 1.5\times40 = 60\ mm$

Pitch circle diameter of pins $D = 4d = 4\times40 = 160\ mm$

Thickness of output flange $t = 0.5d = 0.5\times40 = 20\ mm$

Thickness of protective rim $t_1 = 0.25d = 0.25\times40 = 10\ mm$

The hub resembles a hollow cylinder under the influence of a torsional moment, From Eq. (3.67),

$$J = \frac{\pi\left(d_h^4 - d^4\right)}{32} = \frac{3.14\left(80^4 - 40^4\right)}{32} = 376.8\times10^4\ mm^4$$

$$r = \frac{d_h}{2} = \frac{80}{2} = 40\ mm$$

The torsional shear stress in the hub is given by,

$$\tau = \frac{M_t r}{J} = \frac{435\times10^3\times40}{376.8\times10^4} = 4.617\ N/mm^2$$

$$\therefore \qquad \tau < 15\ N/mm^2\left(\left[\tau\right]_{CI}\right)$$

The shear stress in the flange at the connection with the hub is determined using Eq. (3.57) and is expressed as follows,

$$\tau = \frac{2M_t}{\pi d_h^2 t} = \frac{2\times435\times10^3}{3.14\times80^2\times20} = 2.16\ N/mm^2$$

$$\therefore \qquad \tau < 15\ N/mm^2\left(\left[\tau\right]_{CI}\right)$$

(iv) Diameter of pins:

The number of pins selected is 6. The diameter of the pins is calculated using the following empirical equation:

$$d_1 = \frac{0.5d}{\sqrt{n}} = \frac{0.5\times40}{\sqrt{6}} = 8.164\ mm \simeq 9\ mm$$

The shear stress of the pins is calculated from Eq. (3.70),

$$\tau = \frac{8M_t}{\pi d_1^2 Dn} = \frac{8\times435\times10^3}{3.14\times9^2\times160\times6} = 14.25\ N/mm^2$$

$$\therefore \qquad \tau < 35\ N/mm^2$$

Based on the shear strength, the design is safe.

(v) Dimensions of bushes:

From Eq. (3.72), $D_b^2 = \dfrac{2M_t}{p_b Dn} = \dfrac{2\times435\times10^3}{0.9\times160\times6}$

$$D_b = \sqrt{\frac{2\times435\times10^3}{0.9\times160\times6}} = 31.73\ mm \simeq 35\ mm$$

(vi) Dimensions of keys:

From Design Data Book, the standard cross-section for a flat square key used with a 40 mm diameter is $12\times8\ mm$. The key length is equal to l_h.

$l = l_h = 60\ mm$

The dimensions of the flat key are $12 \times 8 \times 60\ mm$. From Eq. (3.64),

$$\tau = \frac{2M_t}{dbl} = \frac{2 \times 435 \times 10^3}{40 \times 12 \times 60} = 30.2\ N/mm^2$$

$\therefore \qquad \tau < 35\ N/mm^2$

From Eq. (3.64), $\sigma_c = \dfrac{4M_t}{dbl} = \dfrac{4 \times 435 \times 10^3}{40 \times 12 \times 60} = 60.41\ N/mm^2$ $\qquad$ ($t = b$ for square key)

$\therefore \qquad \sigma_c < 70\ N/mm^2$

Considering the shear and crushing strength, the design of key is safe.

3.12. SPUR GEARS

Spur gears are one of the most common types of gears used in mechanical system and it is shown in Fig. 3.14.

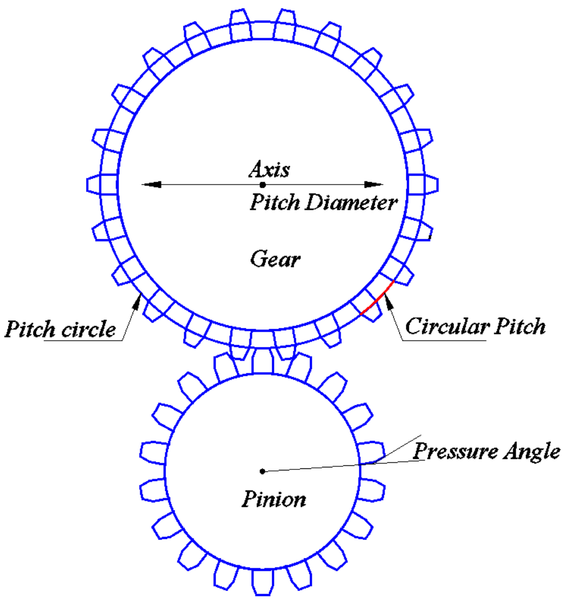


Fig. 3.14: Spur gears – Pinion and gear

Spur gears have teeth that are straight and parallel to the gear's axis. This design allows for efficient power transmission and smooth operation. Spur gears are typically used to transmit motion and power between parallel shafts. Due to their simple tooth geometry and parallel shaft arrangement, spur gears are known for their high efficiency in power transmission. They are capable of transmitting motion with minimal energy loss. Spur gears keep a consistent velocity ratio between the input and output shafts, making them ideal for applications that need precise speed control. Spur gears are well-suited for low to medium-speed applications. At high speeds, they may generate more noise and vibration compared to other types of gears. Spur gears find applications in various industries, including automotive, machinery, robotics, aerospace, and manufacturing. They are commonly used in gearboxes, transmission systems, conveyor systems, printing presses, and many other mechanical devices. The spur gear terminology is shown in Fig 3.15.

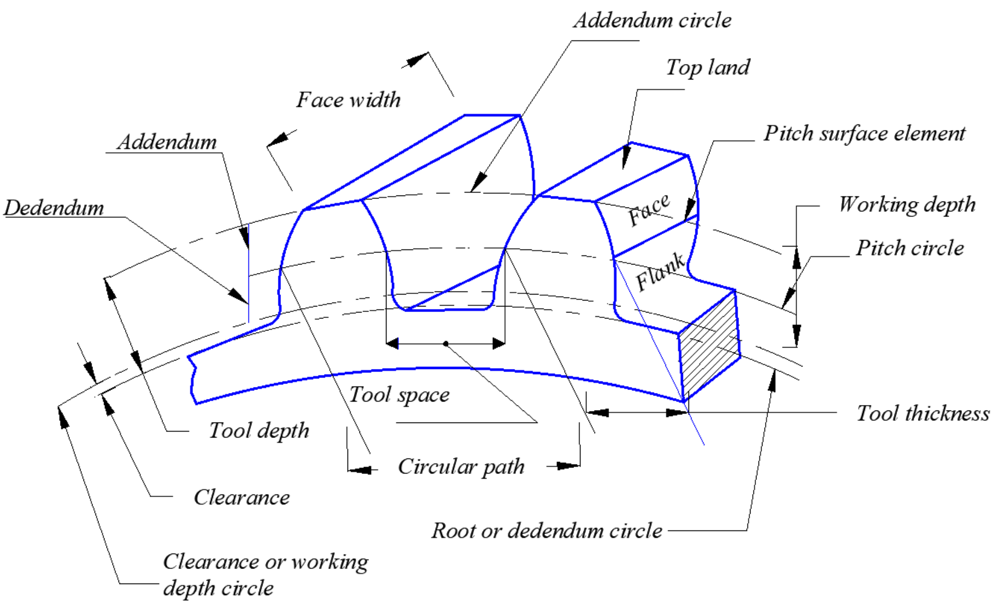


Fig. 3.15: Terms used in gears

The common terminologies associated with spur gears are:

(i) Pitch circle: The pitch circle is a theoretical circle that passes through the point where the teeth of two mating gears mesh together. It represents the effective diameter at which the gears engage and transmit motion.

(ii) Pitch circle diameter: It is an imaginary circle on the gear that represents the point where the pitch diameter is measured. The pitch circle diameter is typically used to specify the gear's size.

(iii) Pitch point: The pitch point is the point at which the pitch circles of two mating gears make contact with each other. It's the point where the gears effectively begin to mesh.

(iv) Module: It represents the ratio of the pitch diameter (D) to the number of teeth (n) on the gear $\left(m = \frac{D}{n}\right)$.

(v) Circular pitch: It represents the distance along the pitch circle from one tooth to the adjacent tooth, measured along the pitch circle.

(vi) Addendum circle: It defines the radial distance from the pitch circle to the highest point of the gear tooth.

(vii) Addendum: The radial distance from the pitch circle to the addendum circle is known as the addendum.

(viii) Dedendum circle: The dedendum circle is an imaginary circle that contacts the base of the gear teeth.

(ix) Dedendum: The radial distance from the pitch circle to the bottom of the gear tooth is called as dedendum.

(x) Clearance: It refers to the intentional gap between the teeth of mating gears. This gap is necessary to ensure smooth meshing and to prevent interference between the gears during operation.

(xi) Full depth: The full depth of a spur gear refers to the total depth of the gear tooth, extending from the tip of the tooth to the bottom of the tooth space, or the root.

(xii) Working depth: The working depth is the distance from the top of a gear tooth to the bottom of the mating tooth on the opposing gear. In other words, it is the depth to which the teeth of two gears mesh together.

(xiii) Space width: The space width is the distance between the outer surface of one tooth and the inner surface of the adjacent tooth, measured along the pitch circle.

(xiv) Tooth thickness: The tooth thickness of a spur gear refers to the width of an individual gear tooth measured along the pitch circle or pitch diameter.

(xv) Backslash: Backlash in spur gears refers to the clearance or play between the mating teeth of two gears when they are engaged.

(xvi) Flank: The surface of the gear tooth that extends from the pitch circle to the tooth tip.

(xvii) Face: The "face" of a spur gear refers to the surface of the gear tooth that is in contact with a mating gear during operation.

(xviii) Pressure line: The pressure line of a spur gear refers to the line along which the force is exerted between the teeth of two mating gears as they come into contact during operation.

(xix) Pressure angle: The pressure angle is the angle between the line of action, which is the imaginary line along which force is transmitted between meshing gears, and the tangent to the pitch circle at the contact point of the teeth of the mating gears.

(xx) Path of contact: It refers to the trajectory along which the teeth of two mating gears come into and go out of contact as they rotate.

(xxi) Arc of contact: The arc of contact is the length of the portion of the tooth profile of one gear that is in contact with the tooth profile of its mating gear at any given instant during their rotation.

3.12.1. Systems of gear teeth

The four systems of gear teeth are commonly used in gear design, each with unique characteristics that make them suitable for different applications.

1. 14.5° Composite System

2. 14 /2° full depth involute system
3. 20° full depth involute system
4. 20° stub involute system

Table 3.2 illustrates the pressure angle, characteristics and applications of four systems of gear teeth.

Table 3.2: Pressure angle, characteristics and applications of four systems of gear teeth

Gear System	Pressure Angle	Tooth Profile	Key Characteristics	Common Applications
14.5° Composite System	14.5°	Composite	Smooth operation, higher contact ratio	Older machinery, low-load applications
14.5° Full Depth Involute	14.5°	Full Depth Involute	Smooth and quiet operation, less risk of undercutting	Precision instruments, quiet operation
20° Full Depth Involute	20°	Full Depth Involute	Better strength, modern standard, balanced performance	Automotive, industrial machinery
20° Stub Involute	20°	Stub Involute	High strength, compact design	High-load, compact systems (heavy machinery, aerospace)

3.12.2. Gear design

The main considerations in gear tooth design are finding the right pitch and face width for the required levels of strength, durability, and manufacturing economy.

The power to be transmitted, the driver's speed, the velocity ratio to be transmitted, and the center distance are the specifications for a gear drive. Machine tools, compressors, reciprocating pumps, hoisting machines and other driven units require a low operating speed than that of the power sources such as turbines, high-speed internal combustion engines, and electric motors. To achieve the low operating speed in the driven units, the speed reduction is required from the drive unit. But sometimes need to enhance speed—centrifuges are a common example of this.

When designing a gear drive for a specific application, the following factors need to be taken into account:

- The gear needs to be strong enough to withstand dynamic loading during operation or starting torques
- the teeth need to have excellent wear characteristics to ensure a very long gear life
- The right blend of materials needs to be selected to provide quiet operation and appropriate gear characteristics
- The drive must be small and it must be correctly aligned
- Appropriate lubrication measures must be implemented

3.12.3. Lewis equation for static beam strength of spur gear teeth

The Lewis equation is considered as the foundational formula for gear design. As illustrated in Fig. 3.16, the gear tooth is considered as a cantilever beam in the Lewis analysis. The bending moment about the tooth's base is brought on by the tangential component (F_t).

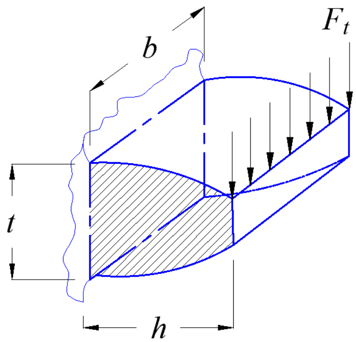


Fig. 3.16: Gear tooth as cantilever

Lewis Equation Lec 15

The following are the assumptions made to drive the Lewis equation:

- Neglecting the radial component's (F_r) influence, which results in compressive stresses.
- The tangential component (F_t) is evenly distributed along the gear's face width.
- Neglecting the impact of stress concentration.
- It is considered that only one set of teeth is in contact and carrying the entire load at any given moment.

The cross-section of the tooth is seen to change from the fixed end to the free end. Consequently, Fig. 3.17 shows the construction of a parabola within the tooth profile, shown by a dotted line. The uniform strength of a parabolic shape beam is one of its advantages. For this beam, the stress is constant or identical throughout all cross sections. The location where the parabola is tangent to the tooth profile is the weakest part of the gear tooth.

The bending moment at the section $T-T$ is written as,

$$M_b = F_t \times h \tag{3.77}$$

Where $I = \left(\frac{1}{12}\right) bt^3$; $y = \frac{t}{2}$

The following yields the bending stresses:

$$\sigma_b = \frac{M_b y}{I} = \frac{(F_t \times h)\left(\frac{t}{2}\right)}{\left[\left(\frac{1}{12}\right) bt^3\right]}$$

Rearranging the terms, $F_t = b\sigma_b\left(\frac{t^2}{6h}\right)$ (3.78)

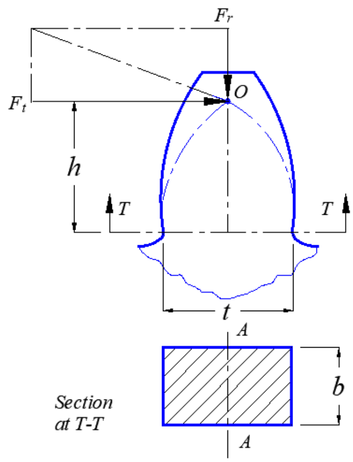


Fig. 3.17: Gear tooth as parabolic beam

$'m'$ is multiplied in the numerator and denominator of the right-hand side, we get

$$F_t = mb\sigma_b\left(\frac{t^2}{6hm}\right) \quad (3.79)$$

Let assume $Y = \left(\frac{t^2}{6hm}\right)$

Eq. (3.79) is rewritten as, $F_t = mb\sigma_b Y$ (3.80)

Y is referred to as the Lewis form factor in the Eq. (3.80). The relationship between the tangential force (F_t) and the related stress is provided by Eq. (3.80). Stress rises in proportion to an increase in tangential force. The associated force is referred to as the beam strength when the stress exceeds the maximum amount that can be allowed for bending strains. Consequently, the greatest tangential force that the tooth can transmit without bending out of shape is known as the beam strength (S_b). Eq. (3.80) is changed by substituting (F_t) by (S_b) in the following manner:

$$S_b = mb\sigma_b Y \quad (3.81)$$

where, S_b = beam strength of gear tooth (N), σ_b = permissible bending stress (N/mm^2)

Eq. (3.81) is known as the Lewis equation for static beam strength of spur gear teeth. Table 3.3 provides the Lewis form factor Y for 20° full-depth involute system.

Table 3.3: Values of the Lewis form factor Y for *20°* full-depth involute system

z	Y	z	Y	z	Y
15	0.289	27	0.348	55	0.415
16	0.295	28	0.352	60	0.421
17	0.302	29	0.355	65	0.425
18	0.308	30	0.358	70	0.429
19	0.314	32	0.364	75	0.433
20	0.320	33	0.367	80	0.436
21	0.326	35	0.373	90	0.442
22	0.330	37	0.380	100	0.446
23	0.333	39	0.386	150	0.458
24	0.337	40	0.389	200	0.463
25	0.340	45	0.399	300	0.471
26	0.344	50	0.408	Rack	0.484

Where z is no. of teeth.

The beam strength needs to be greater than the effective force between the meshing teeth to prevent gear tooth breakage caused by bending. Therefore,

$$S_b \geq F_{eff}$$

Rewriting the Lewis equation,

$$S_b = mb\sigma_b Y \tag{3.82}$$

It is noted that m and b for both gears and pinions are the same. The product $(\sigma_b \times Y)$ determines which of the pinion and gear is weaker when different materials are utilized. When a pinion is compared to a gear, the Lewis form factor Y is always smaller. The pinion is usually weaker than the gear when the same material is used for both.

3.12.4. Design procedure for spur gears

The following procedure is used to design spur gears:

(i) *Calculate the tangential tooth load:*

The tangential tooth load is calculated from the power transmitted by the spur gears and pitch line velocity using the Eq. (3.83), Tangential tooth load, $F_t = \frac{P}{v} \times K_o$ (3.83)

Where P is power transmitted by the spur gears, v is pitch line velocity $= \frac{\pi DN}{60}$ in m/s , K_o is service factor. The service factor is defined as the ratio between the maximum torque M_t, and rated torque $(M_t)_{mean}$. The service factor for speed reduction gear boxes is given in Table. 3.4

Table 3.4: Service factor (K_o) for speed reduction gear boxes

Working condition of source of power	Working condition of driven machine		
	Uniform	**Moderate shock**	**Heavy shock**
Uniform	1.00	1.25	1.75
Low shock	1.25	1.5	2.0
Medium shock	1.5	1.75	2.25

(ii) Calculation of static load (S_b)**:**

Beam Strength $S_b = \pi \times m \times b \times [\sigma_b] \times Y$

Assume $b =$ Face width $= 10 \times m$

From Design Data Book, Form factor $Y = 0.124 - (0.684 / z_1)$, for 14.5^o full depth system; $Y = 0.154 - (0.912 / z_1)$, for $20°$ full depth system; $Y = 0.175 - (0.95 / z_1)$, for $20°$ stub teeth.

(iii) Calculation of dynamic load (F_d) *:*

The tangential tooth load refers to the actual force between meshing teeth when gears rotate at very low speeds, approaching zero velocity. Typically, dynamic forces arise between mating teeth when gears rotate at significant speeds. These dynamic forces result from imperfections in the tooth profile, errors in tooth spacing, bearing misalignment, and gear elasticity. Two methods are used to assess dynamic forces: evaluating the velocity factor during the preliminary design stages and applying *Buckingham's equation* in the final stages for precise dynamic force calculations. In the preliminary stages of design, the empirical relationship for velocity factor is used to calculate the effective load between two meshing teeth (Eq. (3.84)),

$$F_{eff} = \frac{K_o F_t}{C_v} \tag{3.84}$$

Where C_v = Velocity factor

$= \frac{3}{3+v}$ for ordinary and cut gears using form cutters with $v < 10 \ m/s$

$= \dfrac{6}{6+v}$ for accurately hobbed and generated gears with $v < 20\ m/s$

$= \dfrac{5.6}{5.6+\sqrt{v}}$ for precision gears with shaving, grinding and lapping operations

with $v < 20\ m/s$

The Buckingham's equation (Eq. (3.85)) is used to calculate the dynamic load in the final stages of gear design.

$$F_{eff} = F_t + F_d \tag{3.85}$$

Where F_d is dynamic load due to dynamic conditions between two mating gears. The dynamic load is given by,

$$F_d = \frac{21v(Ceb + F_t)}{21v + \sqrt{(Ceb + F_t)}} \tag{3.86}$$

Where C is deformation factor $\left(N/mm^2\right)$, e is sum of errors (mm), b is face width of tooth (mm). The deformation factor C is given by,

$$C = \frac{k}{\left[\dfrac{1}{E_p} + \dfrac{1}{E_g}\right]} \tag{3.87}$$

Where k is form of tooth constant, E_p is modulus of elasticity of pinion $\left(N/mm^2\right)$, E_g is modulus of elasticity of gear $\left(N/mm^2\right)$. The values of k are: $k = 0.107$ (for 14.5° full depth teeth), $k = 0.111$ (for 20° full depth teeth), $k = 0.115$ (for 20° stub teeth).

The error e in Eq. (3.88) is given by,

$$e = e_p + e_g \tag{3.88}$$

Where e_p is error for pinion, e_g is error for gear. The error is dependent on the manufacturing process and the gear's quality. The tolerance is considered as expected error on the gear tooth. The tolerances for adjacent pitch error for twelve different grades from Gr. 1 to Gr. 12 are given in Table 3.5.

$$\text{Tolerance factor } \phi = m + 0.25\sqrt{d_1} \tag{3.89}$$

Where m is module (mm), d_1 is pitch circle diameter (mm).

Table 3.5: Tolerances for adjacent pitch error

Grade	e (microns)	**Grade**	e (microns)
1	$0.80+0.06\phi$	**7**	$11.0+0.90\phi$
2	$1.25+0.10\phi$	**8**	$16.0+1.25\phi$
3	$2.00+0.16\phi$	**9**	$22.0+1.80\phi$
4	$3.20+0.25\phi$	**10**	$32.0+2.50\phi$
5	$5.00+0.40\phi$	**11**	$45+3.55\phi$
6	$8.00+0.63\phi$	**12**	$63.0+3.50\phi$

(iv) Calculation of module (m) *:*

The beam strength should be greater than the effective load between the meshing teeth to avoid the breakage of gear tooth due to bending. Therefore, $S_b > F_{eff}$. Hence, the factor of safety is introduced and beam strength is written as,

$$S_b = F_{eff} \times FOS \tag{3.90}$$

The tangential load $$F_t = \frac{2M_t}{d_1} = \frac{2M_t}{mz} = \frac{2}{mz}\left\{\frac{60\times10^6\times(kW)}{2\pi N}\right\} \tag{3.91}$$

From Eq. (3.84), $$F_{eff} = \frac{K_o F_t}{C_v} = \frac{60\times10^6}{\pi}\left\{\frac{(kW)\times K_o}{mzN\times C_v}\right\} \tag{3.92}$$

From Eq. (3.82), $$S_b = mb\sigma_b Y = m^2\left(\frac{b}{m}\right)\sigma_b Y \tag{3.93}$$

From Eqs. (3.90), (3.91) and (3.92),

$$m = \left[\frac{60\times10^6}{\pi}\left\{\frac{(kW)K_o\times FOS}{zNC_v\left(\frac{b}{m}\right)\sigma_b Y}\right\}\right]^{\frac{1}{3}} \tag{3.94}$$

Eq. (3.94) is used in the preliminary stages of gear design.

(v) *Calculation of Maximum wear load* (F_w) :

$$F_w = d_1 \times b \times Q \times K_w \tag{3.95}$$

From Design Data Book, Q is ratio factor $=\dfrac{2i}{i+1}$, K_w is load stress factor $=\dfrac{\sigma_c^2 \sin\alpha\left(1/E_p + 1/E_g\right)}{1.4}$

Where α is pressure angle in degrees, $K_w = 0.16\left(\frac{BHN}{100}\right)^2$ for both the gears are made of steel with a 20° pressure angle. BHN is the Brinell Hardness Number.

Example 3.12: A pair of spur gears needs to be designed with $20°$ full-depth involute teeth using the Lewis equation. To account for dynamic loading, a velocity factor should be included. The pinion shaft is driven by a $10kW$, $1440rpm$ motor, which has a starting torque of 125% of its rated torque. The speed reduction ratio is 4:1. Both the pinion and the gear are made from plain carbon steel $^{40}C_8$. The permissible be nding stress σ_b, for plain carbon steel 40C8 is $200\ N/mm^2$. The factor of safety is 1.5. Assume pitch line velocity as $5\ m/s$. Design the gears.

Given Data:

Power transmitted $P = 10\ kW$

Speed of the motor $N_1 = 1440\ rpm$

Starting torque $M_t = 1.25 \times (M_t)_{mean}$

Speed reduction ratio $i = 4$

Permissible bending stress for plain carbon steel 40C8 $\sigma_b = 200 N/mm^2$

Factor of safety $FOS = 1.5$

Pitch line velocity $v = 5\ m/s$

Service factor $K_o = 1.25$

Find:

1. Design a spur gear

Solution:

(i) *Calculation of module based on beam strength:*

Since the pinion and gear are made from the same material, the pinion is weaker than the gear. Assume the minimum number of teeth for a 20° pressure angle is 18.

Number of teeth for pinion $z_p = 18$

Number of teeth for gear $z_g = iz_p = 4 \times 18 = 72$

$$M_t = \frac{60 \times 10^6 \times (kW)}{2\pi N_1} = \frac{60 \times 10^6 \times 10}{2\pi \times 1440} = 66348.2\ N - mm$$

From Table 3.2, the Lewis form factor for 18 teeth is 0.308. Therefor, $Y = 0.308$.

$$C_v = \frac{3}{3+v} = \frac{3}{3+5} = \frac{3}{8}$$

Assume $\frac{b}{m} = 10$, From Eq. (3.94),

$$m = \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{(kW) K_o \times FOS}{z_p N C_v \left(\frac{b}{m} \right) \sigma_b Y} \right\} \right]^{\frac{1}{3}} = \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{10 \times 1.25 \times 1.5}{18 \times 1440 \times \left(\frac{3}{8} \right) \times 10 \times 200 \times 0.308} \right\} \right]^{\frac{1}{3}}$$

$m = 3.91\ mm$

(ii) *Selection of module:*

The minimum value of the module is $m = 5\ mm$.

Pitch circle diameter for the pinion $d_p = mz_p = 5 \times 18 = 90\ mm$

Pitch circle diameter for the gear $d_g = mz_g = 5 \times 72 = 360\ mm$

Face width $b = 10m = 10 \times 5 = 50\ mm$

Check for the safe design:

Tangential tooth load $F_t = \frac{2M_t}{d_p} = \frac{2 \times 66348.2}{90} = 1474.4\ N$

Pitch line velocity $v = \frac{\pi d_p N}{60 \times 10^3} = \frac{3.14 \times 90 \times 1440}{60 \times 10^3} = 6.78\ m/s$

$$C_v = \frac{3}{3+v} = \frac{3}{3+6.78} = 0.3067$$

Effective load $F_{eff} = \frac{K_o}{C_v} F_t = \frac{1.25}{0.3067} \times 1474.4 = 6009.13\ N$

From Eq. (3.82), beam strength $S_b = mb\sigma_b Y = 5 \times 50 \times 200 \times 0.308 = 15400\ N$

Factor of safety $FOS = \frac{S_b}{F_{eff}} = \frac{15400}{6009.13} = 2.56$

Since, the $FOS = 2.56$ is higher than 1.5, the design is safe. The module should be $5\ mm$.

Example 3.13: A set of spur gear consists of a 20 teeth pinion meshing with a 40 teeth gear having 20° full-depth involute teeth. The module and face width of the spur gear is 3 mm and 40 mm respectively.

The steel is used for producing both pinion and gear with permissible bending stress of $210\ N/mm^2$. The gear is heat treated with surface hardness of 350 BHN. The pinion operates at 1440 rpm and the service factor is 1.5. Assume that velocity factor accounts for the dynamic load and the factor of safety is 1.5. Determine the power transmitted by the gears.

Given Data:

Speed $N = 1440\ rpm$

Number of teeth in pinion $z_p = 20$

Number of teeth in gear $z_g = 40$

Permissible bending stress for steel $\sigma_b = 210\ \ N/mm^2$

Factor of safety $FOS = 1.5$

Module $m = 3\ \ mm$

Face width $b = 40\ \ mm$

Service factor $K_o = 1.5$

Brinell Hardness Number $BHN = 350$

Find:

1. Power transmitted by the spur gears

Solution:

(i) Beam strength (S_b)*:*

From Table 3.2, the Lewis form factor for 20 teeth is 0.32. Therefore, $Y = 0.32$.

Beam strength $S_b = mb\sigma_b Y = 3 \times 40 \times 210 \times 0.32 = 8064\ N$

(ii) Wear strength (F_w)*:*

Speed reduction ratio $i = \dfrac{40}{20} = 2$

Ratio factor $Q = \dfrac{2i}{i+1} = \dfrac{2 \times 2}{2+1} = 1.33$

$K_w = 0.16\left(\dfrac{BHN}{100}\right)^2$ for both the gears are made of steel with a 20° pressure angle

$$K_w = 0.16\left(\frac{350}{100}\right)^2 = 1.96$$

Pitch circle diameter of the pinion $d_p = mz_p = 4\times20 = 80\ mm$

Wear load $F_w = d_p \times b\times Q\times K_w = 80\times40\times1.33\times1.96 = 8341.76\ N$

(iii) *Effective load* $\left(F_{eff}\right)$

Pitch line velocity $v = \dfrac{\pi d_p N}{60\times10^3} = \dfrac{3.14\times80\times1440}{60\times10^3} = 6.03\ m/s$

Velocity factor $C_v = \dfrac{3}{3+v} = \dfrac{3}{3+6.03} = 0.3322$

Effective load $F_{eff} = \dfrac{K_o}{C_v}F_t = \dfrac{1.5}{0.3322}\times F_t = 4.52F_t\ \ N$

(iv) *Static load:*

In this problem, the wear load is higher than the beam strength. Hence, beam strength is considered for the gear design.

$$S_b = F_{eff}\times FOS$$

$$8064 = 4.52F_t\times1.5$$

$$F_t = \frac{8064}{4.52\times1.5} = 1189.38\ N$$

(v) *Power transmitted* (P) *:*

$$M_t = \frac{F_t d_p}{2} = \frac{1189.38\times80}{2} = 47575.2\ N-mm$$

Power transmitted $P = \dfrac{2\pi N M_t}{60\times10^6} = \dfrac{2\times3.14\times1440\times47575.2}{60\times10^6} = 7.17\ kW$

UNIT SUMMARY

- A shaft is a rotating element, circular in cross section, which is used in rotating machinery to transmit rotary motion and torque from one location to another location
- Mechanical shafts are generally classified into four broad types: Transmission shaft, Axle shaft, Spindle shaft, Machine shaft
- The shaft is subjected to pure bending moment, the bending stress $\sigma_b = \frac{32M_b}{\pi d^3}$
- The shaft is subjected to pure torsional moment, the torsional shear stress $\tau = \frac{16M_t}{\pi d^3}$
- Equivalent torsional moment $M_{te} = \sqrt{M_b^2 + M_t^2}$
- Equivalent bending moment $M_{be} = \frac{1}{2}\left(M_b + M_{te}\right)$
- Hollow shafts exhibit greater strength compared to solid shafts of equivalent weight
- The permissible value of maximum principal stress

$$\sigma_1 = \frac{16}{\pi d_o^3\left(1-C^4\right)} + \left[M_b + \sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}\right]$$

- The permissible value of maximum shear stress $\tau_{\max} = \frac{16}{\pi d_o^3\left(1-C^4\right)}\sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}$
- A key is a machine element which is used to connect transmission shafts with rotating elements such as gears, flywheels and pulleys
- Classification of keys: sunk key, saddle key, tangent key, round key, splines
- Shear stress induced in the key $\tau = \frac{F}{lb}$ **;** Crushing stress induced in key $\sigma_c = \frac{2F}{lt}$
- The keyway cut on the shaft or hub decreases the shaft's load-carrying capacity due to stress concentration near the keyway corners and the reduction in the shaft's cross-sectional area
- A coupling is a machine element that connects two rotating shafts for the purpose of transmitting power
- Muff coupling is a type of rigid coupling and also called as sleeve coupling
- A flange coupling is a mechanical device used to connect two shafts together at their ends to transmit power
- Types of rigid flange couplings: Unprotected flange coupling and protected flange coupling

- Spur gears are among the most common types of gears used in mechanical systems. They feature straight teeth that are parallel to the gear's axis
- Tangential load on gear teeth $F_t = mb\sigma_b Y$; Lewis equation for beam strength $S_b = mb\sigma_b Y$

  Dynamic load $F_d = \dfrac{21v(Ceb + F_t)}{21v + \sqrt{(Ceb + F_t)}}$; Effective load $F_{eff} = F_t + F_d$

- Module of gear teeth $m = \left[\dfrac{60\times10^6}{\pi} \left\{ \dfrac{(kW)K_o \times FOS}{zNC_v\left(\dfrac{b}{m}\right)\sigma_b Y} \right\} \right]^{\frac{1}{3}}$
- Maximum wear load $F_w = d_1 \times b \times Q \times K_w$

EXERCISES

Multiple Choice Questions

1. The motor shaft and pump shaft are constructed from the same material. The diameter of the motor shaft is twice that of the pump shaft. Therefore, the power transmitted by the motor shaft will be _____ compared to that of the pump shaft.

   (a) 2 times (b) 4 times

   (c) 8 times (d) 16 times

2. The shaft is designed on the basis of

   (a) torsional rigidity only (b) strength only

   (c) lateral rigidity only (d) all the above

3. The strength of the two shafts are same, if __________ of both the shaft is same.

   (a) diameter (b) angle of twist

   (c) material (d) twisting moment

4. Which one of the following is not desirable property for shaft material?

   (a) High strength (b) high notch sensitivity

   (c) good machinability (d) good wear resistance

5. If a shaft is subjected to a bending moment M and a twisting moment T, then the equivalent twisting moment is equal to

   (a) $M + T$ (b) $M^2 + T^2$

   (c) $\sqrt{M^2 + T^2}$ (d) $\sqrt{M + T}$

6. When the cross sectional area of the hollow shaft and solid shaft are same, the hollow shaft transmits ______ torque.

(a) same (b) less

(c) more (d) none of the above

7. Generally, _____ the thickness of the sunk key fits into the shaft keyway and remaining _____ in the hub keyway.

(a) half, half (b) ¼, ¾

(c) ¾, ¼ (d) 1 mm, 2mm

8. In a muff coupling, the effective length of the key is

(a) equal to sleeve length (b) equal to half of the sleeve length

(c) double the sleeve length (d) independent of the sleeve length

9. In a protected type flange coupling, the outside diameter of the hub is

(a) equal to shaft diameter (b) equal to half of the shaft diameter

(c) double the shaft diameter (d) independent of the shaft diameter

10. In a bushed-pin type flange coupling, the number of pins is usually

(a) 20 (b) 12

(c) 8 (d) 6

11. The maximum torque is 25 greater than the mean torque, then

(a) $M_t = (M_t)_{mean}$ (b) $M_t = 0.25 \times (M_t)_{mean}$

(c) $M_t = 1.25 \times (M_t)_{mean}$ (d) $M_t = 25 \times (M_t)_{mean}$

12. ______ have teeth that are straight and parallel to the gear's axis.

(a) spur gears (b) bevel gears

(c) helical gears (d) none of the above

13. Which one of the following is the ratio of the pitch diameter to the number of teeth on the gear?

(a) module (b) pitch point

(c) addendum (d) dedendum

14. Lewis equation for static beam strength of spur gear teeth is

(a) $S_b = b\sigma_b Y$ (b) $S_b = mb\sigma_b Y$

(c) $S_b = mb\sigma_b$ (d) $S_b = m^2 b\sigma_b Y$

15. Lewis equation in spur gears is used to find the

(a) tensile stress in bending (b) shear stress

(c) compressive stress in bending (d) fatigue stress

Answers to Multiple Choice Questions

1. (c) 2. (d) 3. (d) 4. (b) 5. (c) 6. (c) 7. (a) 8. (b) 9. (c) 10. (d) 11. (c) 12. (a) 13. (a) 14. (b) 15. (c)

Short and Long Answer Type Questions

1. Write the torsion equation and state the terms involved in it.
2. Write the characteristics of a good shaft material.
3. A solid shaft of diameter $25\ mm$ is transmitting a torque of $900\ N-mm$. Find the angle of twist per unit length by taking $G=80\ GPa$.
4. What is a key? State its function.
5. Mention the types of couplings.
6. What are the requirements of a good coupling?
7. Classify gears.
8. List the various terms used in spur gear terminology.
9. Define module and circular pitch.
10. What is the effect of the keyway on the strength of the shaft?
11. A shaft is subjected simultaneously to a torque of $27500\ N-mm$ and bending moment of $20000\ N-mm$. Find the diameter of the shaft if the maximum normal stress is $60\ N/mm^2$ and maximum shear stress is $30\ N/mm^2$.
12. A solid shaft made of steel is subjected to a B.M of $3000\ N-m$ and a torque of $10000\ N-m$. The shaft material has an ultimate tensile stress of $600\ N/mm^2$, and ultimate shear stress of $500\ N/mm^2$. Considering factor of safety of 6, determine the diameter of the shaft.
13. A solid circular shaft is used to transmit a torque of $12\ N-m$. The angle of twist over a length of 2 m is 2°. Estimate the required diameter of the shaft and shear stress induced in the material. Take $G=0.8\times10^5\ N/mm^2$.
14. Select a rectangular key for transmitting a power of $45\ kW$ at $600\ rpm$, to mount a hub of length $55\ mm$ on a solid circular shaft of diameter $45\ mm$.
15. Design a muff coupling to transmit 7 kW at 15 rpm. Design shear stress $=29\ N/mm^2$ for shaft and key. Shear stress for sleeve is $14\ N/mm^2$. Crushing stress between shaft and key is $9\ N/mm^2$.
16. A pin type flexible coupling has six bolts of $M12\times1$. The rubber bush has a diameter of $35\ mm$ and length of $25\ mm$. The bolt circle diameter is $170\ mm$. The allowable shear stress and normal stress

for bolt are $60\ MPa$ and $110\ MPa$ respectively. Find the power that can be transmitted by this pin. Take allowable bearing pressure for rubber bush as $0.8\ MPa$. The shaft diameter is $22\ mm$ and rotates at $800\ rpm$.

17. Design a protected type flange coupling to transmit power between two shafts $38\ mm$ and $50\ mm$. The allowable shear stress for shaft and bolts is $60\ MPa$. The allowable shear stress and bearing stress for key are $55\ MPa$ and $115\ MPa$ respectively. For CI flange, the allowable shear stress is $5\ MPa$.

18. Design a bushed pin type of flexible flange coupling for the following duty:

    Power to be transmitted = $90\ kW$

    Rotational speed of shaft = 140 rev./min

    Material for flanges = grey cast iron

    Material for both shaft and key = forged steel

    Material for bush = Rubber

    Assuming suitable values for stresses determine the dimensions of the coupling and draw a fully dimensioned sketch.

19. Design a bush type flanged coupling to transmit $14\ kW$ at $1900\ rpm$. Draw a neat a sketch of the coupling. Maximum torque to be transmitted is 30% more than mean torque.

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NPTEL VIDEOS

1. Lecture - 15 Design of Keys and Splines - Design of Machine Elements - I by Prof. G. Chakrborty, Department of Mechanical Engineering, IIT Kharagpur.

Keys and Splines

2. Lecture - 34 Design of Shafts - Design of Machine Elements - I by Prof.B.Maiti, Department of Mechanical Engineering, IIT Kharagpur.

Design of Shafts

3. Lecture - 20 Shaft Couplings - I - Design of Machine Elements - I by Prof. G. Chakrborty, Department of Mechanical Engineering, IIT Kharagpur.

Shaft Couplings - I

4. Lecture - 21 Shaft Couplings - II - Design of Machine Elements - I by Prof. G. Chakrborty, Department of Mechanical Engineering, IIT Kharagpur.

Shaft Couplings - II

4 Design of Power Screws and Springs

UNIT SPECIFICS

Through this unit, the following aspects are discussed:

- Power screws – Thread profiles used, merits and demerits
- Torque required to overcome thread friction, self - locking and overhauling property
- Efficiency of power screws and types of stresses induced
- Design of screw jack and toggle jack
- Springs – Classification, application, terminology, materials and specifications
- Deflection, stresses and energy stored in springs
- Design of helical, tension and compression springs subjected to uniform applied loads
- Leaf springs - Construction and application

RATIONALE

The fourth unit of this book helps the students to have good understanding of the type of threads used in power screws for effective transmission of power. After completing this course, a student will be able to design power screws to either be self-locking or to minimize the torque required for motion, depending on the application's needs. Further reading will help to understand the design procedure of power screw mechanisms i.e. screw jack and toggle jack by considering the stresses induced in power screws. Springs are fundamental components used in various mechanical systems to store and release mechanical energy. This unit also presents the design of helical, tension and compression springs subjected to uniform applied loads for various applications.

PRE-REQUISITES

Strength of materials

UNIT OUTCOMES

List of outcomes of this unit is as follows:

On the successful completion of the unit, students will be able to

U4-O1: Select the suitable threads for various type of threads used in power screws for effective transmission of power

U4-O2: Design power screws to either be self-locking or to minimize the torque required for motion, depending on the application's needs

U4-O3: Design the screw jack and toggle jack by considering the various stresses induced in power screws

U4-O4: Explain the springs, its types, terminology and derive the stress & deflection equations for helical spring

U4-O5: Design the helical, tension and compression springs subjected to uniform applied loads for various industrial applications

Unit 4 Outcomes	**Mapping with Course Outcomes** (1 – weak correlation, 2 – medium correlation, 3 – strong correlation)				
	CO-1	**CO-2**	**CO-3**	**CO-4**	**CO-5**
U4-O1	3	3	1	3	3
U4-O2	3	3	1	3	3
U4-O3	3	3	1	3	3
U4-O4	3	3	1	3	3
U4-O5	3	3	1	3	3

4.1. INTRODUCTION

Power screws are mechanical devices that change rotational motion into linear motion and the other way around. The other name of the power screws is also called as translation screws. Power presses, conveyors, jacks, vices, and other machine tools all make extensive use of power screws. The screw, nut, and element that holds the screw or nut in place are the three basic components of a power screw. The power screws operate in two different ways based on the holding arrangement. One category, the screw rotates in its bearing, while the nut has an axial motion. The example for this category is lead screw of the lathe. In other uses, the screw travels axially while the nut remains fixed. Examples are a machine vice and a screw jack. Among power screw's benefits are:

(i) Compact in design and load carrying capacity is high

(ii) It is easy to design and manufacture, there is no specialized machinery is required

(iii) Power screws provide a mechanical advantage, allowing for the transmission of high axial forces with relatively low input torque. This makes them suitable for applications where a large force is required

(iv) Power screws, especially ball screws, are known for their high precision and accuracy in converting rotary motion into linear motion. This is crucial in applications where precise positioning is required, such as in CNC machines and robotics.

(v) Power screws often require less maintenance and operate smoothly without any noise

(vi) It could have a self-locking feature. This feature is crucial in screw jack applications to stop the load from falling on its own

The disadvantages are:

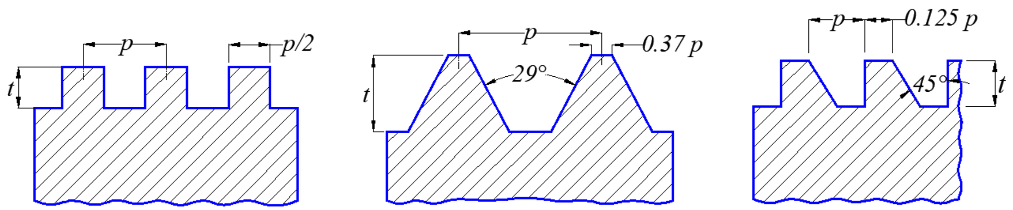
(i) Its 40% efficiency is really low

(ii) High thread friction causes a screw or nut to wear out quickly

The power screws are used for high efficiency and low efficiency applications. Lead screw and presses are examples for high efficiency applications. The example for low efficiency applications are screw jacks, clamps and vices with self-locking property. If the rolling friction is employed instead of sliding friction, the efficiency of power screw can be increased.

4.2. THREAD PROFILES USED FOR POWER SCREWS

Power screws use three different kinds of screw threads. The various screw thread types utilized in power screws are shown in Fig. 4.1.



t = 0.5 p
Square thread

t=0.5 p
Acme thread

t=0.75 p
Buttress thread

Fig. 4.1: Power screw thread profiles

The various types of screw threads are square thread, acme thread, buttress thread. V threads are not used in power screws due to their high friction.

Pow. Scr. Dri. Lec 17

(i) Square thread: Figure 4.1(a) illustrates a square thread that is used to transmit power in both directions. Square threads provide a higher efficiency and there is no radial or bursting pressure on the nut. Hence, the motion of the nut is uniform due to the absence of side thrust. The life of the nut is also increased. The nuts are often cut on a lathe using a single point cutting tool because they are difficult to cut using taps and dies. The machining using the single point cutting tool is costlier as compared to multi-point cutting tools. Square threads have small thickness at the core diameter, which lowers the load carrying capacity of the screw. Wear of the thread surface is a major issue with the power screw's lifespan; when worn out, the nut or screw needs to be replaced. The nominal diameter of the screw thread is the largest diameter of the thread and is typically used to specify the size of the screw. Table 4.1 shows the standard dimensions for square threads.

Table 4.1: Properties of square threads (normal series)

Nominal diameter, $d_o (mm)$	Pitch, $p\ (mm)$
22,24,26,28	5
30,32,36	6
40,44	7
48,50,52	8
55,60	9
65,70,75,80	10
85,90,95,100	12

(ii) Acme thread: The Acme thread is illustrated in Fig. 4.1 (b). Acme is a trapezoidal thread profile characterized by a V angle of 29^o, as specified in ASTM B1.5. Because of its ability to withstand large loads and ease of production, it is the most widely used thread for traversing linear motion (such as leadscrews and power screws). The efficiency of a modified acme thread is marginally lower than that of a square thread due to the slight slope added to its sides. The slope gives the nut some bursting pressure, but it also increases its area in shear. It is utilized in situations where a split nut is necessary and where wear-absorbing capacity is built in, such as the lead screw in a lathe. An adjustable split nut can be used to take up wear. Since acme thread can be manufactured with multipoint cutting tool, so it is more economical to manufacture. The thread thickness at core diameter is higher than square thread, so it can withstand more load. The purpose of the split-type nut is to compensate axial wear on the acme thread surface. The nut's two halves are tightened together when the threads wear out, and it is periodically adjusted in a lathe to make up for wear. Table 4.2 shows the standard proportions for acme threads.

Table 4.2: Proportions of ISO metric trapezoidal threads

Nominal diameter, $d_o (mm)$	Pitch, $p\ (mm)$
24,28	5
32,36	6
40,44	7
48,52	8
60	9
70,80	10
90,100	12

(iii) Buttress thread: Fig. 4.1 (c) shows the buttress thread. A buttress thread is a type of thread profile characterized by asymmetrical flanks. One flank is perpendicular to the thread axis, while the other flank has a slanted or angled profile resembling the shape of a stair-step or the roof of a house. This asymmetrical design provides high resistance to axial forces in one direction while allowing for smooth

movement in the opposite direction. The axial wear at the thread surface can be compensated for with a split-type nut. Because of the greater thickness at the thread's base, the buttress thread is stronger than square threads. Whereas square and acme threads communicate power and motion in both directions, the buttress thread's demerit is that it can only transmit in one direction. Buttress threads are commonly used in applications where a high axial load needs to be supported efficiently, such as in screw jacks, vices, and presses.

4.3. SCREW THREAD TERMINOLOGY

The terminology of the power screw is explained with the help of Fig. 4.2.

i. *Pitch:* The distance between a point on one thread to the corresponding point on the adjacent thread and it is measured parallel to the axis of the screw. The pitch is represented by the letter p.

ii. *Lead:* A screw's lead, which is measured parallel to the screw's axis, is the distance the screw advances in a full turn. The lead is represented by the letter l. The value of lead for single threaded screw is pitch p, and double threaded screw is two times the pitch i.e. $2p$.

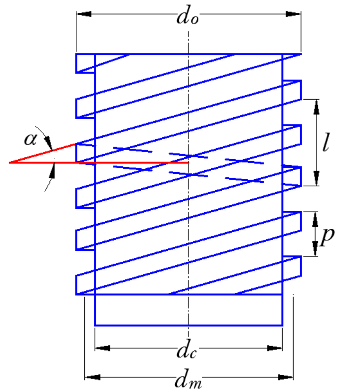


Fig. 4.2: Terminology of power screw

iii. *Nominal Diameter:* Nominal diameter, which is the screw's greatest diameter and is represented by the letter d_o.

iv. *Core Diameter:* The letter d_c stands for the core diameter, which is the smallest screw thread diameter.

v. *Helix Angle:* A screw thread's helix angle is the angle formed between the thread's helix and a plane perpendicular to the screw's axis. The helix angle is denoted by α.

From Fig. 4.2, the core diameter

$$d_c = d_o - \left[\frac{p}{2} + \frac{p}{2}\right] \Rightarrow d_c = (d_o - p) \tag{4.1}$$

Mean diameter $d_m = \frac{1}{2}[d_o + d_c] = \frac{1}{2}[d_o + (d_o - p)]$

$$d_m = (d_o - 0.5p) \tag{4.2}$$

The following assumptions were considered to determine the torque required to overcome the friction between the screw and nut.

1. The screw is considered as an inclined plane with inclination angle α
2. The direction of the load W always acts in vertically downward direction

3. The effort P is applied to the lever to raise or lower the load. This effort acting at the mean radius of the screw which is to raise or lower the load W. The effort P can be multiplied to find the torque needed to raise or lower the load and the mean radius $\left(d_m/2\right)$

Here, we considered two categories for calculating the torque required to overcome the friction viz. (1) torque required to raise the load and (2) torque required to lower the load.

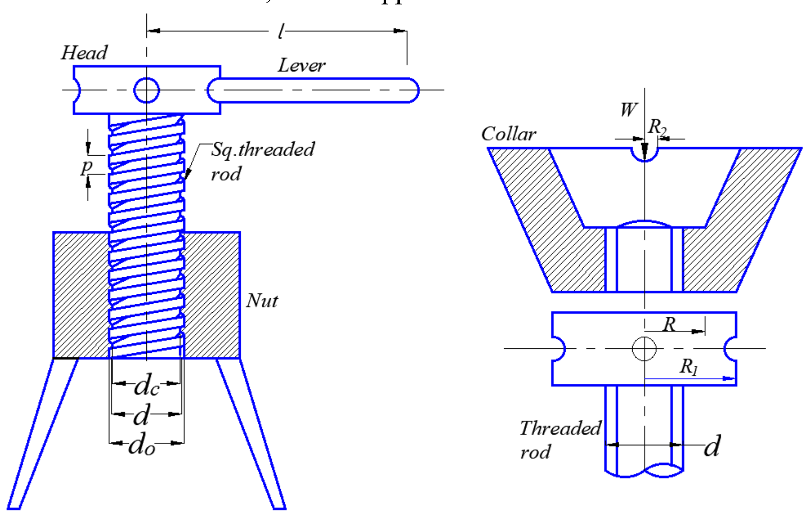
4.4. TORQUE REQUIRED TO OVERCOME THREAD FRICTION

The inclination angle is represented by the letter α, and the screw thread is thought of as an inclined plane. The forces acting at a point on the inclined plane while the load is being raised is given below.

1. Load W: The load acts in a downward direction at all times
2. Normal reaction N: The normal response direction that is acting normal (perpendicular) to the inclined plane
3. Frictional force μN: The motion is opposed by the frictional force, hence it acts opposite direction to the motion direction.
4. Effort P: Perpendicular to the load W acts the effort P.

4.4.1. Torque required to raise the load

By taking into account the scenario where the load is put on the head of the square threaded rod, the torque needed to raise the load can be calculated using the square threaded screws used in the screw jack (Fig. 4.3) as an example. To raise or lower the load, force is applied at the lever's end.



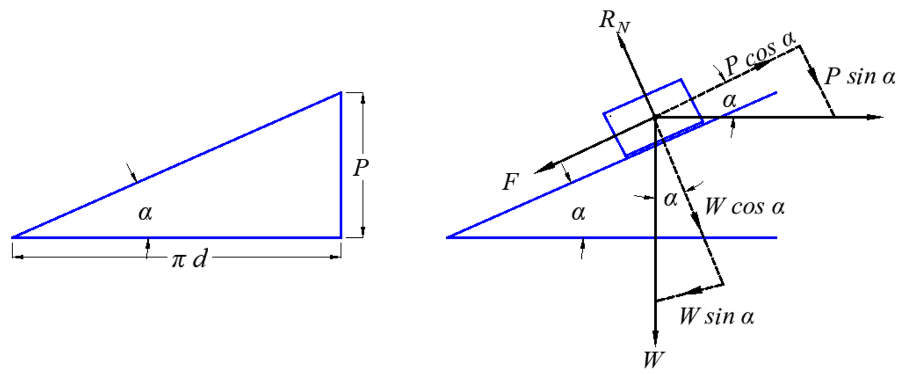
(a) (b)

Fig. 4.3: (a) Screw jack (b) Thrust collar

Using the geometry in Fig. 4.4(a), we obtain,

$$\tan\alpha = \frac{l}{\pi d_m} \tag{4.3}$$

Where α is helix angle, p is pitch of the screw, d_m is mean diameter of the screw. All forces acting on the body is depicted in Fig. 4.4 (b), and the force placed on the screw thread's circumference can be considered as horizontal.



(a) (b)

Fig. 4.4: (a) Geometry of screw (b) Forces acting on the power screw

As the load is being raised, the force of friction $(F = \mu RN)$ acts downward. μ is coefficient of friction between the screw and nut $(\mu = \tan\phi)$ where ϕ is friction angle.

Resolving the forces horizontally,

$$P\cos\alpha = W\sin\alpha + F = W\sin\alpha + \mu R_N \tag{4.3}$$

As the forces are vertically resolved,

$$R_N = P\sin\alpha + W\cos\alpha \tag{4.4}$$

In Eq. (4.3), we obtain by substituting the value of R_N (Eq. (4.4)),

$$P\cos\alpha = W\sin\alpha + F = W\sin\alpha + \mu\left(P\sin\alpha + W\cos\alpha\right)$$

$$= W\sin\alpha + \mu P\sin\alpha + \mu W\cos\alpha$$

$$P\cos\alpha - P\mu\sin\alpha = W\sin\alpha + W\mu\cos\alpha \tag{4.5}$$

Rearranging the Eq. (4.5),

$$P = W \times \frac{\left(\sin\alpha + \mu\cos\alpha\right)}{\left(\cos\alpha - \mu\sin\alpha\right)} \tag{4.6}$$

In Eq. (4.6), substituting the value of $(\mu = \tan\phi)$,

$$P = W \times \frac{\left(\sin\alpha + \tan\phi\cos\alpha\right)}{\left(\cos\alpha - \tan\phi\sin\alpha\right)} \tag{4.7}$$

When we multiply the numerator and denominator of Eq. (4.7) by $\cos\phi$, we obtain,

$$P = W \times \frac{(\sin\alpha\cos\phi + \sin\phi\cos\alpha)}{(\cos\alpha\cos\phi - \sin\phi\sin\alpha)} = W \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)} = W \times \tan(\alpha+\phi) \tag{4.8}$$

The torque required to overcome the thread friction (Eq. (4.9)),

$$(M_t)_s = P \times \frac{d_m}{2} = W\tan(\alpha+\phi) \times \frac{d_m}{2} \tag{4.9}$$

Where d_m is mean diameter of the screw.

A thrust collar takes up the axial load and produces the torque required to overcome friction at the collar.

$$(M_t)_c = \frac{2}{3} \times \mu_c \times W\left[\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right] \tag{4.10}$$

(Assuming uniform pressure conditions)

$$(M_t)_c = \mu_c \times W\left[\frac{R_1 + R_2}{2}\right] = \mu_c \times W \times R \tag{4.11}$$

(Assuming uniform wear conditions)

Where, R_1 *and* R_2 are outer and inner radii of the collar

R is Mean radius of the collar $\frac{R_1 + R_2}{2}$

μ_c is coefficient of friction of the screw

Total torque required to overcome friction

$$M_t = (M_t)_s + (M_t)_c \tag{4.12}$$

4.4.2 Torque required to lower the load

A little analysis reveals that the force of friction acts upwards as the weight is lowered. Fig. 4.5 depicts every force operating on the body.

Resolving the horizontal forces,

$$P\cos\alpha = F - W\sin\alpha$$

$$= \mu R_N - W\sin\alpha \tag{4.13}$$

and resolving the vertical forces,

$$R_N = W\cos\alpha - P\sin\alpha \tag{4.14}$$

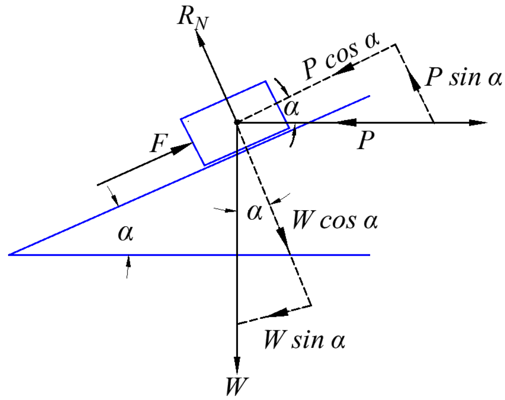


Fig. 4.5: Forces acting on the body when the load is lowered

We obtain, by replacing this value of R_N (Eq. (4.14)) in Eq. (4.13),

$$P\cos\alpha = \mu\left(W\cos\alpha - \mathrm{P}\sin\alpha\right) - W\sin\alpha$$

$$= \mu W\cos\alpha - \mu \mathrm{P}\sin\alpha - W\sin\alpha$$

$$P\cos\alpha + \mu P\sin\alpha = \mu W\cos\alpha - W\sin\alpha \tag{4.15}$$

Rearranging the Eq. (4.15),

$$P = W\times\frac{\left(\mu\cos\alpha - \sin\alpha\right)}{\left(\cos\alpha + \mu\sin\alpha\right)} \tag{4.16}$$

When we change the value of $\mu = \tan\phi$ in Eq. (4.16), we obtain,

$$P = W\times\frac{\left(\tan\phi\cos\alpha - \sin\alpha\right)}{\left(\cos\alpha + \tan\phi\sin\alpha\right)} \tag{4.17}$$

Multiplying by $\cos\phi$ on both the numerator and denominator (Eq. (4.17)), we get,

$$P = W\times\frac{\left(\sin\phi\cos\alpha - \cos\phi\sin\alpha\right)}{\left(\cos\phi\cos\alpha + \sin\phi\sin\alpha\right)}$$

$$P = W\times\frac{\sin\left(\phi-\alpha\right)}{\cos\left(\phi-\alpha\right)} = W\tan\left(\phi-\alpha\right) \tag{4.18}$$

∴ The amount of torque required to reduce friction between the nut and screw,

$$M_t = P\times\frac{d_m}{2} = W\tan\left(\phi-\alpha\right)\frac{d_m}{2} \tag{4.19}$$

4.5. SELF-LOCKING OF POWER SCREWS

Power screws can exhibit self-locking behavior under certain conditions, which means they can resist external forces and prevent back-driving without the need for additional locking mechanisms. Self-locking is desirable in many applications to maintain position or prevent undesired motion when the screw is not actively being turned. According to Eq. (4.20), the torque at the screw's circumference to lower the end is provided by,

$$M_t = W\tan(\phi-\alpha)\times\frac{d_m}{2} \tag{4.20}$$

The torque needed to lower the load in the Eq. (4.20) is positive, meaning that some effort is needed to do so; this is a self-locking phenomenon, if the friction angle is more than the helix angle $(\phi>\alpha)$. However, it's important to note that not all power screws are self-locking. For instance, Acme and square threads are commonly used in applications where self-locking is desired, while other thread forms like V-threads may not be self-locking and might require additional measures such as a braking mechanism or a separate locking device to prevent back-driving.

4.6. OVERHAULING OF POWER SCREWS

"Overhauling" of power screws refers to a condition where the load or external forces on the screw can cause it to rotate freely in one direction (usually to lower a load or allow for easy adjustment) without the need for actively turning the screw. Eq. (4.21) provides the torque at the screw's circumference to lower the end,

$$M_t = W\tan(\phi-\alpha)\times\frac{d_m}{2} \tag{4.21}$$

The torque needed to reduce the weight in the Eq. (4.21) is negative, if the friction angle is less than the helix angle $(\phi<\alpha)$, meaning that the load will begin to move downward without any effort on the part of the operator. This type of screw is known as an overhauling screw.

Overhauling is generally undesirable in applications where maintaining position or preventing undesired motion is crucial. However, in certain situations, such as in screw jacks or lifting mechanisms, controlled overhauling can be advantageous for quick adjustment or repositioning.

To prevent overhauling and ensure stability in applications where it's undesirable, various methods can be employed, such as using self-locking threads (e.g., Acme threads), adding braking mechanisms, incorporating anti-backlash nuts, or applying external clamping devices. These measures help to maintain the desired position and prevent unintended movement of the screw under load.

4.7. EFFICIENCY OF POWER SCREWS

The efficiency of square threaded screws may be defined as the ratio between the ideal effort (i.e. the effort required to move the load, neglecting friction) to the actual effort (i.e. the effort required to move the load taking friction into account).

The ratio of the ideal effort *i.e. the effort needed to transfer the load without considering friction* to the actual effort *i.e. the effort needed to move the weight while accounting for friction* can be used to determine the efficiency of square threaded screws.

The force used to raise the load at the screw's circumference is,

$$P = W\tan(\alpha+\phi) \tag{4.22}$$

ϕ would equal zero, if there had been no friction between the screw and the nut. Then, using Eq. (4.23), we can determine the value of effort P_0 required to raise the load.

Substituting $\phi = 0$ in Eq. (4.22))

$$P_0 = W\tan\alpha \tag{4.23}$$

Therefore, efficiency of power screw,

$$\eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W\tan\alpha}{W\tan(\alpha+\phi)} = \frac{\tan\alpha}{\tan(\alpha+\phi)} \tag{4.24}$$

The efficiency of a screw jack is observed to be independent of the load raised on Eq. (4.24). It solely depends on ϕ and α. Additionally, because $(\tan\alpha = l/\pi d_m)$, α is dependent on the lead (l) and the screw's mean diameter (d_m). Hence, the following three variables affect a square threaded power screw's efficiency:

- Lead of the screw
- Mean diameter of the screw
- Coefficient of friction

4.7.1. Maximum efficiency of square threaded screw

A square threaded screw can be made more efficient in two ways.

- Through appropriate lubrication, the coefficient of friction between the screw and the nut can be reduced
- By employing multiple-start threads, the helix angle can be increased to between 40° and 45°

A screw with this kind of helix angle also loses its self-locking ability.

Rewrite the Eq. (4.24), we get

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\sin\alpha/\cos\alpha}{\sin(\alpha+\phi)/\cos(\alpha+\phi)} = \frac{\sin\alpha\times\cos(\alpha+\phi)}{\cos\alpha\times\sin(\alpha+\phi)}$$

After multiplying the denominator and numerator by two, we obtain:

$$\eta = \frac{2\sin\alpha \times \cos(\alpha+\phi)}{2\cos\alpha \times \sin(\alpha+\phi)} = \frac{\sin(2\alpha+\phi)-\sin\phi}{\sin(2\alpha+\phi)+\sin\phi} \tag{4.25}$$

$$\ldots\begin{bmatrix} \because 2\sin A\cos B = \sin(A+B)+\sin(A-B) \\ 2\cos A\sin B = \sin(A+B)-\sin(A-B) \end{bmatrix}$$

The efficiency given by Eq. (4.25) will be maximum when $\sin(2\alpha+\phi)$ is maximum, *i.e.* when

$$\therefore \quad \sin(2\alpha+\phi)=1 \qquad \text{when} \quad 2\alpha+\phi=90^\circ$$

$$2\alpha = 90^\circ - \phi \tag{4.26}$$

$$\alpha = 45^\circ - \phi/2 \tag{4.27}$$

Substituting the value of 2α (Eq. (4.26)) in Eq. (4.25), we have maximum efficiency,

$$\eta_{\max} = \frac{\sin(90^\circ-\phi+\phi)-\sin\phi}{\sin(90^\circ-\phi+\phi)+\sin\phi} = \frac{\sin 90^\circ - \sin\phi}{\sin 90^\circ + \sin\phi}$$

$$\eta_{\max} = \frac{1-\sin\phi}{1+\sin\phi} \tag{4.28}$$

Example 4.1: The nominal diameter of vertical single start square threaded screw is $40\ mm$ with $7\ mm$ mm pitch. The load raised by the screw is $2\,kN$ by means of hand wheel. The values of coefficient of friction at the screw threads is 0.15. Find the torque required to overcome the friction between screw and nut.

Given Data:

Load raised by the screw $W = 2\ kN$

Nominal diameter of the screw $d_o = 40\ mm$

Pitch of the screw $p = 7\ mm$

Lead for single start square threaded screw $l = p = 7\ mm$

Coefficient of friction at the screw threads $\mu = \tan\phi = 0.15$

Find:

Torque required to overcome the friction between screw and nut

Solution:

From Eq. (4.2),

$$d_m = d_o - 0.5p = 40 - 0.5(7) = 36.5\ mm$$

From Eq. (4.3),

$$tan\alpha = \frac{l}{\pi d_m} = \frac{p}{\pi d_m} = \frac{7}{\pi(36.5)} = 0.0611$$

The tangential force required at the circumference of the screw,

$$P = W\tan(\alpha+\phi) = W \times \left(\frac{\tan\alpha + \tan\phi}{1-\tan\alpha\tan\phi}\right)$$

$$P = 2\times10^3 \times \left(\frac{0.0611+0.15}{1-0.0611\times0.15}\right) = 422.90\ N$$

The amount of torque needed to reduce the friction between the nut and screw

$$M_t = P\times\frac{d_m}{2} = 422.9\times\frac{36.5}{2} = 7177.9\ N-mm$$

Example 4.2: The nominal diameter of vertical double start square threaded power screw is $44\ mm$ with $7\ mm$ pitch. The load acting on the screw is $8\ kN$ and mean diameter of the collar is $45\ mm$. Coefficient of friction is 0.12. Find

1. Torque required to rise the load
2. Torque required to lower the load
3. Efficiency
4. Is the screw of self-locking type?

Given Data:

Load $W = 8\ kN$

Nominal diameter of the screw $d_o = 44\ mm$

Pitch of the screw $p = 7\ mm$

Lead for single double square threaded screw $l = 2p = 14\ mm$

Coefficient of friction at the screw threads $\mu_s = \mu_c = \tan\phi = 0.12$

Mean diameter of the collar $D = 45\ mm$ or $R = 22.5\ mm$

Find:

1. Torque required to rise the load
2. Torque required to lower the load
3. Efficiency
4. Is the screw of self-locking type?

Solution:

From Eq. (4.2)

$d_m = d_o - 0.5p = 44 - 0.5(7) = 47.5\ mm$

From Eq. (4.3),

$$tan\,\alpha = \frac{l}{\pi d_m} = \frac{14}{\pi(47.5)} = 0.0939$$

(i) Torque required to rise the load:

The tangential force required at the circumference of the screw,

$$P = W\tan(\alpha+\phi) = W \times \left(\frac{\tan\alpha + \tan\phi}{1-\tan\alpha\tan\phi}\right)$$

$$= 8\times10^3 \times \left(\frac{0.0939+0.12}{1-0.0611\times0.12}\right) = 1723.84\ N$$

The torque required to overcome the screw friction,

$$(M_t)_s = P\times\frac{d_m}{2} = 1723.84\times\frac{47.5}{2} = 40941.2\ N-mm$$

The torque required to overcome the collar friction,

$$(M_t)_c = \mu_c WR = 0.12\times8\times10^3\times22.5 = 21600\ N-mm$$

The total torque required to overcome the friction including collar friction,

$$M_t = (M_t)_s + (M_t)_c = 40941.2 + 21600 = 62541.2\ N-mm$$

(ii) Torque required to lower the load:

The tangential force required at the circumference of the screw,

$$P = W\tan(\phi-\alpha) = W \times \left(\frac{\tan\alpha - \tan\phi}{1+\tan\alpha\tan\phi}\right)$$

$$= 8\times10^3 \times \left(\frac{0.12-0.0939}{1+0.0611\times0.12}\right) = 207.28\,N$$

The torque required to overcome the screw friction,

$$(M_t)_s = P\times\frac{d_m}{2} = 207.28\times\frac{47.5}{2} = 4922.91\ N-mm$$

The torque required to overcome the collar friction,

$$(M_t)_c = \mu_c WR = 0.12\times8\times10^3\times22.5 = 21600\ N-mm$$

The total torque required to overcome the friction including collar friction,

$$M_t = (M_t)_s + (M_t)_c = 4922.91 + 21600 = 26522.91\ N-mm$$

(iii) Efficiency:

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \tan\alpha \times \left(\frac{1-\tan\alpha\tan\phi}{\tan\alpha+\tan\phi}\right) = 0.0939 \times \left(\frac{1-0.0939\times0.12}{0.0939+0.12}\right) = 0.434$$

Efficiency $\eta = 43.4\%$

(iv) Is the screw of self-locking type?

$tan\,\alpha = 0.0939$ $\quad\quad \therefore \alpha = 0.0936$

$\tan\phi = 0.12$ $\quad\quad \therefore \phi = 0.119$

Since $\phi > \alpha$, the screw is self-locking and not overhauling.

4.8. TYPES OF STRESSES IN POWER SCREWS

The stress analysis of power screw and nut is based upon the following assumptions.

1. It is assumed that the load supported by both the screw and the nut is evenly spread across their entire thread engagement. However, the first few threads in engagement carry the major portion of the load due to elastic deflection.
2. Assume uniform bearing pressure throughout the thread engagement achieved through proper lubrication.
3. The effects of fillet radii, surface finish, class of fit, etc. on the actual stress distribution are neglected.

The various stresses induced in the power screw are discussed below.

1. *Direct stress:* The direct tensile or compressive stresses is developed in a power screw due to axial load. The magnitude of the direct stress is equal to the ratio between axial load (W) and minimum cross sectional area of the thread (A_c) i.e. corresponding area of the minor or core diameter (d_c). This relationship is only valid when the unsupported length of the screw distance between load and nut is small.

2. *Bearing pressure:* The bearing pressure induced in a power screw refers to the pressure exerted on the contacting surfaces of the screw and nut threads due to the applied load (Fig. 4.6).

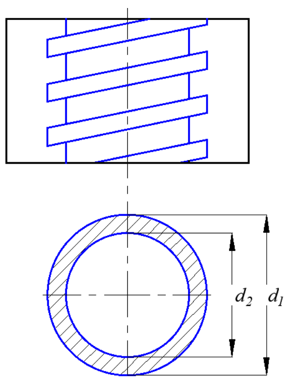


Fig. 4.6: Area under bearing pressure

Bearing pressure is a critical consideration in power screw design as it directly affects the wear, friction, and overall performance of the screw assembly. Excessive bearing pressure can lead to premature wear, galling, or even failure of the threads.

The bearing pressure assumed to be uniform and taken the low values in order to handle the inaccuracy due to its assumption. The number of threads n can be calculated from Eq. (4.29):

$$n=\frac{4W}{p_b\pi\left(d_1^2-d_2^2\right)} \tag{4.29}$$

Where d_1 is the major diameter, d_2 is the minor diameter, p_b is the bearing pressure $\left(5-25\ N/mm^2\right)$.

3 *Transverse shear stress:* In power screws, the threads of both the screw and the nut experience a transverse shearing stress. The transverse shear stress for rectangular cross section is given as (Eq. (4.30)),

$$\tau=\frac{3W}{2A} \tag{4.30}$$

In Eq. (4.30), A is shear area

Shear area $A=n\pi d_2 t$ for the screw

Shear area $A=n\pi d_1 t$ for the nut

Where n is number of threads, t is thread thickness at core diameter of the screw

4. *Torsional shear stress:* The screw experiences torsional shear stress due to the external torque, applied to lift the load. The following relation (Eq. (4.31)) can be used to evaluate the torsional shear stress;

$$\tau=\frac{16T}{\pi d_2^3} \tag{4.31}$$

5. *Buckling stress:* If the axial load on the power screw is compressive and the distance between the load and the nut, where the screw is unsupported, is short, then designing based on average compressive stress may be appropriate. But if the unsupported length is large, the screw must be designed as a column with appropriate end conditions. The crippling or buckling force can be calculated using the following Rankine formula:

$$P_{cr}=\frac{\sigma_c A}{1+a\left(\frac{l}{k}\right)^2} \tag{4.32}$$

where σ_c is the crippling or buckling stress, a is the Rankine constant, l is the effective length of the column as per end conditions, k is the radius of gyration.

Example 4.3: An axial force of $12\ kN$ acts upon a power screw with double start square threads, nominal diameter of $25\ mm$ and pitch of $5\ mm$. The screw collar has an outside diameter of $50\ mm$ and an inner diameter of $20\ mm$. It is to assume that the coefficients of thread and collar friction are 0.25 and 0.2, respectively. The screw has a 15 rpm. Considering consistent wear at the collar and a $5.8\ N/mm^2$ maximum thread bearing pressure, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.

Given Data:

Load $W = 12\ kN$

Nominal diameter of the screw $d_o = 25\ mm$

Pitch of the screw $p = 5\ mm$

Lead for double square threaded screw $l = 2p = 10\ mm$

Coefficient of friction at the thread $\mu_s = \tan\phi = 0.25$

Coefficient of friction at the collar $\mu_c = \tan\phi = 0.2$

Outside diameter of the collar $d_1 = 50\ mm$ or $R_1 = 25\ mm$

Inner diameter of the collar $d_2 = 20\ mm$ or $R_2 = 10\ mm$

Speed $N = 15\ rpm$

Bearing pressure $p_b = 5.8\ N/mm^2$

Find:

1. Torque required to rotate the screw
2. Stress in the screw
3. Number of threads of nut in engagement

Solution:

From Eq. (4.2)

$$d_m = d_o - 0.5p = 25 - 0.5(5) = 27.5\ mm$$

From Eq. (4.3),

$$tan\,\alpha = \frac{l}{\pi d_m} = \frac{10}{\pi(27.5)} = 0.1158$$

(i) Torque required to rotate the screw:

The tangential force required at the circumference of the screw,

$$P = W\tan(\alpha+\phi) = W\times\left(\frac{\tan\alpha+\tan\phi}{1-\tan\alpha\tan\phi}\right)$$

$$= 12\times10^3\times\left(\frac{0.1158+0.25}{1-0.1158\times0.25}\right) = 4520.5\ N$$

The torque required to overcome the screw friction,

$$(M_t)_s = P\times\frac{d_m}{2} = 1723.84\times\frac{27.5}{2} = 62156.43\ N-mm$$

The torque required to overcome the collar friction,

$$(M_t)_c = \mu_c W\left(\frac{R_1+R_2}{2}\right) = 0.2\times12\times10^3\times\left(\frac{25+10}{2}\right) = 42000\ N-mm$$

The total torque required to overcome the friction including collar friction,

$$M_t = (M_t)_s + (M_t)_c = 62156.43 + 42000 = 104.1\times10^3\ N-mm$$

(ii) The stress in the screw:

We know that, the core diameter of the screw

$$d_c = d_o - p = 25 - 5 = 20\ mm$$

Cross sectional area of the screw

$$A_c = \frac{\pi}{4}d_c^2 = \frac{\pi}{4}\times20^2 = 314\ mm^2$$

Direct stress $\sigma_c = \dfrac{W}{A} = \dfrac{12\times10^3}{314} = 38.22\ N/mm^2$

Shear stress $\tau = \dfrac{16M_t}{\pi d_c^3} = \dfrac{16\times104.1\times10^3}{3.14\times20^3} = 66.31 N/mm^2$

Maximum shear stress in the screw

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma_c^2+4\tau^2} = \frac{1}{2}\sqrt{(38.22)^2+4\times(66.31)^2}$$

$$= 138.02 N/mm^2 = 138.02\ MPa$$

(iii) Number of threads of nut in engagement:

From Eq. (4.26), the number of nut threads engagement

$$n = \frac{4W}{p_b\pi\left(d_1^2-d_2^2\right)} = \frac{4\times12\times10^3}{5.8\times3.14\times\left(50^2-20^2\right)} = 1.25 \simeq 2$$

The number of nut threads engagement $n = 2$

4.9. DESIGN OF SCREW JACK

The schematic diagram of the screw jack is shown in Fig. 4.7. The different parts of the screw jack are listed below.

1. Square threaded screw
2. Nut and collar for nut
3. Head at the top of the screw for handle
4. Cup at the top of the head to carry the load
5. Body
6. Handle

The following procedure is used to design the screw jack for the load W.

1. Design of screw:

Consider the screw under pure compression, find the core diameter of the screw using Eq. (4.33),

$$\sigma_c = \frac{W}{A} = \frac{W}{\frac{\pi}{4} \times d_c^2} \tag{4.33}$$

Where, core diameter of the screw $d_c = \sqrt{\frac{4W}{\pi \sigma_c}}$

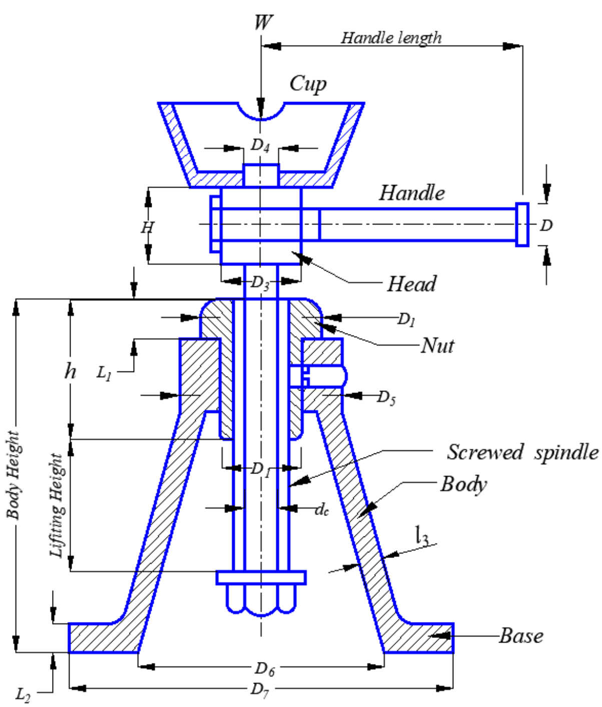


Fig. 4.7: Schematic diagram of screw jack

2. Torque required to rotate the screw:

The torque required to lift the load (Eq. (4.34)) is,

$$M_t = P \times \frac{d_m}{2}$$

$$M_t = W \tan(\alpha + \phi) \times \frac{d_m}{2} \tag{4.34}$$

3. Shear stress due to torque applied:

Shear stress induced due to toque M_t applied (Eq. (4.35)),

$$\tau = \frac{16 M_t}{\pi d_c^3} \tag{4.35}$$

4. Principal stresses:

We can find the maximum principal stress (tensile or compressive), using maximum principal stress theory,

$$(\sigma_c)_{\max} = \frac{1}{2}\left[\sigma_c + \sqrt{\sigma_c^2 + 4\tau^2}\right] \tag{4.36}$$

and maximum shear stress,

$$\tau_{\max} = \frac{1}{2} \times \sqrt{\sigma_c^2 + 4\tau^2} \tag{4.37}$$

For safety $\sigma_{c(\max)} < \sigma_{(Given)}$ & $\tau_{\max} < \tau_{(Given)}$

5. Design of nut:

The height of nut can be found by considering the bearing pressure on nut, (Eq. (4.38)) i.e.

$$p_b = \frac{W}{\frac{\pi}{4}\left[d^2 - d_c^{\,2}\right] \times n} \tag{4.38}$$

Where n is no. of threads in contact with screwed spindle.

Therefore, height of the nut, $h = n \times p$ (4.39)

Where p is pitch of the threads

6. Check for shear stresses in the screw and nut:

Shear stress in screw $(\tau_s) = \frac{W}{\pi d_c t \times n}$ (4.40)

Shear stress in nut $(\tau_n) = \frac{W}{\pi d t n}$ (4.41)

Where t is thickness of the screw, $= p/2$

For safety of the screw & nut threads τ_s and $\tau_n < \tau_{(Given)}$

7. Dimensions of inner diameter (D_1), outer diameter (D_2) and thickness (t_1) of the nut collar:

The tearing strength of the nut is considered to calculate the inner diameter (D_1) of the nut collar. We know that,

$$\sigma_{t(nut)} = \frac{W}{\frac{\pi}{4}\left(D_1^2 - d^2\right)} \tag{4.42}$$

From the Eq. (4.42), we can find the inner diameter (D_1) of the nut collar.

The crushing strength of the nut is considered to calculate the outer diameter (D_2) of the nut collar. We know that,

$$\sigma_{cr(nut)} = \frac{W}{\frac{\pi}{4}\left(D_2^2 - D_1^2\right)} \tag{4.43}$$

From the above equation, we can find the outer diameter (D_2) of the nut collar.

The shearing strength of the nut is considered to calculate the thickness (t_1) of the nut collar. We know that,

$$\tau_{(nut)} = \frac{W}{\pi D_1 t_1} \tag{4.44}$$

From Eq. (4.44), we can find the thickness (t_1) of the nut.

8. Design of screw head:

The diameter of head (D_3) on the top of the screw and for the cup is equal to

$$D_3 = 1.75d \tag{4.45}$$

The chamfer at the top of the cup is seated on the head. The cup is fitted with a pin of diameter $D_4 = \frac{D_3}{4}$ approximately.

9. Find the torque required to overcome friction (collar friction) at top of the screw:

Assume uniform pressure condition,

$$M_{tc} = \frac{2}{3} \times \mu_c \times W \times \left[\frac{R_3^3 - R_4^3}{R_3^2 - R_4^2}\right] \tag{4.46}$$

Assume uniform wear condition,

$$M_{tc} = \mu_c \times W \times R \tag{4.47}$$

Where $R = \frac{R_3 + R_4}{2}$

10. Design of handle:

Length of the handle:

The total torque is considered to design the handle, the total torque is calculated as (Eq. (4.48)),

$$M_{t\_total} = M_t + M_{tc} \tag{4.48}$$

Where, M_{t_total} is total torque required to raise the load. Assume the force applied on the handle is 300 N to 400 N,

Effective length of handle required = $L_E = \frac{M_{t\_total}}{Force\ \ applied\ \ on\ \ the\ \ handle}$

Length of the handle = $= L = L_E +$ Grip length

11. Diameter of the handle:

The diameter of the handle $\left(d_h\right)$ can be calculated from the following relation,

$$\sigma_b = \frac{M}{\frac{\pi}{32} \times d_h^3} \tag{4.49}$$

Height of the nut $H = 2d_h$. Where, M is maximum bending moment acting on the handle, $\sigma_b =$ Bending stress induced in the handle (Eq. 4.50) i.e.

$$\sigma_b = \frac{M}{Z} \tag{4.50}$$

12. Buckling load of the screw:

As per the formula developed by J B Johnson, the buckling or critical buckling of the load is (Eq. (4.51)),

$$W_{cr} = A_c \times \sigma_{yc} \left[1 - \frac{\sigma_{yc}}{4C\pi^2 E}\left(\frac{L}{k}\right)\right] \tag{4.51}$$

Where C is end fixity coefficient which is 0.25 for one end is fixed and other end is free, σ_{yc} is yield stress, k is radius of gyration which is normally taken as $0.25d_c$, $A_c = \frac{\pi}{4}d_c^2$ area of screw, E is modulus of elasticity.

Factor of safety against buckling failure is $N_{cr} = \frac{W_{cr}}{W}$

For safe design of screw under buckling is $N_{cr} \geq N_f$

13. Dimensions of the screw jack body:

(i) diameter of body at top $= D_5 = 1.5D_2$

(ii) Thickness of body $= t_3 = 0.25d_o$

(iii) Inside diameter at bottom $= D_6 = 2.25D_2$

(iv) Outside diameter at bottom $= D_7 = 1.75D_6$

(v) Thickness at the base $= t_2 = 2t_1$

(vi) Height of body $= H_b = Max.\ lift + Ht.\ of\ nut + 100\ mm$ extra for clearance

$$H_b = (H_l + h + 100) \tag{4.52}$$

Example 4.4: The load lifted by a screw is $70\ kN$ through a height of $350\ mm$. The tensile and compressive strength of the screw material are $200\ MPa$. The tensile and compressive strength of the nut (phosphor-bronze material) are $100\ MPa$ and $90\ MPa$ respectively. The shear strength of the screw and nut are $120\ MPa$ and $80\ MPa$ respectively. The bearing pressure between the nut and the screw is not to exceed $18\ N/mm^2$. Design a screw jack.

Given Data:

Load $W = 70\ kN = 70\times10^3\ N$

Height $H = 350\ mm$

Take factor of safety $= 2$

Tensile and compressive strength of the screw

$$(\sigma_t)_s = (\sigma_c)_s = \frac{200}{F.S} = \frac{200}{2} MPa = 100\ N/mm^2$$

Shear strength of the screw $(\tau)_s = \dfrac{120}{F.S} = \dfrac{120}{2} = 60 MPa = 60\ N/mm^2$

Tensile strength of the nut $(\sigma_t)_n = \dfrac{100}{F.S} = \dfrac{100}{2} = 50\ MPa = 50\ N/mm^2$

Compressive strength of the nut $(\sigma_c)_n = \dfrac{90}{F.S} = \dfrac{90}{2} = 45\ MPa = 45\ N/mm^2$

Shear strength of the nut $(\tau)_n = \dfrac{80}{F.S} = \dfrac{80}{2} = 40 MPa = 40\ N/mm^2$

Bearing pressure $p_b = 18\ N/mm^2$

Find:

1. Design the screw jack

Solution:

(i) Design of screw:

Consider the screw under pure compression, find the core diameter of the screw using Eq. (4.33),

$$(\sigma_c)_s = \frac{W}{\frac{\pi}{4}\times d_c^2}$$

Core diameter of the screw $d_c = \sqrt{\frac{4W}{\pi(\sigma_c)_s}} = \sqrt{\frac{4\times70\times10^3}{3.14\times100}} = 29.53 \simeq 30\ mm$

From Design Data Book,

Core diameter $d_c = 31\ mm$; Outside diameter of the spindle $d = 38\ mm$; Pitch of threads $p = 7\ mm$

Mean diameter of the screw $d_m = \frac{d+d_c}{2} = \frac{38+31}{2} = 34.5\ mm$

and $\tan\alpha = \frac{p}{\pi d_m} = \frac{7}{3.14\times34.5} = 0.06462$

Take coefficient of friction between screw and nut $\mu = \tan\phi = 0.15$

Torque required to rotate the screw in the nut $(M_t)_s = P\times\frac{d_m}{2}$

The tangential force required at the circumference of the screw,

$$P = W\tan(\alpha+\phi) = W\times\left(\frac{\tan\alpha+\tan\phi}{1-\tan\alpha\tan\phi}\right)$$

$$= 70\times10^3\times\left(\frac{0.06462+0.15}{1-0.06462\times0.15}\right) = 685.15\ N$$

The torque required to overcome the screw friction,

$$(M_t)_s = P\times\frac{d_m}{2} = 685.15\times\frac{34.5}{2} = 11818.84\ N-mm$$

Compressive stress due to axial load $\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}d_c^2} = \frac{70\times10^3}{\frac{\pi}{4}(31)^2} = 92.79\ N/mm^2$

Shear stress due to the torque $\tau = \frac{16(M_t)_s}{\pi d_c^3} = \frac{16\times11818.84}{3.14\times(31)^3} = 2.02\ N/mm^2$

Maximum principal stress (tensile or compressive),

$$(\sigma_c)_{\max} = \frac{1}{2}\left[\sigma_c + \sqrt{\sigma_c^2 + 4\tau^2}\right] = \frac{1}{2}\left[92.79 + \sqrt{(92.79)^2 + 4\times(2.02)^2}\right] = 92.84\ N/mm^2$$

Maximum shear stress $\tau_{\max} = \frac{1}{2}\times\sqrt{\sigma_c^2 + 4\tau^2} = \frac{1}{2}\left[\sqrt{(92.79)^2 + 4\times(2.02)^2}\right] = 46.44\ N/mm^2$

$\therefore\ \sigma_{c(\max)} < (\sigma_c)_s\ \&\ \tau_{\max} < (\tau)_s$

Consequently, since these maximum stresses are within limits, the screw design for the spindle is safe.

(ii) Design of nut:

From Eq. (4.35), bearing pressure $p_b = \dfrac{W}{\frac{\pi}{4}\left[d^2 - d_c^2\right]\times n}$

$$n = \frac{4W}{\pi\left[d^2 - d_c^2\right]\times p_b} = \frac{4\times 70\times 10^3}{3.14\times\left[(38)^2 - (31)^2\right]\times 18} = 10.26$$

Number of threads of nut in engagement $n \simeq 12$

Height of the nut $h = n\times p = 12\times 7 = 84\ mm$

(iii) Check for shear stresses in the screw and nut:

From Eq. (4.40), Shear stress in screw $(\tau_s) = \dfrac{W}{\pi d_c t\times n} = \dfrac{70\times 10^3}{3.14\times 31\times 3.5\times 12} = 17.15\ N/mm^2$

From Eq. (4.41)

Shear stress in nut $(\tau_n) = \dfrac{W}{\pi dtn} = \dfrac{W}{\pi dtn} = \dfrac{70\times 10^3}{3.14\times 38\times 3.5\times 12} = 14\ N/mm^2$

Shear stress of screw & nut (τ_s) and $(\tau_n) < \tau_{(Given)}$

Since these stresses are within permissible limit, therefore design for nut is safe.

(iv) Dimensions of inner diameter (D_1), outer diameter (D_2) and thickness (t_1) of the nut collar:

From the Eq. (4.42), we can find the inner diameter (D_1) of the nut collar,

$$(\sigma_t)_n = \frac{W}{\frac{\pi}{4}\left(D_1^2 - d^2\right)}$$

$$50 = \frac{4\times 70\times 10^3}{3.14\left(D_1^2 - 38^2\right)}$$

$$D_1^2 = \frac{4\times70\times10^3}{3.14\times50} + 38^2 = 3227.44$$

$$D_1 = 56.81\ mm \simeq 57\ mm$$

The crushing strength of the nut is considered to calculate the outer diameter (D_2) of the nut collar. From Eq. (4.43),

$$(\sigma_c)_n = \frac{W}{\frac{\pi}{4}\left(D_2^2 - D_1^2\right)}$$

$$45 = \frac{70\times10^3}{\frac{\pi}{4}\left(D_2^2 - 57^2\right)}$$

$$D_2^2 = \frac{4\times70\times10^3}{\pi\times45} + 57^2 = 5230.6$$

$$D_2 = 72.32\ mm \simeq 73\ mm$$

The shearing strength of the nut is considered to calculate the thickness (t_1) of the nut collar. From Eq. (4.44),

$$(\tau)_n = \frac{W}{\pi D_1 t_1} \qquad \therefore t_1 = \frac{W}{\pi D_1 (\tau)_n}$$

$$t_1 = \frac{W}{\pi D_1 (\tau)_n} = \frac{70\times10^3}{3.14\times57\times40} = 9.78\ mm \simeq 10\ mm$$

(v) Design of screw head:

From Eq. (4.45), the diameter of head (D_3) on the top of the screw and for the cup is equal to $D_3 = 1.75d$

$$D_3 = 1.75\times38 = 66.5\ mm \simeq 68\ mm$$

The chamfered at the top of the cup is seated on the head. The cup is fitted with a pin of diameter $D_4 = D_3/4$ approximately.

$$D_4 = D_3/4 = 68/4 = 17\ mm \simeq 18\ mm$$

(vi) Find the torque required to overcome friction (collar friction) at top of the screw:

From Eq. (4.46), for uniform pressure condition,

$$(M_t)_c = \frac{2}{3}\times\mu_c\times W\times\left[\frac{R_3^3 - R_4^3}{R_3^2 - R_4^2}\right]$$

$$\left(M_t\right)_c = \frac{2}{3}\times 0.15\times 70\times 10^3\times\left[\frac{(34)^3-(9)^3}{(34)^2-(9)^2}\right] = 251.2\times 10^3\ N-mm$$

(vii) Design of handle:

Length of the handle

Total torque applied on the handle $M_t = \left(M_t\right)_s + \left(M_t\right)_c$

$$M_t = 118.2\times 10^3 + 251.2\times 10^3 = 369.4\times 10^3\ N-mm$$

Effective length of handle required = $L_E = \dfrac{M_{t\_total}}{Force\ applied\ on\ the\ handle}$

Assume the force applied on the handle is $300\ N$

$$L_E = \frac{M_t}{Force\ applied\ on\ the\ handle} = \frac{369.4\times 10^3}{300} = 1231.3\ mm \simeq 1232\ mm$$

Length of the handle $L = L_E + Grip\ length = 1232 + 68 = 1300\ mm$

(viii) Diameter of the handle:

From Eq. (4.49), the diameter of the handle $\left(d_h\right)$ can be calculated.

$$\sigma_b = \frac{M}{\frac{\pi}{32}\times d_h^3} \qquad \therefore d_h^3 = \frac{32M}{\pi\times\sigma_b}$$

The material of the handle is assumed same as that of screw, therefore bending stress $\sigma_b = \left(\sigma_t\right)_s = 100\ N/mm^2$

Bending moment $M = Force\ Applied \times Length\ of\ the\ lever$

$$M = 300\times 1300 = 390\times 10^3\,N-mm$$

Diameter of the handle $\therefore d_h^3 = \dfrac{32M}{\pi\times\sigma_b} = \dfrac{32\times 390\times 10^3}{3.14\times 100} = 39745.2$

Diameter of the handle $d_h = 34.13\ mm \simeq 36\ mm$

Height of the nut $H = 2d_h = 2\times 36 = 72\ mm$

(ix) Dimensions of the screw jack body:

Diameter of body at top $D_5 = 1.5D_2 = 1.5\times 73 = 109.5 \simeq 110\ mm$

Thickness of body $t_3 = 0.25d = 0.25\times 38 = 9.5 \simeq 10\ mm$

Inside diameter at bottom $D_6 = 2.25D_2 = 2.25 \times 73 = 164.25 \simeq 165\ mm$

Outside diameter at bottom $D_7 = 1.75D_6 = 1.75 \times 165 = 288.8 \simeq 290\ mm$

Thickness at the base $t_2 = 2t_1 = 2 \times 10 = 20\ mm$

Height of body $H_b = Max.\ lift + Ht.\ of\ nut + 100\ mm$ extra for clearance

$H_b = (H + h + 100) = 350 + 84 + 100 = 534\ mm$

4.10. DESIGN OF TOGGLE JACK

A toggle is a tool used for lifting large, heavy machinery during maintenance. Fig. 4.8 shows the schematic diagram of a toggle jack. It consists of a mechanism that employs a toggle linkage to amplify relatively small input forces into much larger output forces. The various parts of the toggle jack are: platform, metal screw rod, two nuts, eight links, eight pins and head. A metal screw rod that fits into the jack allows to raise and lower the jack. There are two nuts are placed at the ends of the screw rod. Four links are connected to each nut with two pins. A head is connected to upper links with two pins for carrying the load. The platform is connected to lower links with two pins. The operator turns the metal screw rod in a clockwise direction to raise the linkages, in turn load is lifted which is kept on the head. The metal screw rod is turned in opposite direction to lower the load.

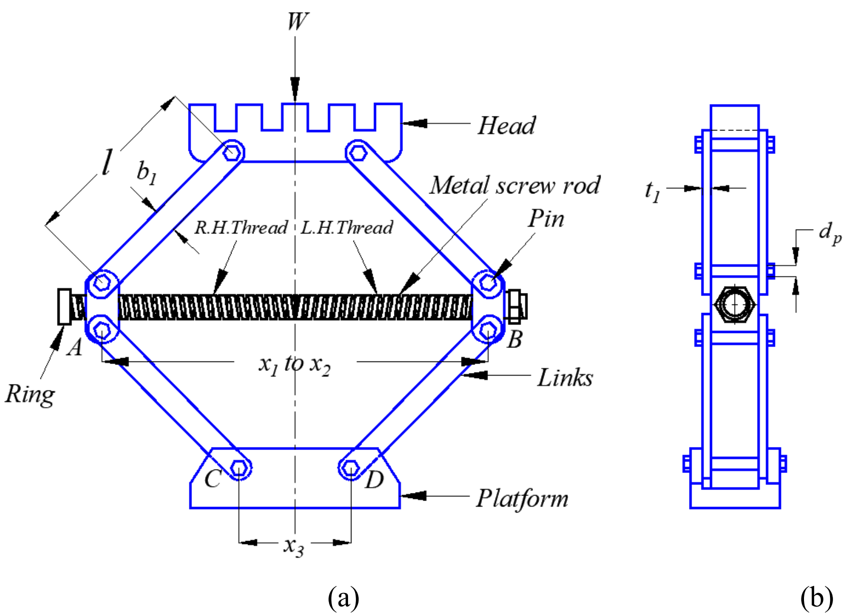


Fig. 4.8: Schematic diagram of toggle jack (a) Front view of the toggle jack (b) Side view of the toggle jack

Let l is length of the link (mm); x_1 is distance between the centre lines of nuts when the jack is in the top position (mm); x_2 is distance between the centre lines of nuts when the jack is in the bottom position (mm); x_3 is distance between the link pins in the platform (mm); d_p is diameter of the pin (mm); t_1 is thickness of the link (mm); b_1 width of the link (mm).

1. Design of square threaded screw:

It is considered that when the jack is in the bottom position, the square threaded screw experiences its maximum load. Because each nut bears half of the total load on the jack, the square threaded screw is being pulled, and the link CA is being tensioned as shown in Fig. 4.9.

The magnitude of the force acting on the square threaded screw is given by (Eq. 4.53)),

$$F = \frac{W}{2\tan\theta} \tag{4.53}$$

Where, θ be the angle of inclination at the bottom position of the link *CA* with the horizontal.

There is a similar force acting on the other nut, the total force on metal screw threaded rod is written as, $F_{total} = 2F$.

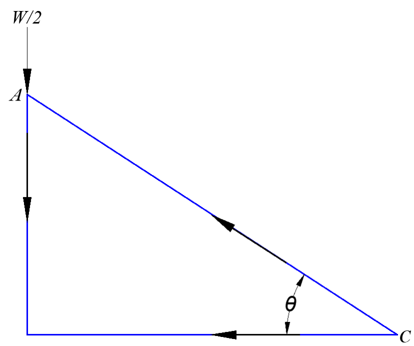


Fig. 4.9: Direction of force on the link CA when the link jack in bottom position

We know that, the total force acting on the screw $F_{total} = \frac{\pi}{4}(d_c)^2 \sigma_t$ (4.54)

The core diameter (d_c) of the screw can be calculated from the equation (Eq. 4.54).

Nominal or outer diameter of the screw $d_o = d_c + p$ (4.55)

Mean diameter of the screw $d_m = d_o - p/2$ (4.56)

2. Principal stresses induced in the screw:

We know that, the effort applied at the circumference of the screw to lift the load (Eq. (4.57)), is

$$P = F_{total} \times \tan(\alpha+\phi) = F_{total} \times \left(\frac{\tan\alpha + \tan\phi}{1-\tan\alpha\tan\phi}\right) \tag{4.57}$$

Torque required to overcome the thread friction and rotate the screw (Eq. (4.58)),

$$M_t = P\left(d_m / 2\right) \tag{4.58}$$

Based on the normal stress theory, maximum principal (tensile) stress (Eq. (4.59))

$$\sigma_{t(\max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{\left(\sigma_t\right)^2 + 4\tau^2} \tag{4.59}$$

Where, direct compressive stress induced in the screw,

$$\sigma_t = \frac{W}{\frac{\pi}{4} d_c^2} \tag{4.60}$$

Induced torsional shear stress in the screw,

$$\tau = \frac{16 M_t}{\pi\left(d_c\right)^3} \tag{4.61}$$

Based on the maximum shear stress theory, maximum shear stress

$$\tau_{\max} = \frac{1}{2}\left[\sqrt{\sigma_t^2 + 4\tau^2}\right] \tag{4.62}$$

For safe design, $\sigma_{t(\max)} \le \sigma_t$ and $\tau_{\max} \le \tau$

Check the principal stresses $\left(\sigma_{t(\max)}\ and\ \tau_{\max}\right)$ induced are less than the permissible values, if yes design is safe.

3. Design of nut:

The bearing pressure is considered for designing the nut. We know that the bearing pressure on the nut,

$$p_b = \frac{F_{total}}{\frac{\pi}{4}\left(d_o^2 - d_c^2\right) \times n} \tag{4.63}$$

Where n is no. of threads in contact with the screw and it is calculated from Eq. (4.63) by considering the bearing pressure.

Thickness of the nut $t = n \times p$ and width (b) of nut can be taken as, $1.5 d_o$.

4. Length of the screwed portion of the screw

Length of screw = dist. between the center lines of nuts when toggle jack at the top position + thickness of nut + (2 × thickness of ring)

5. Design of pins:

Since the load on the pin is equal to the force acting on the square threaded screw, and the pin is in double shear. Therefore, the pins are designed under double shear condition. Hence,

$$F = 2 \times \frac{\pi}{4} \times \left(d_p\right)^2 \times \tau_p \tag{4.64}$$

The diameter of the pin $\left(d_p\right)$ can be calculated from the Eq. (4.64).

Example 4.5: A toggle jack, as illustrated in Fig. 4.10, is designed to lift a load of $5\ kN$. The distance between the centerlines of the nuts is $60\ mm$ when the jack is in the top position and $220\ mm$ when it is in the bottom position. The length of each link is $120\ mm$, and the jack uses eight symmetrical links. The distance between the link pins at the base is $40\ mm$. All links, screws, and pins are made from mild steel, with permissible stresses of $100\ MPa$ in tension and $50\ MPa$ in shear. The bearing pressure on the pins is restricted to $25\ N/mm^2$. The pitch of the square threads is $7\ mm$, and the coefficient of friction between threads is 0.25. Design the toggle jack accordingly.

Given Data:

Load $W = 5\ kN = 5 \times 10^3\ N$

Length of the link $l = 120\ mm$

Permissible tensile stress $\sigma_t = 100\ MPa = 100\ N/mm^2$

Permissible shear stress $\tau = 50\ MPa = 50\ N/mm^2$

Bearing pressure $p_b = 25\ N/mm^2$

Pitch of square threads $p = 7\ mm$

Coefficient of friction between threads $\mu_s = \tan\phi = 0.25$

Find:

1. Design the toggle jack

Solution:

(i) Design of square threaded screw:

It is considered that when the jack is in the bottom position, the square threaded screw experiences its maximum load.

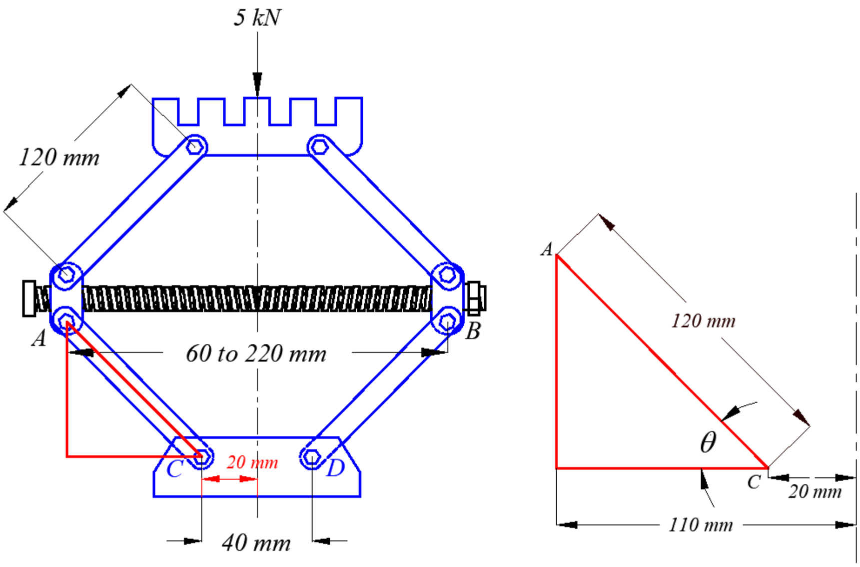


Fig. 4.10: Schematic diagram of toggle jack

$$\cos\theta = \frac{110-20}{120} = 0.75 \qquad \theta = 41.409$$

Where, θ be the angle of inclination of the link CA with the horizontal.

From Eq. (4.53), the magnitude of the load acting on the square threaded screw is,

$$F = \frac{W}{2\tan\theta} = \frac{5\times10^3}{2\times\tan(41.409)} = 2834.79\ N$$

Total tensile pull on the square threaded rod $F_{total} = 2\times F = 2\times2834.79 = 5669.58\ N$

From Eq. (4.54), total force acting on the screw,

$$F_{total} = \frac{\pi}{4}(d_c)^2\,\sigma_t$$

$$\therefore (d_c)^2 = \frac{4F_{total}}{\pi\sigma_t} = \frac{4\times5669.58}{3.14\times100} = 72.22$$

$$d_c = 8.5\ mm \simeq 10\ mm$$

The core diameter may be taken as $d_c = 14\ mm$, since the screw is subjected to torsional stress.

Nominal or outer diameter of the screw $d_o = d_c + p = 14 + 7 = 21\ mm$

Mean diameter of the screw = $d_m = d_o - p/2 = 21 - 7/2 = 17.5\ mm \simeq 18\ mm$

(ii) Principal stresses induced in the screw:

We know that, the effort applied at the circumference of the screw to lift the load (Eq. 4.57)), is

$$P = F_{total} \times \tan(\alpha + \phi) = F_{total} \times \left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi} \right)$$

$$\tan\alpha = \frac{p}{\pi d_m} = \frac{7}{3.14 \times 18} = 0.1238$$

$$P = 5669.58 \times \left(\frac{0.1238 + 0.25}{1 - 0.1238 \times 0.25} \right) = 2204.53 \;\; N$$

Torque required to overcome the thread friction and rotate the screw (Eq. 4.58)),

$$M_t = P(d_m / 2) = 2204.53 \times (18/2) = 19840.77 \;\; N - mm$$

Based on the normal stress theory, maximum principal (tensile) stress (Eq. 4.59)

$$\sigma_{t(\max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

Where, direct compressive stress induced in the screw (Eq. (4.60)),

$$\sigma_t = \frac{F_{total}}{\frac{\pi}{4} d_c^2} = \frac{4 \times 5669.58}{3.14 \times (14)^2} = 36.85 \;\; N / mm^2$$

Induced torsional shear stress in the screw (Eq. (4.61)),

$$\tau = \frac{16 M_t}{\pi (d_c)^3} = \frac{16 \times 19840.77}{3.14 \times (14)^3} = 36.84 \, N / mm^2$$

$$\sigma_{t(\max)} = \frac{36.85}{2} + \frac{1}{2}\sqrt{(36.85)^2 + 4 \times (36.84)^2}$$

$$= 59.62 \; N / mm^2$$

Based on the maximum shear stress theory, maximum shear stress (Eq. (4.62)),

$$\tau_{\max} = \frac{1}{2}\left[\sqrt{\sigma_t^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(36.85)^2 + 4 \times (36.84)^2}\right] = 41.19 \;\; N / mm^2$$

$$\sigma_{t(\max)} \le \sigma_t \qquad 59.62 < 100 \;\; N / mm^2$$

$$\tau_{\max} \le \tau \qquad 41.19 < 50 \; N / mm^2$$

The principal stresses $\left(\sigma_{t(\max)} \; and \; \tau_{\max}\right)$ induced are less than the permissible values, hence, design is safe.

(iii) Design of nut:

The bearing pressure is considered for designing the nut. We know that the bearing pressure on the nut,

$$p_b = \frac{F_{total}}{\frac{\pi}{4}\left(d_o^2 - d_c^2\right) \times n}$$

$$n = \frac{4F_{total}}{\pi\left(d_o^2 - d_c^2\right) \times p_b}$$

No. of threads in contact with the screw $n = \dfrac{4 \times 5669.58}{3.14 \times \left((21)^2 - (14)^2\right) \times 25} = 1.18$

Take $n = 4$ to ensure optimal stability and avoid the screw from locking in the nut, Thickness of the nut $t = n \times p = 4 \times 7 = 28\ mm$

Width (b) of nut $b = 1.5d_o = 1.5 \times 21 = 31.5 \simeq 32\ mm$

(iv) Length of the screwed portion of the screw:

Length of screw = dist. between the center lines of nuts when toggle jack at the top position + thickness of nut + (2 $\times$ thickness of ring)

$L = 60 + 28 + 2 \times 8 = 104\ mm$ (Take thickness of the ring as $8\ mm$)

(v) Design of pins:

Since the load on the pin is equal to the force acting on the square threaded screw, and the pin is in double shear. Therefore, the pins are designed under double shear condition. Hence,

$$F = 2 \times \frac{\pi}{4} \times \left(d_p\right)^2 \times \tau_p$$

$$\left(d_p\right)^2 = \frac{F}{2 \times \frac{\pi}{4} \times \tau_p} = \frac{4F}{2 \times \pi \times \tau_p} = \frac{4 \times 2834.79}{2 \times 3.14 \times 50} = 36.11$$

The diameter of the pin $d_p = 6.01\ mm \simeq 8\ mm$

4.11. SPRINGS

Springs are mechanical devices typically used to store and release mechanical energy. They are flexible components that can undergo deformation and return to their original shape when the force causing the deformation is removed. Springs are widely used in various applications to provide tension, compression, or torsion forces. The primary functions of a spring are:

1. Springs are used in toys, clocks, circuit breakers, and starters, among other devices, to store mechanical energy
2. Springs are used to absorb shocks, vibrations, and impacts, e.g., vehicle suspension springs, railway buffer springs, and vibration mounts for machinery
3. To apply a specific torque or force and control motion, e.g., cam and follower mechanism: maintain contact between the cam and follower; Engine valve mechanism: Return the rocker arm to its normal position
4. To measure the force e.g., springs are used in weighing balance and scales.

4.11.1. Classification of springs

Mechanical springs are available in various types, the most important of which may be classified as,

Helical Lec 19

1. Helical compression or extension springs
2. Helical torsion springs
3. Spiral springs
4. Leaf springs
5. Belleville springs

(i) Helical compression or extension springs:

Fig. 4.11 (a) and (b) show the photographic view of extension and compression spring respectively. Helical compression or extension springs are typically made from a length of wire wound in the shape of a helix, forming a coil. The major stress developed due to twisting of helical compression or extension spring is torsional shear stress. The load being applied along the axis of the helix. The linear behavior is existing during the deflection of the spring. Helical compression springs are used in shock absorbers, retractable pens, spring mattress etc. Extension springs are used in washing machines, harvesters, tractors, ploughers, stretchers, surgical lights, toy applications etc.

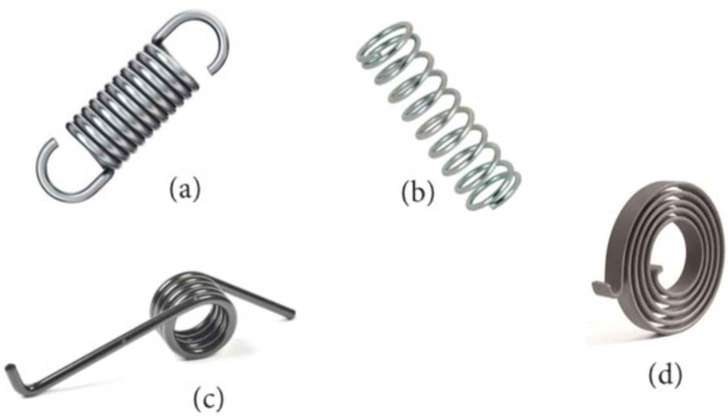
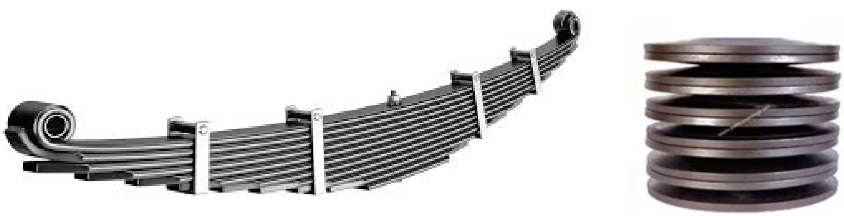


Fig. 4.11: (a) Extension spring (b) Compression spring (c) Torsion spring (d) Spiral spring



(a) (b)

Fig. 4.12: (a) Semi elliptic leaf spring (b) Belleville springs

(ii) Helical torsion springs:

Fig. 4.11 (c) shows the photographic view of torsion spring. These springs are similar to helical compression springs. The major stresses developed due to bending of helical torsion spring are tensile and compressive stresses. The torque being applied about the axis of the helix. The spring exhibits circular behavior during deflection. Helical torsion springs are used in clocks, cloth pins, automotive valves, clutches, gear shifters, medical immobilization devices, hospital beds, several dental applications, wheelchair lifts etc.

(iii) Spiral springs:

Fig. 4.11 (d) shows the photographic view of spiral spring. Spiral springs are formed by winding a flat strip of material (usually metal) around a central axis in a spiral pattern. The strip is wound tightly to create multiple coils, and the ends are often attached to some form of hub or anchor point. This design allows the spring to store energy when twisted about its axis. The major stresses developed due to bending of spiral springs are tensile and compressive stresses. The deflection is angular. Spiral springs have the following applications: Toys, mechanical watch, seat recliner, type writers and parking meters etc.

(iv) Leaf springs:

Fig. 4.12 (a) shows the photographic view of semi elliptic leaf spring. Leaf springs are constructed from several thin strips or leaves of spring steel, usually of decreasing length, stacked on top of each other and clamped together. The longest leaf, called the main leaf, forms the base, while shorter leaves, known as helper or secondary leaves, are stacked on top and decrease in length towards the center. The leaves are held together by center bolts or clamps. The major stresses developed in the leaf springs are tensile and compressive stresses. The deflection is linear. Leaf springs are used in railway carriage suspension system, heavy automobile suspension system, positions the automobile axle and control the vehicle weight etc.

(v) Belleville springs:

Fig. 4.12 (b) shows the photographic view of Belleville spring. Belleville springs consist of one or more disc-shaped washers with a conical or cupped profile. The convex sides of washers stack together and fit into each other's concave sides to form an alternating pattern of convex and concave surfaces. This arrangement allows the springs to compress axially when subjected to an axial load. The major stresses developed in the Belleville springs are tensile and compressive stresses. The deflection is linear. The practical applications of Belleville springs in punch and die sets, clutch brake mechanisms, bearing assemblies, ball seat valves, maintaining the load on the bolted connections etc.

4.11.2. Applications of springs

Springs have a wide range of applications across various industries due to their ability to store and release mechanical energy. Here are some common applications of springs:

1. Coil springs and leaf springs, are used in vehicle suspension systems to absorb shocks and vibrations, providing a smoother ride.

2. To engage and disengage clutch plates and brake discs, facilitating smooth operation and precise control.
3. Valve springs are used to keep engine valves closed when needed and to open them at the appropriate time, ensuring proper engine performance.
4. Springs are used to tension conveyor belts, ensuring smooth and consistent movement of materials along the production line.
5. To provide the necessary force for pressing and stamping operations, such as in metal forming and shaping processes.
6. Springs are employed in machinery mounts and isolators to reduce vibrations and minimize the transmission of shock and noise to surrounding structures.
7. Springs are used in various weapon systems, such as firearms, missile launchers, and artillery, to provide recoil control and assist in loading and firing mechanisms.
8. Springs are used in various aerospace components, such as actuators, hinges, and latches, to provide mechanical support and actuation.
9. Springs are used in implantable medical devices, such as pacemakers and prosthetic limbs, to provide mechanical support and functionality.
10. Springs are used in electrical switches and contacts to provide consistent contact force, ensuring reliable conductivity and performance.
11. Springs are used in battery contacts and connectors to maintain electrical contact and provide flexibility in battery insertion and removal.

4.11.3. Spring terminology

Spring terminology encompasses a variety of terms used to describe the design, characteristics, and behavior of springs. Here is some common spring terminology:

Spring Terminology

1. *Coil:* A single loop of wire forming a helical shape in a spring.
2. *Wire diameter:* The diameter of the wire used to make the coils of a spring. The wire diameter is denoted as 'd'.
3. *Coil diameter:* The outer diameter of the coil in a helical spring.
4. *Pitch:* The distance between adjacent coils in a helical spring.
5. *Solid length:* The height of a compression spring when it is fully compressed. In this case, the spring is completely compressed and no further compression is possible. The solid length is given by,

   Solid length of the spring $\left(l_s\right) = n \times d$ (4.65)

   Where n is total no of coils, d is wire diameter
6. *Free length:* The free length of a spring (Fig. 4.10) refers to its length when it is not under any external force or load. This is the natural length of the spring, also known as its uncompressed length or relaxed length. Free length is given by,

   Free length of the spring $\left(l_f\right) = Compressed\, length + \delta_{\max}$

$$= Solid\,length + total\,axial\,gap + \delta_{\max}$$

$$= n \times d + 0.15\delta_{\max} + \delta_{\max} \tag{4.66}$$

Where n is total no of coils, d is wire diameter, $\delta_{\max}$ is maximum compression

7. *Spring index:* The ratio between the mean diameter of a helical spring coil and the diameter of the wire used to make the spring. It is calculated using the formula:

Spring Index, $C = \dfrac{D}{d}$ (4.67)

Where, D is mean diameter of the coil, d is diameter of the wire. The spring index is an important factor in spring design as it influences various properties of the spring, including stress distribution, spring rate, and the number of active coils. A higher spring index generally results in a softer spring with more turns, while a lower spring index tends to produce a stiffer spring with fewer turns.

Springs

8. *Spring Rate:* The spring rate, often denoted by k, is a measure of the stiffness of a spring. It represents the amount of force required to compress or extend the spring by a certain distance.

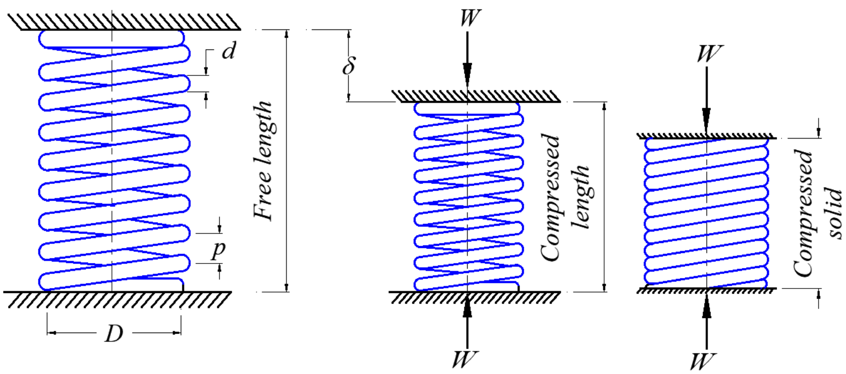


Fig. 4.13: Comp ression spring nomenclature

9. The spring rate is typically expressed in units of force per unit length, such as Newtons per millimeter (N/mm).

Spring Rate, $k = \dfrac{W}{\delta}$ (4.68)

Where, W is load, δ is deflection of the spring. The spring rate depends on various factors, including the geometry of the spring (such as the wire diameter, coil diameter, and number of coils), the material properties of the spring material (such as Young's modulus), and the type of ends of the spring (such as closed and ground ends, or open ends).

4.11.4. Materials and specifications

One crucial aspect of spring design is selecting the appropriate spring material. The spring's material ought to possess exceptional ductility, resilience, fatigue strength, and creep resistance. It mostly depends on the type of service that they are utilized for, such as mild, moderate, or severe duty. Depending on the materials size, spring index, and required qualities, hot or cold working techniques can be used to manufacture the springs. The understanding of tensile and yield strength of the various alloy materials is essential for choosing the right materials for spring design. The factors considered for selection of material for the spring wire are:

Spring Materials

1. The force exerted on the spring
2. The operating stress of the spring
3. The restrictions on spring mass and volume
4. Fatigue life of the spring
5. The spring's operational environment, including its temperature and corrosive atmosphere
6. The degree of distortion experienced during the spring's creation

Below are some of the common materials used for springs:

1. Hard-drawn wire (Cold-drawn spring steel):

Hard-drawn wire is a type of spring steel produced through cold drawing. It is the most affordable spring material and is suitable for applications that involve low stress and static loads. The key properties of the hard – drawn wire are

(a) Lowest cost among spring materials

(b) The wire is drawn through dies to achieve the desired diameter and properties

(c) Suitable for low-stress applications

(d) Provides adequate elasticity

(e) Corrosive resistance is limited; not ideal for corrosive environments without protective coatings.

The applications of the hard – drawn wire springs are used in non – critical components in automotive sectors and low stress applications such as household appliances, toys and gadgets etc. It is not suitable for use at subzero temperatures or temperatures above 120°C. The specifications of hard – drawn wire are ASTM A227 material standard, tensile strength of $830-1600\ N/mm^2$, Modulus of elasticity of $210\times10^3\ N/mm^2$.

2. Oil-tempered wire:

Oil-tempered wire is a popular spring material that undergoes a heat treatment process involving oil quenching and tempering. This process imparts improved strength and elasticity to the wire, making it suitable for a wide range of applications. The properties of the oil-tempered wire are :

(a) High tensile strength, suitable for medium to high-stress applications

(b) Good elasticity and resistance to deformation

(c) Better fatigue life compared to hard-drawn wire, making it suitable for dynamic applications

(d) Corrosive resistance is moderate; typically requires additional coatings or treatments for use in corrosive environments

Oil-tempered wire is a versatile and cost-effective choice for many spring applications, offering a good balance of strength, elasticity, and fatigue resistance for a variety of industrial, automotive, and general-purpose applications. It is not suitable for fatigue or sudden loads, subzero temperatures, or temperatures above 180°C. The specifications of oil-tempered wire are ASTM A229 material standard, tensile strength of $1600-2100\ N/mm^2$, Modulus of elasticity of $210\times10^3\ N/mm^2$.

3. Chrome vanadium:

Chrome vanadium steel is a high-quality spring material known for its excellent strength, toughness, and resistance to fatigue. It is alloyed with chromium and vanadium to enhance its mechanical properties, making it a preferred choice for demanding high strength, excellent fatigue resistance, and the ability to withstand higher temperatures. Chrome vanadium steel is a versatile material for automotive, aerospace, industrial, and performance applications. The important properties of the chrome vanadium are

(a) High tensile strength, making it suitable for heavy-duty applications

(b) Excellent elasticity and resilience

(c) Superior fatigue resistance, ideal for dynamic and cyclic loads

(d) Can withstand higher temperatures compared to many other spring materials

(e) Corrosive resistance is moderate; may require protective coatings for use in corrosive environments

This alloy spring steel is utilized in high-stress conditions and is effective at temperatures up to 220°C. The specifications of chrome vanadium steel are ASTM A231 material standard, tensile strength of $1400-2100\ N/mm^2$, modulus of elasticity of $210\times10^3\ N/mm^2$.

4. Chrome silicon:

Chrome silicon is a high-performance spring material known for its excellent mechanical properties, particularly its resilience and durability under high stress and varying temperatures. It is alloyed with chromium and silicon to enhance its strength and fatigue resistance, making it suitable for demanding applications. Its combination of strength, resilience, and durability makes it invaluable in automotive, aerospace, and industrial applications. The important properties of the chrome silicon are

(a) High tensile strength, providing robust performance under heavy loads

(b) Superior elasticity and resilience, maintaining spring integrity over long periods

(c) Exceptional fatigue resistance, ideal for applications involving cyclic loading

(d) Effective over a wide temperature range, from subzero to high temperatures

(e) Good corrosion resistance, suitable for many industrial and automotive environments

It provides exceptional performance for extended service life, handling shock loading, and operating effectively at temperatures up to 250°C. The specifications of chrome silicon steel are ASTM A401 material standard, tensile strength of $1400-1700\ N/mm^2$, modulus of elasticity of $210\times10^3\ N/mm^2$.

5. *Music wire:*

Music wire, also known as high carbon steel wire, is a popular and versatile spring material renowned for its strength and resilience. It is primarily composed of high carbon steel alloyed with manganese to enhance its mechanical properties. Music wire is a robust and reliable choice for applications requiring strength, resilience, and versatility in spring design. The important properties of the music wire are

(a) High tensile strength, making it suitable for high-stress applications

(b) Good elasticity and resilience, maintaining spring integrity under repeated loading

(c) Moderate fatigue resistance, suitable for many general-purpose applications

(d) Corrosive resistance is low to moderate; may require coatings for protection in corrosive environments

This spring material is the most widely used for small springs. The specifications of music wire are ASTM A228 material standard, tensile strength of $1600-2750\ N/mm^2$, modulus of elasticity of $210\times10^3\ N/mm^2$.

6. *Stainless steel:*

Stainless steel is a widely used spring material known for its excellent corrosion resistance, strength, and durability. It is available in various grades, each offering unique properties suitable for different applications. The various grades of stainless steel materials used for manufacturing of spring are 302/304 stainless steel, 316 stainless steel and 17-7 PH stainless steel.

7. *Phosphor bronze / spring brass*

Phosphor bronze and spring brass are copper-based alloys known for their excellent corrosion resistance, electrical conductivity, and moderate strength. These materials are commonly used in applications requiring reliable performance in corrosive environments and good electrical properties. This is why it is commonly used for contacts in electrical switches. Spring brass is also suitable for use at subzero temperatures. The properties of the phosphor bronze / spring brass are

(a) high resistance to corrosion, particularly in marine and industrial environments

(b) moderate tensile strength, suitable for medium-stress applications

(c) good elasticity and resilience

(d) excellent electrical conductivity, making them ideal for electrical and electronic applications.

(e) good fatigue resistance, suitable for cyclic loading applications

The specifications of phosphor bronze and spring brass are: ASTM B159 for phosphor bronze and ASTM B134 for spring brass material standard, tensile strength of $620-830\ N/mm^2$ for phosphor bronze, and $480-760\ N/mm^2$ for spring brass and modulus of elasticity of $103\times10^3-110\times10^3\ N/mm^2$.

4.12. STRESSES IN SPRINGS

The stresses experienced by a spring depend on several factors including the type of loading (compression, tension, torsion), the geometry of the spring, the material properties, and the operating conditions. The types of stresses commonly encountered in springs:

Wahl's Correction Factor

1. Pure torsional shear stress:

The pure torsional shear stress is an indirect shear stress produced inside the spring subjected to torsion. The torsional shear stress is given by,

$$\tau_1 = \frac{8WD}{\pi d^3} \tag{4.69}$$

Where τ_1 is torsional shear stress, W is axial load, D is mean diameter of the coil, d is diameter of the wire.

2. Direct shear stress:

The direct shear stress is produced due to the effect of direct shearing force considering the curvature stress effect. The direct shear stress is given by,

$$\tau_2 = K_s \frac{8WD}{\pi d^3} \tag{4.70}$$

Where K_s is shear stress correction factor. $K_s = \left(1+\frac{0.5}{C}\right)$, Therefore, spring index $C=\frac{D}{d}$

3. Combined torsional, direct and curvature shear stress:

The combined torsional, direct and curvature shear stress is given by,

$$\tau_3 = K \frac{8WD}{\pi d^3} \tag{4.71}$$

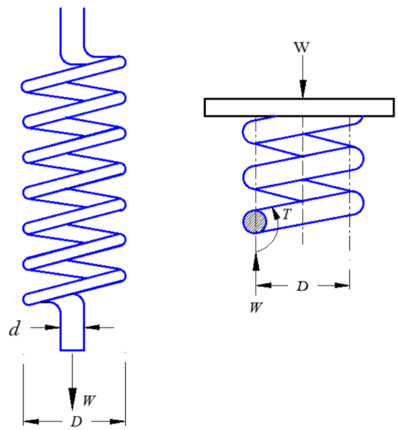
Where K is Wahl's correction factor. $K=\left(\frac{4C-1}{4C-4}+\frac{0.615}{C}\right)$

4.12.1. Springs subjected to axial load W

The closed coil helical spring made of circular wire subjected to axial load W is considered to derive the torsional shear stress induced in the spring and it is shown in Fig. 4.14.

In Fig. 4.14, D is mean diameter of the spring; d is diameter of the wire; n is number of coils; G is modulus of rigidity for the spring material; W is axial load on the spring; τ is torsional shear stress induced in the wire; C is spring index $=\dfrac{D}{d}$; p is pitch of the coils, and δ is deflection of the spring due to axial load W.

The small portion of the spring element is considered and as shown in Fig. 4.14. Assume the load (W) tends to rotate the wire because of the twisting moment (T) that the wire has created due to applied axial load. Thus torsional shear stress is induced in the wire.



(a) (b)

Fig. 4.14: (a) Closed coil helical spring (b) Free body diagram of coil spring subjected to torsional shear and a direct shear

The twisting moment (M_t) can be written as,

Spring Index

$$M_t = W \times \frac{D}{2} \tag{4.72}$$

Using torsion formula,

$$\frac{M_t}{J} = \frac{\tau_1}{r} \tag{4.73}$$

We get, torsional shear stress

$$\tau_1 = \frac{M_t r}{J} = M_t\left(\frac{r}{J}\right) = M_t\left(\frac{d/2}{\pi d^4/32}\right) = \frac{16M_t}{\pi d^3} \tag{4.74}$$

Substitute the value $M_t = W \times \dfrac{D}{2}$ in Eq. (4.74),

We get, Torsional shear stress $\tau_1 = \dfrac{8WD}{\pi d^3}$ (4.75)

In addition to the torsional shear stress (τ_1), direct shear stress (τ_2) due to axial load (W) and stress due to the curvature of the wire (τ_3) also induced in the spring.

The direct shear stress due to load W is the ratio between load applied and cross sectional area of the wire.

Direct shear stress due to load $(\tau_2) = \dfrac{W}{\frac{\pi}{4}d^2}$

$$\tau_2 = \frac{4W}{\pi d^2} \tag{4.76}$$

Maximum shear stress induced in the wire = Torsional shear stress + Direct shear stress

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8WD}{\pi d^3}\left(1+\frac{d}{2D}\right)$$

$$= \frac{8WD}{\pi d^3}\left(1+\frac{1}{2C}\right) = K_S \times \frac{8WD}{\pi d^3} \tag{4.77}$$

(Substituting $\dfrac{D}{d} = C$)

where $K_S =$ Shear stress factor $= 1 + \dfrac{1}{2C}$

The effect of direct shear $\left(\dfrac{8WD}{\pi d^3}\left(1+\dfrac{1}{2C}\right)\right)$ is noticed in the Eq. (4.77) without considering the effect of wire curvature and applicable for springs with small spring index C. Wahl's stress factor (K) is introduced, when the effects of both direct shear as well as curvature of the wire are considered.

∴ Maximum shear stress induced in the wire,

$$\tau_3 = K \times \frac{8WD}{\pi d^3} = K \times \frac{8WC}{\pi d^2} \tag{4.78}$$

where $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

4.12.2. Energy stored in the spring

We are aware that the purpose of the springs is to store energy equivalent to the work, an external load does on them. By neglecting the strain energy due to direct shear W, the strain energy stored in the spring can be written as,

Energy stored in the spring $U = \dfrac{\tau_1^2}{4G} \times Volume\ of\ the\ spring$

$$U = \frac{\left(\frac{8WD}{\pi d^3}\right)^2}{4G} \times \left(\frac{\pi d^2}{4}.(\pi D n)\right)$$

$$U = \frac{4W^2D^3n}{Gd^4} \tag{4.79}$$

Where G is the modulus of rigidity.

4.12.3. Deflection of the spring

Let δ is assumed as deflection of the spring. The work done by the load can be written as,

Work done by the load $= \frac{1}{2}W.\delta$

By equating work done to the strain energy stored in the spring,

$$\frac{1}{2}W.\delta = \frac{4W^2D^3n}{Gd^4}$$

$$\therefore \delta = \frac{8WD^3n}{Gd^4} \tag{4.80}$$

4.12.4. Stiffness of the spring

The load required to create a unit of deflection is the known as spring's stiffness. The stiffness of the spring normally denoted as k. This is also called as spring constant.

The stiffness of the spring $k = \frac{W}{\delta}$

$$k = \frac{Gd^4}{8D^3n} \tag{4.81}$$

4.12.5. Design of helical spring

The following procedure is used to design the helical springs.

1. Calculate the maximum spring force (W) and the required deflection (δ) of the spring for the given applications.
2. Choose the suitable spring material and find out the ultimate tensile strength from the data book. Then, find out the permissible shear stress for the wire using the following relation, $\tau = 0.30S_{ut}$
3. Assume the suitable spring index (C) value. Normally the spring index value for industrial applications is 8 to 10. The spring index for springs in valves and clutches is 5. The spring index value is always 3 and above.
4. Calculate the Wahl factor using the relation, $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$
5. Calculate the wire diameter (d) from the relation, $\tau_1 = \frac{8WD}{\pi d^3}$
6. Calculate mean coil diameter (D) using the relation, $D = Cd$

7. Determine the no. of active coils (n) from the Eq. (4.80) $n=\frac{\delta G d^4}{8WD^3}$

8. Determine the solid length of the spring by the following relation, $(l_s)=n\times d$

9. Calculate the actual deflection of the spring by Eq. (4.80), $\delta=\frac{8WD^3 n}{Gd^4}$

10. The gap between the adjacent coils is assumed as 0.5 to 2 mm, when the spring is under maximum load. The total gap is calculated from the relation, Total gap = $(n-1)\times$ gap between two adjacent coils

11. Calculate the free length of the spring using the relation, Free length = solid length + total gap + δ

12. Calculate the pitch of the coil using the relation, $p=\frac{free\,length}{(n-1)}$

13. Calculate the spring rate k using the relation, $k=\frac{Gd^4}{8D^3 n}$

Example: 4.6: A helical spring of wire diameter $5\ mm$ and spring index 6 is acted by an initial load of $500\ N$. After compressing it further by $8\ mm$ the stress in the wire is $450\ MPa$. Find the number of active coils. Take modulus of rigidity $G=80000\ MPa$. The initial load of $500\ N$ is acting on a $5\ mm$ diameter of helical spring of wire with spring index 6. The stress induced in the spring is $450\ MPa$, after compressing it further by $8\ mm$. Calculate the number of active turns. Take modulus of rigidity $G=80000\ MPa$.

Given Data:

Diameter of wire $d=5\ mm$

Spring index $C=6$

Initial load $W_i=500\ N$

Shear stress $\tau_1=450\ N/mm^2$

Modulus of rigidity $G=80000\ MPa=80\times10^3\ N/mm^2$

Deflection $\delta=8\ mm$

Find:

1. Number of active coils

Solution:

Mean diameter of the spring $D=C\times d=6\times5=30\ mm$

Wahl's factor $K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6}$

$K = 1.2525$

Torsional shear stress $\tau_1 = K\frac{8WD}{\pi d^3}$

$\therefore\ 450 = 1.2525 \times \frac{8\times W\times30}{3.14\times5^3}$

$W = \frac{450\times3.14\times5^3}{1.2525\times8\times30} = 587.57\,N$

The stiffness of the spring $k = \frac{W-W_i}{\delta} = \frac{587.57-500}{8} = 10.95\ N/mm$

No. of active coils $n = \frac{Gd^4}{8kD^3} = \frac{80\times10^3\times5^4}{8\times10.95\times30^3} = 4.22 \simeq 5\ turns$

Example: 4.7: The mean diameter of the helical spring coils is 12 times the wire diameter. The spring is to be designed to absorb $300\ J$ energy with an extension of $150\ mm$. The maximum shear stress is not to exceed $140\ N/mm^2$. Determine the mean diameter of the spring, diameter of the wire and the number of turns. Take the modulus of rigidity of the material of the spring as $80\ kN/mm^2$.

Given Data:

Mean diameter of helical spring $D = 12d$

Energy stored $U = 300\ J$

Extension of the spring $\delta = 150\ mm$

Maximum shear stress $\tau_1 = 140\ N/mm^2$

Modulus of rigidity $G = 80\ kN/mm^2 = 80\times10^3\ N/mm^2$

Find:

1. Diameter of the wire
2. Mean diameter of the spring
3. Number of turns

Solution:

1. Diameter of the spring wire:

Spring index $C = \frac{D}{d} = 12$

Energy stored in the spring $U = \frac{1}{2} W\delta$

$$\therefore W = \frac{2U}{\delta} = \frac{2 \times 300 \times 1000}{150} = 4000\ N$$

Shear stress $\tau_1 = \frac{8WD}{\pi d^3} = \frac{8 \times 4000 \times 12 \times d}{3.14 \times d^3}$

$$140 = \frac{8 \times 4000 \times 12 \times d}{3.14 \times d^3}$$

$$d = \sqrt{\frac{8 \times 4000 \times 12}{3.14 \times 140}} = 29.55\ mm \simeq 30\ mm$$

Diameter of the wire $d \simeq 30\ mm$

2. Mean diameter of the spring:

Mean diameter of the spring $D = 12d = 12 \times 30 = 360\ mm$

3. Number of turns:

The stiffness of the spring $k = \frac{W}{\delta} = \frac{4000}{150} = 26.67\ N/mm$

No. of active coils $n = \frac{Gd^4}{8kD^3} = \frac{80 \times 10^3 \times 30^4}{8 \times 26.67 \times 360^3} = 6.51 \simeq 7\ turns$

Example: 4.8: For the following operating conditions, design an IC engine's valve spring: The load acting on the spring when the valve is open and closed conditions are $390\ N$ and $240\ N$ respectively. Maximum diameter of spring is $24\ mm$. The length of the spring when the valve is open and closed conditions are $38\ mm$ and $48\ mm$ respectively. The spring material's maximum allowable shear stress is $400\ MPa$. Take Modulus of rigidity $G = 80\ kN/mm^2$.

Given Data:

Spring-loaded load when the valve is open $W_1 = 390\ N$

Spring-loaded load when the valve is closed $W_2 = 240\ N$

Maximum diameter of spring $D_1 = 24\ mm$.

Spring's length while the valve is open $l_1 = 38\ mm$

Spring's length while the valve is closed $l_2 = 48\ mm$

Maximum permissible shear stress $\tau = 400\ MPa = 400\ N/mm^2$

Modulus of rigidity $G = 80\ kN/mm^2 = 80 \times 10^3\ N/mm^2$

Find:

1. Design a valve spring of an IC engine

Solution:

1. Mean diameter of the spring:

Mean diameter of the spring coil $D = D_1 + d = 24 + d$

The maximum load is taken to calculate the diameter of the spring wire. Hence, the maximum twisting moment of the spring,

$$M_t = W_1 \times \frac{D}{2} = 390 \times \frac{(24+d)}{2}$$

Also, the maximum twisting moment $M_t = \frac{\pi}{16} \times \tau \times d^3$

$$(4680 + 195d) = \frac{\pi}{16} \times 400 \times d^3$$

$$78.5d^3 - 195d - 4680 = 0$$

$$d = 4.12\ mm$$

The nearest standard size of wire is SWG 7 having $d = 4.47\ mm$

The diameter of the spring wire by considering Wahl's stress factor K

Spring index $C = \frac{D}{d} = \frac{(24+d)}{d} = \frac{(24+4.47)}{4.47} = 6.4$

Wahl's stress factor $K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6.4 - 1}{4 \times 6.4 - 4} + \frac{0.615}{6.4} = 1.235$

Maximum shear stress $\tau_1 = K \frac{8WD}{\pi d^3}$

$$\therefore\ 400 = 1.235 \times \frac{8 \times 390 \times 28.47}{3.14 \times d^3}$$

$$d^3 = 1.235 \times \frac{8 \times 390 \times 28.47}{3.14 \times 400} = 70.721$$

$$d = 4.14\ mm$$

The standard size of wire is SWG 7 having $d = 4.47\ mm$

Mean diameter of the spring $D = (24 + d) = 24 + 4.47 = 28.47\ mm$

Outer diameter of the spring $D_o = D + d = 28.47 + 4.47 = 32.94\ mm$

2. No. of turns of the spring:

The load acting on the spring $\left(W_1 - W_2\right)$ is $150\ N$ to cause the deflection $\left(\delta\right)$ of the spring $\left(l_2 - l_1\right)$ $10\ mm$.

Stiffness of the spring $k = \dfrac{W}{\delta} = \dfrac{150}{10} = 15\ N/mm$

No. of active coils $n = \dfrac{Gd^4}{8kD^3} = \dfrac{80\times10^3\times\left(4.47\right)^4}{8\times15\times\left(28.47\right)^3} = 11.53 \simeq 12\ turns$

3. Free length of the spring:

The maximum deflection for the maximum load of $390\ N$,

$$\delta_{\max} = \frac{10}{150}\times390 = 26\ mm$$

Free length of the spring $l_f = nd + \delta_{\max} + 0.15\delta_{\max}$

$$l_f = 12\times4.47 + 26 + 0.15\times26 = 83.54\ mm$$

4. Pitch of the coil:

Pitch of the coil $p = \dfrac{Free\ length}{\left(n-1\right)} = \dfrac{83.54}{\left(12-1\right)} = 7.59\ mm$

Example: 4.9: A spring with 15 twists must be contained in a 13 mm diameter casing. The spring balance is used to measure values between 0 and 500 N across a 40 mm length scale. The modulus of rigidity of the spring is $85\ kN/mm^2$. Design a spring balance and also evaluate the maximum shear stress induced.

Given Data:

Load $W = 500\ N$

Deflection of spring $\delta = 40\ mm$.

No. of turns $n = 15\ mm$

Modulus of rigidity $G = 85\ kN/mm^2 = 85\times10^3\ N/mm^2$

Find:

1. Design a spring balance
2. Evaluate the maximum shear stress induced in the spring

Solution:

1. Design of the spring:

The spring is to be enclosed in a casing of $13\ mm$ diameter, therefore the outer diameter of the spring coil $(D_o = D + d)$ should be less than $13\ mm$.

Deflection of the spring $\delta = \dfrac{8WD^3 n}{Gd^4}$

$$40 = \frac{8\times500\times D^3\times15}{85\times10^3\times d^4}$$

$$\frac{D^3}{d^4} = \frac{85\times10^3\times40}{8\times500\times15} = 56.67$$

$$\frac{C^3}{d} = 56.67 \qquad \therefore C^3 = 56.67\times d \qquad \text{Assume } d = 2\ mm$$

$$C = \sqrt[3]{56.67\times2} = 4.84$$

and $\quad D = C\times d = 4.84\times2 = 9.68\ mm$

Outer diameter of the spring coil $D_o = D + d = 9.68 + 2 = 11.68\ mm$

$D_o = 11.68\ mm$ is less than the casing diameter of $13\ mm$, hence, the assumed diameter of the spring wire $2\ mm$ is correct.

2. Maximum shear stress induced:

Wahl's factor $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C} = \dfrac{4\times4.84-1}{4\times4.84-4} + \dfrac{0.615}{4.84} = 1.322$

Maximum shear stress induced $\tau_1 = K\dfrac{8WD}{\pi d^3}$

$$\tau_1 = 1.322\times\frac{8\times500\times9.68}{3.14\times(2)^3} = 2037.7\ N/mm^2$$

Example: 4.10: A mass of rail wagon is $22\ tonnes$ moving with a velocity of $2.5\ m/s$. Two buffers with springs of 300 mm diameter is used to rest the rail wagon. The maximum deflection of springs is $255\ mm$. The allowable shear stress in the spring material is $610\ MPa$. Design the spring for the buffers.

Given Data:

Mass $m = 22\ tonnes = 22000\ kg$

Velocity $v = 2.5\ m/s$

Mean diameter of the spring $D = 300\ mm$

Maximum deflection $\delta = 255\ mm$

Allowable stress $\tau = 610\ MPa$

Modulus of rigidity $G = 85\ kN/mm^2 = 85\times10^3\ N/mm^2$

Find:

1. Design the spring for buffers

Solution:

(i) Design of wire

Kinetic energy of the wagon $= \frac{1}{2}mv^2 = \frac{1}{2}\times22000\times(2.5)^2$

$= 68.75\times10^3\ N-m = 68.75\times10^6\ N-mm$ (i)

Energy stored in the springs $U = \frac{1}{2}W\delta\times2\ buffer\ springs$

$U = W\delta = W\times255 = 255\times W$ (ii)

Equating Eq. (i) and (ii),

$255W = 68.75\times10^6$

$\therefore W = \dfrac{68.75\times10^6}{255} = 269.607\times10^3\ N$

Torque transmitted by the spring

$(M_t)_s = W\times\dfrac{D}{2} = 269.607\times10^3\times\dfrac{300}{2} = 40.4\times10^6\ N-mm$

Also, torque transmitted by the spring $(M_t)_s = \dfrac{\pi}{16}\tau d^3$

$40.4\times10^6 = \dfrac{3.14}{16}\times610\times d^3$

$d^3 = \dfrac{16\times40.4\times10^6}{3.14\times610} = 337475.2$

Diameter of the spring wire $d = 69.6\ mm \simeq 70\ mm$

(ii) No. of turns of the spring:

Stiffness of the spring $k = \dfrac{W}{\delta} = \dfrac{269.607\times10^3}{255} = 1057.3\ N/mm$

No. of active coils $n = \dfrac{Gd^4}{8kD^3} = \dfrac{85\times10^3\times(70)^4}{8\times1057.3\times(300)^3} = 8.94 \simeq 9\ turns$

(iii) Free length of the spring:

Free length of the spring $l_f = nd + \delta + 0.15\delta$

$$l_f = 9\times 70 + 255 + 0.15\times 255 = 923.25\ mm$$

1. *Pitch of the coil:*

Pitch of the coil $p = \dfrac{Free\ length}{(n-1)} = \dfrac{923.25}{(9-1)} = 115.4\ mm$

4.13. CONSTRUCTION OF LEAF SPRINGS

A leaf spring is a particular kind of spring composed of several plates, or leaves, arranged in progressively larger stacks on top of one another. The various types of leaf springs are: elliptic, semi-elliptic, quarter, elliptic, and transverse. Fig. 4.15 shows the construction of a semi-elliptical leaf spring. The center clamp, rebound clips, and metal plates or leaves are the major components of a leaf spring. The leaf spring is made up of several metal plates, sometimes referred to as leaves, stacked on top of one another in decreasing size order. The semi-elliptic appearance of the leaf spring is caused by a curvature known as the camber applied to the leaves.

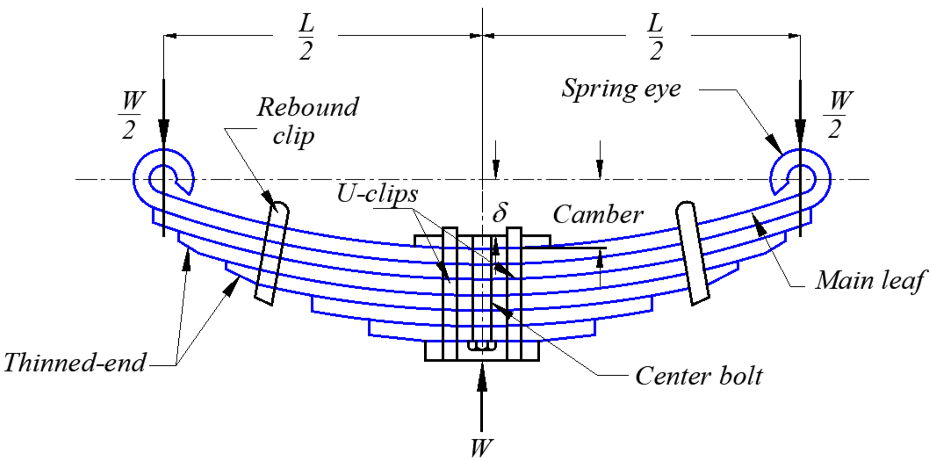


Fig. 4.15: Construction of semi-elliptical leaf spring

Fig. 4.15 illustrates how the ends of the master leaf are curled. The term "eye" refers to these coiled ends of the master leaf. The second master leaf, or nearly full-length leaf, is positioned directly beneath the master leaf to support it. In the construction of semi – elliptic leaf spring, minimum 2 number of full length leafs are used. *"Graduated leaves"* refers to the leftover leaves. The purpose of the central clamp is to secure all of the leaves together. It has a center bolt and U bolts to prevent the leaves from falling apart.

Leaf Lec 21

Rebound clips are made of steel bands that are positioned at specified positions on either side of the central clamp with the main goal of preventing the leaf spring from breaking during a rebound. When a leaf spring is pressurized, such as when a bus or truck axle is bumped, it often does not break. Nevertheless, the leaf spring may break if the force is abruptly released. This is because each

leaf is supported by the leaf below it while the spring is squeezed from below. However, if the load were suddenly released, the leaves would not find support without the rebound clips, which hold them together and offer support. When the U bolts are tightened, the leaf spring is positioned such that it is sandwiched between the axle at its top and the plate at its bottom. The location of the leaf spring is beneath the axle. Next, the axle is passed through the U bolts. Two eyes make up the leaf spring; one eye is fixed, and the other is fastened to the shackle. Each eye has bushings attached to it that are composed of anti-friction materials like rubber, brass, nylon, etc. The leaf spring shackle is a free-swinging part that has one end fixed to the frame and the other to the leaf spring's eye. The shackle's purpose is to give the leaf spring flexibility and keep it from snapping under pressure.

4.13.1. Working of leaf spring

The axle has a tendency to move up and down when the bus or truck passes over a bump. But since a leaf spring is there, it supports the axle and lessen shocks and vibrations. Let's take an example to better understand how it works. Assume that when a truck crosses a speed bump, the axle and wheel are driven upward. Therefore, because of its semi-elliptical shape, the leaf spring goes up with the axle as it does, absorbing and transferring the upward force to the chassis frame. When force is applied, the leaf spring's shackle facilitates its expansion.

4.14. DESIGN OF LEAF SPRINGS

4.14.1. Stresses in leaf springs

Fig. 4.15 shows a leaf spring carrying a vertical load W, which is equally shared by end reactions $\frac{W}{2}$. Let

W is load on the spring; R is initial radius of curvature of plates; δ is initial central deflection; b is width of each plate; t is thickness of each plate; n is number of leaves; L is span of the spring; σ is bending stress.

Section modulus for a single plate or laminate $=\frac{bt^2}{6}$

For n laminates, $Z = n \times \frac{bt^2}{6}$

Maximum bending moment $M = \frac{WL}{4}$

From bending equation, $\frac{M}{I} = \frac{\sigma}{y}$

$$\therefore M = \sigma \times \frac{I}{y} = \sigma \times Z$$

$$\frac{WL}{4} = \sigma \times n \times \frac{bt^2}{6}$$

Stress $\sigma = \dfrac{3WL}{2nbt^2}$ (4.82)

4.14.2. Deflection of the spring

Total strain energy = Resilience due to bending x Volume of the equivalent plate

Resilience due to bending $= \dfrac{\sigma^2}{6E}$

Volume of the equivalent plate $= \dfrac{nb}{2} Lt$

Total strain energy $U = \dfrac{\sigma^2}{6E} \times \dfrac{nb}{2} Lt$

Substituting the value of σ from Eq. (4.82),

$$U = \frac{\left(\frac{3WL}{2nbt^2}\right)^2}{6E} \times \frac{nb}{2} \times Lt \tag{4.83}$$

Also, strain energy $U = \dfrac{1}{2} W\delta$ (4.84)

Equating the Eqns. (4.83) and (4.84),

$$\frac{1}{2} W\delta = \frac{\left(\frac{3WL}{2nbt^2}\right)^2}{6E} \times \frac{nb}{2} \times Lt$$

$$\delta = \frac{3WL^3}{8Enbt^3} \tag{4.85}$$

4.14.3. Stiffness of the spring

Stiffness of the spring, $k = \dfrac{W}{\delta} = \dfrac{8Enbt^3}{3L^3}$ (4.86)

Example: 4.11: The dimensions of a leaf spring is $50\,mm$ wide, $8\,mm$ thick and $700\,mm$ span length. Determine the number of leaves required to carry a central load of $40\,kN$. The maximum allowable stress in the leave is $215\ \text{N/mm}^2$. Find the maximum deflection under this load.

Given Data:

Load $W = 40\ kN$

Length of the leaves $L = 700\ mm$

Width $b = 50\ mm$

Thickness of the leaves $t = 8\ mm$

Maximum Allowable stress $\tau = 215\ N/mm^2$

Find:

1. Maximum deflection

Solution:

1. Maximum deflection:

Maximum Allowable stress $\sigma = \frac{3}{2}\frac{WL}{nbt^2}$

$$215 = \frac{3}{2} \times \frac{40 \times 10^3 \times 700}{n \times 50 \times 8^2} \qquad \therefore\ n = 61 \text{ leaves}$$

Maximum deflection under the central load,

$$\delta_{\max} = \frac{3}{8}\frac{WL^3}{nbt^3E} = \frac{3}{8}\left(\frac{45 \times 10^3 \times 700^3}{61 \times 50 \times 8^3 \times 200 \times 10^3}\right) = 18.53\,mm$$

4.15. APPLICATIONS OF LEAF SPRINGS

Leaf springs have been used in various industries due to their simple yet effective design. Here are some common applications of leaf springs:

1. *Automotive suspension:* Leaf springs have been widely used in automotive suspension systems, especially in trucks, buses, and older model cars. They provide support and dampening for the vehicle, helping to absorb shocks and maintain stability on rough roads.
2. *Railroad cars:* Leaf springs are employed in railroad cars to provide suspension and support for the weight of the car and its cargo. They help to absorb the vibrations and impacts encountered during rail transportation.
3. *Trailers and towed vehicles:* Leaf springs are commonly used in trailers and towed vehicles to support the weight of the cargo and provide a smoother ride. They are especially popular in utility trailers, boat trailers, and recreational vehicles.
4. *Agricultural equipment:* Leaf springs are utilized in various agricultural equipment, including tractors, ploughs, and harvesters, to provide suspension and support for heavy loads. They help to absorb the shocks and vibrations encountered during field work.

5. *Military vehicles:* Leaf springs have historically been used in military vehicles due to their durability and reliability in rough terrain. They are commonly found in military trucks, armored vehicles, and tanks.
6. *Industrial machinery:* Leaf springs find applications in various industrial machinery and equipment where suspension and shock absorption are required. They are used in applications such as forklifts, cranes, and heavy-duty machinery.
7. *Lifting devices:* Leaf springs are also used in lifting devices such as scissor lifts and platform lifts to provide support and stability while lifting heavy loads.
8. *Custom and vintage vehicles:* Leaf springs are sometimes used in custom and vintage vehicle builds due to their simplicity and retro aesthetic appeal. They can be found in hot rods, custom motorcycles, and classic cars.
9. *Recreational vehicles:* Leaf springs are utilized in recreational vehicles such as camper vans, motorhomes, and off-road vehicles for their ability to provide stable suspension support, especially when traveling over uneven terrain.

UNIT SUMMARY

- *Power screws* are mechanical devices that change rotational motion into linear motion and the other way around
- Classification of screw threads: Square thread, Acme thread, Buttress thread
- Torque required to raise the load

$$M_t = \left[W \tan(\alpha + \phi) \times \frac{d_m}{2} \right] + \left[\mu_c \times W \left[\frac{R_1 + R_2}{2} \right] = \mu_c \times W \times R \right]$$

- Torque required to lower the load $M_t = W \tan(\phi - \alpha) \frac{d_m}{2}$
- *"Overhauling"* of power screws refers to a condition where the load or external forces on the screw can cause it to rotate freely in one direction (usually to lower a load or allow for easy adjustment) without the need for actively turning the screw.
- *Efficiency of square threaded screws:* The ratio between the ideal effort (i.e. the effort required to move the load, neglecting friction) to the actual effort (i.e. the effort required to move the load taking friction into account)

$$\eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

- *The variables affecting the efficiency of a square thread:* Lead of the screw, Mean diameter of the screw, Coefficient of friction

- *Maximum efficiency* of power threaded screw $\eta_{\max} = \dfrac{1-\sin\phi}{1+\sin\phi}$
- *Types of stresses in power screws:* Direct stress, Bearing pressure, Transverse shear stress, Torsional shear stress, Buckling stress
- *Springs* are mechanical devices typically used to store and release mechanical energy
- *Classification of springs:* Helical compression or extension springs, Helical torsion springs, Spiral springs, Leaf springs, Belleville springs
- Maximum shear stress induced in the wire, $\tau_3 = K \times \dfrac{8WC}{\pi d^2}$
- Energy stored in the spring $U = \dfrac{4W^2D^3n}{Gd^4}$
- Deflection of the coil spring $\delta = \dfrac{8WD^3n}{Gd^4}$
- Stiffness of the coil spring $k = \dfrac{Gd^4}{8D^3n}$
- A leaf spring is a particular kind of spring composed of several plates, or leaves, arranged in progressively larger stacks on top of one another
- Bending stress in the leaf spring $\sigma = \dfrac{3WL}{2nbt^2}$
- Stiffness of the leaf spring $\delta = \dfrac{3WL^3}{8Enbt^3}$
- Stiffness of the leaf spring, $k = \dfrac{W}{\delta} = \dfrac{8Enbt^3}{3L^3}$

EXERCISES

Multiple Choice Questions

1. Core diameter of screw thread is

   (a) $d+p$ (b) $d+2p$

   (c) $d-p$ (d) $d-2p$

2. The largest diameter of the screw is called as

   (a) nominal diameter (b) core diameter

   (c) pitch circle diameter (d) inner diameter

3. The efficiency of power screw to be self-locking type should be

(a) >50% (b) <50%

(c) =50% (d) <25%

4. The efficiency of the power screw is

(a) $\eta = \dfrac{2\tan\alpha}{\tan(\alpha-\phi)}$ (b) $\eta = \dfrac{\tan\alpha}{\tan(\alpha-\phi)}$

(c) $\eta = \dfrac{\tan(\alpha+\phi)}{\tan\alpha}$ (d) $\eta = \dfrac{\tan\alpha}{\tan(\alpha+\phi)}$

5. The pitch of single start screw is $6\ mm$, then the lead of double start screw is

(a) $6\ mm$ (b) $3\ mm$

(c) $12\ mm$ (d) $24\ mm$

6. The torque required to lower load in screw jack is

(a) $M_t = W\tan(\phi-\alpha)\dfrac{d_m}{2}$ (b) $M_t = W\tan(\phi+\alpha)\dfrac{d_m}{2}$

(c) $M_t = \dfrac{W}{2}\tan(\phi-\alpha)\dfrac{d_m}{2}$ (d) $M_t = W\tan(\alpha-\phi)\dfrac{d_m}{2}$

7. Height of the nut $=$

(a) $n \times p$ (b) $n+p$

(c) $n-p$ (d) n/p

8. Bearing pressure on nut is

(a) $p_b = \dfrac{W}{\frac{\pi}{4}\left[d_o^2 + d_c^{\ 2}\right]\times n}$ (b) $p_b = \dfrac{W}{\frac{\pi}{4}\left[d_o^2 - d_c^{\ 2}\right]\times n}$

(c) $p_b = \dfrac{W}{\frac{\pi}{4}\left[d_o^2 - d_c^{\ 2}\right]}$ (d) $p_b = \dfrac{W}{\left[d_o^2 - d_c^{\ 2}\right]\times n}$

9. Which one of the following condition is correct, in order to exhibit the self-locking behavior in a screw jack?

(a) $\phi > \alpha$ (b) $\phi < \alpha$

(c) $\phi = \alpha$ (d) none of the above

10. Springs are mechanical devices typically used to store and release

(a) mechanical energy (b) thermal energy

(c) electrical energy (d) thermos-mechanical energy

11. Solid length of the spring

(a) $2n \times d$ (b) $n + d$

(c) $n \times d$ (d) $\frac{n \times d}{2}$

12. Spring index $C =$

(a) $\frac{D}{d}$ (b) $D \times d$

(c) $D \times 2d$ (d) $D + 2d$

13. The deflection of the spring is measured as $50\ mm$ when the force applied on the spring is $2\ kN$. Find the spring rate.

(a) $20\ N-mm$ (b) $80\ N-mm$

(c) $60\ N-mm$ (d) $40\ N-mm$

14. Which one of the following effect is considered for Wahl's correction factor?

(a) direct shear effects (b) curvature effects

(c) both direct shear and curvature effects (d) helix angle

15. Deflection of the spring $\delta =$

(a) $\frac{WD^3 n}{8Gd^4}$ (b) $\frac{8WD^3 n}{Gd^4}$

(c) $\frac{WD^3 n}{Gd^4}$ (d) $\frac{8WG^3 n}{Dd^4}$

16. The minimum number of full length leaves in a leaf spring is

(a) 1 (b) 3

(c) 2 (d) 4

17. The breadth and thickness of the leaf spring are denoted as b and t. The section modulus can be written as

(a) $\frac{bt^2}{6}$ (b) $\frac{bt^2}{4}$

(c) $\frac{bt^2}{32}$ (d) $\frac{bt^2}{64}$

18. The leaves other than the master and second master leaf in a semi elliptic leaf spring is known as ____________.

(a) spring eye (b) graduated leaves

(c) master Leaf (d) rebound clip

19. In either direction, what screw thread type is used for power transmission?
    (a) square threads (b) acme threads
    (c) buttress threads (d) multiple threads
20. _________ screws are ideal for power transmission.
    (a) high load lifting capacity (b) low efficiency
    (c) high efficiency (d) high mechanical advantage

Answers to Multiple Choice Questions

1. (c) 2. (a) 3. (b) 4. (d) 5. (c) 6. (a) 7. (a) 8. (b) 9. (a) 10. (a) 11. (c) 12. (a) 13. (d) 14. (c) 15. (b) 16. (c) 17. (a) 18. (b) 19. (a) 20. (c)

Short and Long Answer Type Questions

1. List the applications of power screws.
2. The nominal diameter and pitch of the screw are $20\ mm$ and $6\ mm$. Find the mean diameter of the screw.
3. Write the types of screw threads and explain in detail.
4. Sketch the thread profile of power screws.
5. List the advantages and disadvantages of the power screw.
6. What is the thread's self-locking feature and when is it required?
7. Describe the procedure to design toggle jack.
8. "Power screw overhauling": what is it? For overhauling, what are the requirements?
9. Classify springs.
10. What are the uses of helical springs?
11. List materials used for springs.
12. Define stiffness of spring and mention its unit in SI system.
13. What is Wahl's correction factor?
14. List the functions of spring.
15. Draw a leaf spring and indicate its parts.
16. A triple-threaded square screw with a nominal diameter of $45\ mm$, and a pitch of $7\ mm$ is utilized for lifting a load. The collar has an outer diameter of $90\ mm$ and an inner diameter of $60\ mm$. The coefficient of friction between the thread and collar surface is assumed to be $0.15.$. The screw is used to lift a load of $13\ kN$. Using the uniform wear theory for collar friction, calculate the following:
    (i) The torque required to lift the load;

(ii) The torque required to lower the load; and

(iii) The force needed to lift the load if applied at a radius of $450\ mm$.

17. In a machine tool application, an operating nut attached to a screw is used to move the tool holder, which advances at a rate of 5 m/min. The screw features a nominal diameter of 48 mm and a single-start square thread with an 8 mm pitch. The operating nut exerts a force of 500 N to drive the tool holder. The friction collar has a mean radius of 40 mm, and the coefficient of friction at both the thread and collar surfaces is 0.15. Determine the following:

(i) power required to drive the screw; and

(ii) the efficiency of the mechanism

18. A double-threaded ISO metric trapezoidal power screw with a nominal diameter of 90 mm and a pitch of $12\ mm$ is used to lift a load of $210\ kN$. Given that the coefficient of friction at the screw threads is 0.15, calculate:

(i) the torque required to lift the load,

(ii) the torque required to lower the load, and

(iii) the efficiency of the screw, assuming collar friction is negligible.

19. A double start square threaded power screw of $30\ mm$ nominal diameter and $5\ mm$ pitch is acted upon by an axial load of $14\ kN$. The outer diameter of screw collar is $50\ mm$ and inner diameter of screw collar is $20\ mm$. The coefficient of friction between thread and collar is taken as 0.25 and 0.15 respectively. The screw rotates at $15\ rpm$. Assuming uniform wear condition at the collar and allowable thread bearing pressure of $6\ N/mm^2$, Find:

(i) the torque required to rotate the screw;

(ii) the stress in the screw; and

(iii) the number of threads of nut in engagement with screw

20. The screw jack consists of a square-threaded steel screw with nominal diameter of $50\ mm$ rotates in a bronze nut and lifts a load of $40\ kN$. The pitch of the screw is $9\ mm$. Assume the permissible bearing pressure at the threads is $15\ N/mm^2$. Find:

A screw jack, featuring a square-threaded steel screw with a nominal diameter of $50\ mm$, rotates within a bronze nut to lift a load of $40\ kN$. The screw has a pitch of $9\ mm$. Given that the allowable bearing pressure on the threads is $15\ N/mm^2$, determine:

(i) the required length of the nut; and

(ii) the transverse shear stress in the nut

21. A helical spring has an outer diameter of $70\ mm$ and is made from wire with a diameter of $6 mm$. The allowable shear stress is $300\ MPa$, and the modulus of rigidity is $80\ kN/mm^2$. Calculate the spring's axial load capacity and the deflection per active coil.

22. A close-coiled helical spring has a stiffness of $1.6\ N/mm$ under a maximum load of $55\ N$. The maximum shearing stress in the wire is $125\ N/mm^2$, and the solid length of the spring (when the coils are fully compressed) is $45\ mm$. Take $C = 4.5\times10^4\ N/mm^2$. Determine:
    (i) the diameter of the wire,
    (ii) the mean diameter of the coils, and
    (iii) the number of coils required

23. A close-coiled helical spring has a stiffness of $850\ N/m$ in compression and can support a maximum load of $40\ N$. The solid length of the spring (when the coils are fully compressed) is $42\ mm$, and the maximum shearing stress is $120\ N/mm^2$. Determine the following:
    (i) the wire diameter
    (ii) the mean coil radius, and
    (iii) the number of coils. The modulus of rigidity of the spring material is given as $0.4\times10^5\ N/mm^2$

24. Design a helical valve spring to handle an operating load range of approximately $80\ to\ 125\ N$ with a deflection of $7\ mm$ across this range. The spring index is given as $C = 10$. The permissible shear stress of the spring material is $420\ MPa$, and its modulus of rigidity is $G = 70\ kN/mm^2$. Take Wahl's correction factor $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$.

25. A railway wagon weighing $40 kN$ and traveling at a speed of $7 km$ per hour needs to be stopped using four buffer springs. The maximum allowable compression for each spring is $210 mm.$. Calculate the number of turns required in each spring, given that the mean diameter of the spring is $140 mm$ and the wire diameter is $24\ mm.$. Take $G = 80\ kN/mm^2$.

26. A semi-elliptic leaf spring in a car suspension system comprises thirteen leaves of varying lengths, including the master leaf, along with three additional full-length leaves. The distance between the centers of the two eyes of the spring is 1 meter. The spring experiences a load of $70\ kN$. The ratio of the width to the thickness of each leaf in the spring is 9:1. The modulus of elasticity of the material used for the leaves is $200\ kN/mm^2$, and the stress induced in each leaf is $400\ kN/mm^2$. Determine the width and thickness of the leaves.

27. The semi-elliptical laminated vehicle spring, which has 7 leaves and a width of $65\,mm$, is designed to support a load of $5\,kN$. Two of the leaves extend the full length of the spring. The spring measures 1 meter in length and is secured to the vehicle axle with two U-bolts spaced $75\,mm$ apart. The U-bolts firmly anchor the center of the spring, effectively acting like a band with a width equal to the distance between the bolts. The permissible stress for the spring material is $400\ MPa$. Determine

    1. thickness of the spring leaves
    2. deflection of the spring

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NPTEL VIDEOS

1. Lecture – 18 - Power screws - Design of Machine Elements - I by Prof. G. Chakrborty, Department of Mechanical Engineering, IIT Kharagpur.

Power Screws

2. Lecture – 19 – Design of power screws - Design of Machine Elements - I by Prof. G. Chakrborty, Department of Mechanical Engineering, IIT Kharagpur.

Design of Power Screws

3. Lecture – 27 & 29– Design of springs - Design of Machine Elements - I by Prof. G. Maiti, Department of Mechanical Engineering, IIT Kharagpur.

Design of Springs 1 | Design of Springs 2

Lecture 27 | Lecture 29

5. https://www.thomasnet.com/articles/metals-metal-products/types-of-spring-materials/

Spring Materials

5 Design of Fasteners and Ergonomics Considerations

UNIT SPECIFICS

Through this unit, the following aspects are discussed:

- Stresses in screwed fasteners – Initial stresses due to tightening forces, stresses due to external forces and combination of these
- Bolts of uniform strength
- Design of bolted joints under eccentric loading conditions
- Design of parallel and transverse fillet welds for axially loaded symmetrical sections
- Advantages and disadvantages of screwed and welded joints
- Ergonomics of design: Man–Machine relationship; design of equipment for control, environment & safety;
- Aesthetics of design: Shape, size, color and surface finish

RATIONALE

The fifth unit of this book will help the students to have a good understanding of the type of stresses induced during the screw fastening due to applied load to determine their dimensions. After completing this unit, a student will be able to design bolted joints subjected to eccentric loading and design parallel and transverse fillet welds subjected to axially loaded symmetric section. The student will also be able to integrate ergonomic and aesthetic considerations in design to create fasteners that not only perform well but also enhance the user experience and visual appeal of the final product.

PRE-REQUISITES

Engineering mechanics, Strength of materials

UNIT OUTCOMES

List of outcomes of this unit is as follows:

On the successful completion of the unit, students will be able to

U5-O1: Determine the dimensions of screw fasteners under various stresses due to applied load

U5-O2: Design the bolted joints under eccentric loading conditions

U5-O3: Design the parallel and traverse fillet welds subjected to axial load

U5-O4: Design products and environments that are comfortable, efficient, and safe for humans to use

U5-O5: Apply aesthetic considerations in design to enhance the user experience and visual appeal of the final product

Unit 5 Outcomes	Mapping with Course Outcomes (1 – weak correlation, 2 – medium correlation, 3 – strong correlation)				
	CO-1	**CO-2**	**CO-3**	**CO-4**	**CO-5**
U5-O1	3	3	3	3	3
U5-O2	3	3	3	3	3
U5-O3	3	3	3	3	3
U5-O4	3	3	3	3	3
U5-O5	1	1	1	1	-

5.1. INTRODUCTION

Fasteners are mechanical devices used to join or fix two or more components together securely. They come in various shapes, sizes, and materials, each tailored to specific applications. Fasteners play a vital role in industries such as construction, automotive, aerospace, manufacturing, and electronics. Fig. 5.1 illustrates the different types of fasteners.


(a)


(b)


(c)


(d)


(e)

Fig. 5.1: (a) Screws (b) Bolt and Nut (c) Washers (d) Rivets (e) Nails

1. Screws: Screws are some necessary fasteners for many tasks, such as attaching electrical components and joining wooden boards. They are available in many shapes and sizes, ranging from coarse deck screws to fine ones. Whether for indoor or outdoor use, each is made for a certain function.

2. Bolts and Nuts: Fasteners such as bolts and nuts are frequently used to join disparate parts. To connect the pieces, the bolt is inserted and fastened with a nut on the end.

3. Washers: One kind of fastener that is utilized in numerous sectors is the washer. Typically, they are positioned beneath axle bearings, nuts, and joints to lessen friction and stop vibration-induced component loosening. Additionally, washers aid in component isolation and possible leak prevention. The different kinds of washers have different shapes based on what they are used for, but they all have the same function.

4. Rivets: One kind of permanent fastener that is utilized in many different sectors and applications is the rivet. This permanent joint cannot be removed or reused once it is riveted. Traditionally, rivets are used to secure metal plates and sheets together. The rivet heads are either spherical or hexagonal, and they usually have a long mandrel or shank with a cylindrical shape. Rivets are frequently employed where dependability and permanency are needed because of their superior strength over other fastener types.

5. Nails: Nail fasteners are one of the most common types of fasteners. These nails are come in various sizes and shapes to suit different applications due to their efficiency, ease of use, and cost-effectiveness. Nails create a strong, temporary or semi-permanent joint and are often used for various applications in construction, machinery, electronics, and other fields.

Among all the fastener kinds, screw fasteners are the most adaptable. They have strong holding power due to their threaded shafts, and unlike bolts, they don't need anything to keep them in place. Bolts and nuts combine to form screw joints, which are used to fasten, adjust, assemble, check, and replace machine parts.

Threaded fasteners, such as screws, usually need a matching nut or pre-tapped hole. Similar to screws, bolts typically include a separate nut to secure the joint and are larger in size.

5.2. STRESSES IN SCREWED FASTENERS

Determining the dimensions of screw fastenings requires analyzing the stresses caused by both static and dynamic loading. To design for static loading, it is necessary to know both the initial tightening and the external loadings. The following stresses in screw fastenings must be taken into account under static loading:

1. Initial stresses caused by tightening forces
2. Stresses caused by external forces
3. Stresses caused by combination of tightening and external forces

5.2.1. Initial stresses caused by tightening forces

When a nut is tightened on to a screw, the following stresses are induced:

(a) Tensile stresses caused by stretching of the bolt

(b) Torsional shear stress resulting from frictional resistance at the threads

(c) Shear stress distributed across the threads

(d) Compressive or crushing stress on the threads

(e) Bending stress if the surfaces under the bolt head or nut are not perfectly aligned with the bolt axis.

5.2.1.1. Tensile stress

Bolts are designed using a large factor of safety to account the indeterminate stresses, as it is not possible to accurately determine the aforementioned stresses. The tensile stress can be written as,

$$\sigma_t = \frac{P_i}{A} = \frac{2840 d_o}{\frac{\pi}{4}\left(\frac{d_m + d_c}{2}\right)^2} \tag{5.1}$$

The value of $P_i = 2840 d_o$ is an initial load for making a joint fluid tight like steam engine cylinder cover joints. Whereas $P_i = 1420 d_o$ can be used for fastening other than fluid tight joints.

Where P_i is initial tension in bolts due to tightening, d_m is mean diameter or pitch of the screw, d_c is core diameter $d_m = 0.84 d_o$, d_o is nominal diameter of the nut.

5.2.1.2. Torsional shear stress

Torsional shear stress is a type of stress that occurs in a bolt material subjected to a twisting or torsional force. Consider the torsion equation,

$$\frac{M_t}{J} = \frac{\tau_s}{r} = \frac{G\theta}{l} \tag{5.2}$$

Torsional shear stress $\tau_s = \frac{M_t r}{J} = \frac{M_t}{\frac{\pi}{32} \times d_c^4} \times \frac{d_c}{2}$ (5.3)

Torsional shear stress $\tau_s = \frac{16 M_t}{\pi d_c^3}$ (5.4)

Twisting torque $M_t = \frac{\pi}{16} \times \tau_s \times d_c^3$ (5.5)

Where J is polar moment of inertia, M_t is twisting torque

5.2.1.3. Shear stress distributed across the threads

This stress is induced in screw fasteners at core diameter and in nut threads at nominal diameter due to axial load acting on them.

The average shear stress for screw (τ_{sc}) is written as,

$$\tau_{sc} = \frac{W}{\pi d_c b n} \tag{5.6}$$

The average shear stress for nut (τ_n) is written as,

$$\tau_n = \frac{W}{\pi d_o b n} \tag{5.7}$$

Where d_c is core diameter of screw thread, b is width of thread at root, n is number of thread in contact.

5.2.1.4. Compressive or crushing stress on the threads

The compressive or crushing stress exist between the thread of screw and nut. The formula to calculate Compressive or crushing stress on the threads is:

$$\sigma_c = \sigma_{cr} = \frac{W}{\pi\left(d_o^2 - d_c^2\right)n} \tag{5.8}$$

5.2.1.5. Bending stress

The bending stresses that are created in the bolt shank when the connected parts' outside surfaces are not parallel. The bending stresses (σ_b) can be calculated by formula,

$$\sigma_b = \frac{xE}{2l} \tag{5.9}$$

Where x is difference in height between the extreme corner of the nut or head, E is modulus of elasticity, l is length of shank of the bolt.

5.2.2. Stresses caused by external forces

External forces exerted on bolts and screws cause tensile stresses; on sometimes, however, the bolts also experience shear loads. The weakest point of a bolt will be near the thread's root when it is subjected to an axial tensile load.

5.2.2.1. Tensile stress

Let d_c be the diameter at the root of the thread, then the tensile stress due to external forces is given by,

$$\sigma_t = \frac{W}{\frac{\pi}{4}d_c^2} \tag{5.10}$$

The diameter at the root of the thread will be calculated from the Eq. (5.10). Then, refer to the table of screw threads and identify the most suitable standard diameter, when the types of threads are selected.

Assume n is number of bolts used to resist the external forces acting on the member, tensile stress is given by,

$$\sigma_t = \frac{W}{\frac{\pi}{4}d_c^2 \times n} \tag{5.11}$$

Take root or core diameter of the thread $d_c = 0.84 d_o$

5.2.2.2. Shear stress

Shear stress is induced in a bolt when it prevents the relative movement between the parts in the direction normal to the axis of the bolt.

The shear stress (τ) due to shear load is given by, $\tau = \dfrac{W_s}{\frac{\pi}{4} d_c^2}$ (5.12)

Assume n is number of bolts used to resist the shear load acting on the member, then, the shear stress (τ) is given by,

$$\tau = \frac{W_s}{\frac{\pi}{4} d_c^2 \times n} \tag{5.13}$$

5.2.3. Stresses caused by combination of tensile and shear stresses

For cylinder cover joints in steam engines, the bolts need to withstand both the initial tightening load and the steam load. The resultant load on each bolt will be influenced by the relative elastic yielding of the bolt compared to the connected components. When the connected components are much more yielding than the bolt, the resultant load on the bolt is the sum of the initial tension and the external load. The resultant load is generally calculated using the formula:

$$W_R = W_i + KW_2 \tag{5.14}$$

Where W_R is Resultant load on the bolt, W_i is initial tightening load on the bolt, W_2 is external load on the bolt.

$$K = \frac{k_b}{k_b + k_c} \tag{5.15}$$

Where k_b is the stiffness constant for the bolt, k_c is the stiffness constant for compressed member.

The values of K which is used for preliminary calculations for the bolt design are listed in the Table 5.1.

Table 5.1: Value of K

Type of Joint	K
A metal-to-metal joint secured with through bolts	0.00 to 0.10
Hard copper gasket with long through bolts	0.25 to 0.50
Soft copper gasket with long through bolts	0.50 to 0.75
Soft packaging with through bolts	0.75 to 1.00
Soft packaging with studs	1.00

Example 5.1: Find the safe tensile load for a bolt of $M24$, if safe tensile stress is $50\ MPa$. Take the cross sectional area as $353\ mm^2$.

Given Data:

Bolt size = $M24$

Safe tensile stress $\sigma_t = 50\ MPa = 50\ N/mm^2$

Cross sectional area $A_c = 353\ mm^2$

Find:

15. Safe tensile load

Solution:

Safe tensile stress $\sigma_t = \frac{Load}{Area} = \frac{W}{A_c}$

Safe tensile load $W = \sigma_t \times A_c = 50 \times 353 = 17650\ N = 17.65\ kN$

Example 5.2: A bolt carries a tensile load of $6\ kN$ and the tightening load is $2\ kN$. Determine the size of the bolt taking allowable stress as $70\,MPa$ for the bolt.

Given Data:

External tensile load $W_2 = 6\ kN = 6000N$

Initial tightening load on the bolt $W_i = 2\ kN = 2000\ N$

Allowable shear stress on the bolt $\sigma_{sb} = 70\ MPa = 70\ N/mm^2$

Find:

16. Size of the bolt

Solution:

The resultant load acting on the bolt $W_R = W_i + KW_2$

Assume soft copper gasket is used, take the value from Table 5.1 as $K = 0.5$.

$W_R = 2000 + 0.5 \times 6000 = 5000\ N$

Allowable shear stress on the bolt $\sigma_{sb} = \frac{W_R}{\frac{\pi}{4} d_c^2} = \frac{W_R}{A_c}$

Rearranging the above equation $A_c = \frac{W_R}{\sigma_{sb}}$

$$A_c = \frac{6000}{70} = 85.71\ mm^2$$

Select the standard bolt M12 from Design Data Book for $A_c = 85.71\ mm^2$.

Example 5.3: A 50 kN load is lifted using an eye bolt. Determine the bolt's nominal diameter if the tensile stress is limited to 90 MPa. Assume coarse threads.

Given Data:

External tensile load $W = 50\ kN = 50 \times 10^3\, N$

Allowable tensile stress on the bolt $\sigma_t = 90\ MPa = 90\ N/mm^2$

Find:

17. Nominal diameter of the bolt

Solution:

Tensile stress on the bolt $\sigma_t = \dfrac{W}{\frac{\pi}{4} d_c^2}$

$$d_c^2 = \frac{4W}{\pi\sigma_t} = \frac{4 \times 50 \times 10^3}{3.14 \times 90} = 707.71$$

Core diameter $d_c = 26.6\ mm$

From Design Data Book, we find that the standard core diameter (d_c) is 28.706 mm and the corresponding nominal diameter (d_o) is 33 mm.

Example 5.4: A generator weighing $12\ kN$ is to be provided with eye bolt in the housing for lifting purpose. Find the size of the bolt it is made of C-40 steel. If the ultimate tensile strength of C-40 steel is $500\ MPa$ and the factor of safety is 6. Take allowable tensile stress on the bolt $\sigma_t = 90\ MPa$. *Given Data:*

External tensile load on the bolt $W =$ Weight of the generator $W = 12\ kN = 12 \times 10^3\, N$

Allowable tensile stress on the bolt $\sigma_t = 90\ MPa = 90\ N/mm^2$

Ultimate tensile strength of C-40 steel bolt, $\sigma_{ut} = 500\ MPa = 500\ N/mm^2$

Factor of safety $FOS = 6$

Find:

1. Nominal diameter of the bolt

Solution:

Allowable permissible stress $\sigma_t = \dfrac{Ultimate\ tensile\ strength}{FOS}$

$$\sigma_t = \frac{500}{6} = 83.33\ N/mm^2$$

Also, $\sigma_t = \frac{W}{A_c} = \frac{12\times10^3}{A_c}$

$$A_c = \frac{12\times10^3}{\sigma_t} = \frac{12\times10^3}{83.33} = 144\ mm^2$$

Select the standard bolt M16 from Design Data Book for $A_c = 144\ mm^2$.

5.3. BOLTS OF UNIFORM STRENGTH

A bolt of uniform strength is designed to have consistent stress distribution along its length, ensuring that no single section is weaker than the others. This concept aims to eliminate stress concentrations and improve the overall performance and reliability when the bolts experience shock or impact loads. For instance, the connecting rod bolts or cylinder head bolts in an internal combustion engine experiences shock or impact loads.

It is important to consider the resilience of the bolt to prevent the failure of bolts at the threads when bolts are used under shock or impact loads. The ability of a material to absorb energy during elastic deformation and release it when the load is removed is called resilience. A resilient bolt, for instance, absorbs energy without permanent deformation within its elastic limit and releases it once the load is removed. This characteristic, often referred to as the 'spring property', allows the bolt to function like a spring, absorbing shocks and vibrations. The energy absorbed during elastic deformation is proportional to the square of the material's stress.

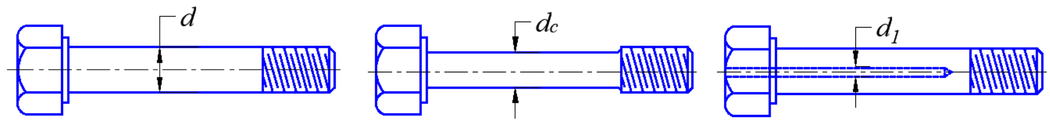
Fig. 5.2 (a) shows the shape of ordinary bolt. The diameter of the shank and major diameter of the threaded portion of the ordinary bolt are same and it is denoted as d. The core diameter of the threaded portion is d_c. The tensile force acts on the bolt axially, when the bolt experiences shock or impact loads. The threaded portion is the weakest area in the ordinary bolt, since the diameter d_c is less than the shank diameter d. Therefore, the stress induced in the threaded portion is higher than the shank area. Hence, the energy absorbed in the threaded portion of the bolt is higher.

To enhance the shock-absorbing capacity of the bolt, reduce the diameter of the shank portion to be equal or even be smaller than the core diameter of the threads. Additionally, since the energy absorbed by the shank increases linearly with its length, extending the length of the shank can further improve the bolt's resilience.

Therefore, there are two ways for increasing the shock absorbing the capacity of bolts.

1. Reduce the shank diameter to the thread core diameter or even smaller
2. Increase the length of the shank portion of the bolt

A bolt that maintains the same stress level across various cross-sections, thereby absorbing maximum energy, is known as a bolt of uniform strength.



(a) (b) (c)

Fig. 5.2: Bolts of uniform Strength (a) Original bolt (b) Reduced diameter of the shank (c) Drilled hole in the shank

Two methods exist to reduce the cross-sectional area of the shank and convert a standard bolt into a bolt of uniform strength, as shown in Fig. 5.2 (b) and (c). The first method involves reducing the shank diameter to match the core diameter of the threads, as depicted in Fig. 5.2 (b). This results in the cross-sectional area of the shank being equal to that of the threaded portion. The second method entails drilling a hole to reduce the shank's cross-sectional area, as illustrated in Fig. 5.2 (c). The diameter of the hole is determined by equating the cross sectional area of the shank portion and cross sectional area of the threaded portion. Therefore,

$$\frac{\pi}{4}d^2 - \frac{\pi}{4}(d_1)^2 = \frac{\pi}{4}(d_c)^2$$

$$d_1 = \sqrt{d^2 - (d_c)^2} \qquad (5.16)$$

Where d is nominal diameter, d_c is core diameter of the threads, d_1 is diameter of the hole

5.4. DESIGN OF BOLTED JOINTS SUBJECTED TO ECCENTRIC LOADING

In various applications, a machine component experiences loading that generates both a bending moment and direct normal or shear loading concurrently. This type of loading is commonly referred to as eccentric loading. In this section the bolted joints subject to eccentric loading is discussed. Bolted joints subject to eccentric loading are used in various applications, such as wall brackets and pillar cranes. The eccentric load may be

Bol. Joi. Lec 14

1. Load acting parallel to the axis of the bolts
2. Load acting perpendicular to the axis of the bolts
3. In the plane containing the bolts

5.4.1. Load acting parallel to the axis of the bolts

As shown in Fig. 5.3, let us consider a bracket with a rectangular base that is fixed to a steel structure using four bolts with same diameter. It is considered that every bolt experiences a direct tensile load of $W_{t1} = \frac{W}{n}$ where n is the number of bolts. The external load (W) applied on the bracket tends to rotate about the edge $T - T$. The stretching of each bolt varies based on its distance from the tilting edge. The symbol

Ecc. Bolted Lec 32

" w " represents the load per unit distance on a bolt due to the turning effect of the bracket. W_1 and W_2 are the load acting on bolts at a distances l_1 and l_2 from the tilting edge.

Load acting on each bolt at a distance l_1 ,

$$W_1 = w \times l_1 \tag{5.17}$$

Moment about the tilting edge $= w \times l_1 \times l_1 = \; = w \times l_1^2$ (5.18)

Load acting on each bolt at a distance l_2 ,

$$W_2 = w \times l_2 \tag{5.19}$$

Moment about the tilting edge $= w \times l_2 \times l_2 = \; = w \times l_2^2$ (5.20)

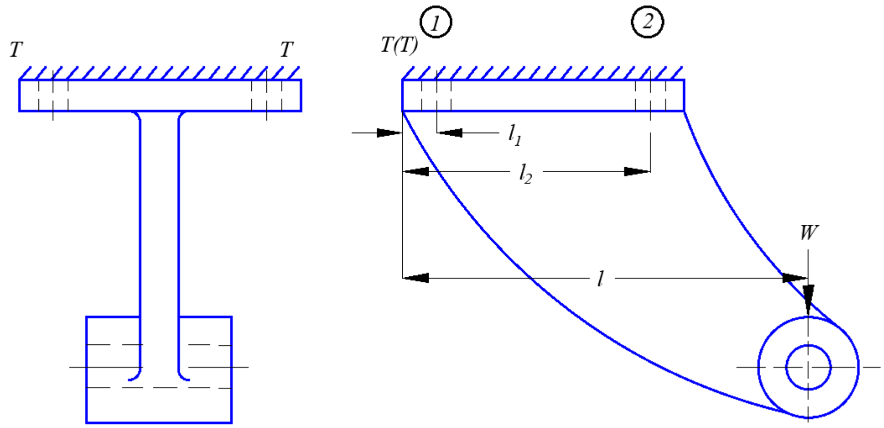


Fig. 5.3: Eccentric load acting parallel to the axis of bolts

The entire moment of the force applied to the bolts about the tilting edge,

$$= 2w \times l_1^2 + 2w \times l_2^2 \tag{5.21}$$

Moment due to load W about the tilting edge $= W \times l$ (5.22)

From equations (5.21) and (5.22) we have,

$$W \times l = 2w \times l_1^2 + 2w \times l_2^2$$

$$w = \frac{W \times l}{2\left[l_1^2 + l_2^2\right]} \tag{5.23}$$

It can be observed that the more load is carried by the bolts nearer to the loading point. Hence, the bolts at a distance l_2 are highly loaded.

Tensile load on each bolt at a distance l_2 ,

$$W_{t2} = W_2 = w \times l_2 = \frac{W \times l \times l_2}{2\left[l_1^2 + l_2^2\right]} \tag{5.24}$$

As well as, the overall tensile load on the heavily loaded bolt,

$$W_t = W_{t1} + W_{t2} \tag{5.25}$$

Take σ_t tensile stress for the bolt material, and d_c is the core diameter of the bolt, then total tensile load,

$$W_t = \frac{\pi}{4} \times d_c^2 \times \sigma_t \tag{5.26}$$

From the equations (5.25) and (5.26), the value of d_c may be calculated.

Example 5.5: Following data is given for the bracket shown in Fig. 5.4.: $W = 12\ kN$, $l_1 = 50\ mm$, $l_2 = 325\ mm$. There are 4 identical bolts with major diameter $d = 25\ mm$ are used to fix the bracket and it is not pre-loaded. The load is applied parallel to the axis of the bolts, and the maximum allowable tensile stress for the bolt material is $50\ N/mm^2$. Find the size of the bolts.

Given Data:

External tensile load on the bracket $W = 12\ kN = 12 \times 10^3\ N$

Major diameter of the bolt $d = 25\ mm$

Allowable tensile stress on the bolt $\sigma_{tb} = 50\ N/mm^2$

Distance of bolts 1 and 4 from the tilting edge $A - B$, $l_1 = 50\ mm$

Distance of bolts 2 and 3 from the tilting edge $A - B$, $l_2 = 50 + 300 = 350\ mm$

Find:

1. Size of the bolt

Solution:

Since the load is acting parallel to the axis of bolts, therefore direct tensile load on each bolt,

$$W_{t1} = \frac{W}{n} = \frac{12 \times 10^3}{4} = 3000\ N$$

Since the bracket has a turning effect, the load on each bolt per mm of distance from the edge A-B,

$$w = \frac{W \times l}{2\left[l_1^2 + l_2^2\right]} = \frac{12 \times 10^3 \times (50 + 50 + 300 + 50)}{2\left[50^2 + 350^2\right]} = 21.6\ N/mm$$

Since the bolts 2 and 3 are far away from the tilting edge A-B and these bolts are heavily loaded.

Maximum tensile load on each of bolts 2 and 3,

$W_{t2} = w \times l_2 = 21.6 \times 350 = 7560\ N$

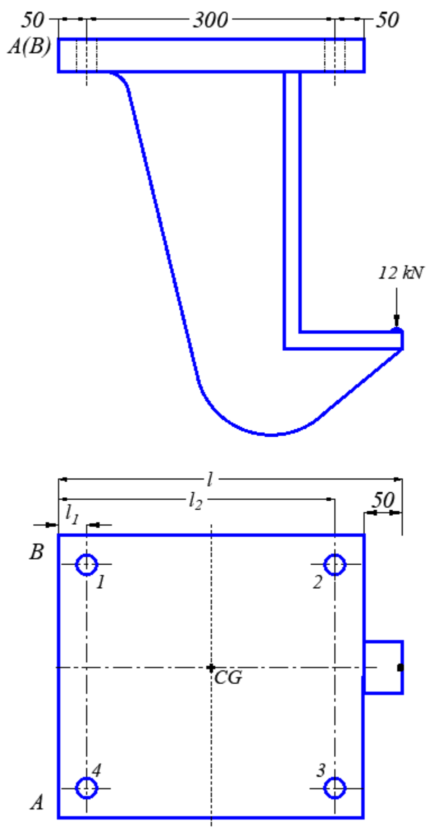


Fig. 5.4: Bracket - I

Total tensile load on each of the bolts 2 and 3,

$W_t = W_{t1} + W_{t2} = 3000 + 7560 = 10560\ N$

Also, tensile load on the bolt $W_t = \frac{\pi}{4}\left(d_c^2\right)\sigma_{tb}$

$$10560 = \frac{\pi}{4} \times \left(d_c^2\right) \times 50$$

$$d_c^2 = \frac{4 \times 10560}{3.14 \times 50} = 269.04\ mm^2$$

$$d_c = 16.40\ mm$$

From Design Data Book, we find that the standard core diameter $\left(d_c\right)$ is 16.933 mm and the corresponding nominal diameter $\left(d\right)$ is 20 mm.

5.4.2. Load acting perpendicular to the axis of the bolts

Fig. 5.5 shows the wall bracket subject to an eccentric load acting perpendicular to the bolt's axis. It is assumed that every bolt experiences a direct shear load of $W_{t1} = W_s = \frac{W}{n}$ where W is eccentric load, n is the number of bolts. The eccentric load W tries to rotate the bracket about the tilting edge $T-T$ in the clockwise direction.

Hence, the maximum tensile load on a heavily loaded bolt $\left(W_t\right)$ i.e. bolt 1 and 2, may be calculated as discussed in the previous section.

$$W_{t2} = W_t = \frac{W \times l \times l_2}{2\left[l_1^2 + l_2^2\right]} \tag{5.27}$$

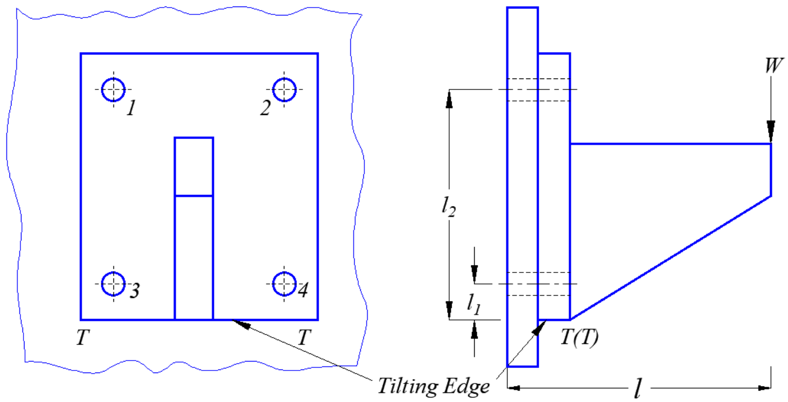


Fig. 5.5: Eccentric load acting perpendicular to the axis of bolts

When the bolts are subjected to combination of shear and tensile load, the equivalent tensile load can be written as,

$$\left(W_t\right)_{eq} = \frac{1}{2}\left[W_t + \sqrt{\left(W_t\right)^2 + 4\left(W_s\right)^2}\right] \tag{5.28}$$

The equivalent tensile load can be written as,

$$\left(W_t\right)_{eq} = \frac{\pi}{4} d_c^2 \sigma_t \tag{5.29}$$

Calculate the core diameter $\left(d_c\right)$ from equation (5.29). The relation used to find the nominal diameter $\left(d_o\right)$ is $\frac{d_c}{0.84}$.

Example 5.6: A bracket is fixed to the steel structure using four identical bolts arranged in two rows (with two bolts in each row), positioned $50\ mm$ and $375\ mm$ from the lower edge of the bracket. The bracket experiences maximum vertical load of $14\ kN$ applied at a distance of $410\ mm$. If the bolt material's allowable tensile stress is $85\ MPa$, then calculate the bolt sizes.

Given Data:

Eccentric load $W_{el} = 14\ kN = 14\times10^3\ N$

Eccentric distance $l = 410\ mm$

Distance between the lower edge and bottom row of the bolt $l_1 = 50\ mm$

Distance between the lower edge and top row of the bolt $l_2 = 375\ mm$

Allowable tensile stress on the bolt $\sigma_{tb} = 85\ MPa = 85\ N/mm^2$

Number of bolts $n = 4$

Find:

1. Size of the bolt

Solution:

Direct shear load on each bolt $W_s = \dfrac{W}{n} = \dfrac{14\times10^3}{4} = 3500\ N$

The bolts will experience tensile stress due to the turning moment, as force W will try to tilt the bracket clockwise about the lower edge. The bolts 3 and 4 are far away from the tilting edge and these bolts are heavily loaded.

Maximum tensile load carried by bolts 3 or 4,

$$W_t = \frac{W\times l\times l_2}{2\left[l_1^2 + l_2^2\right]} = \frac{14\times10^3\times410\times375}{2\left[50^2+375^2\right]} = 7519.65\ N$$

Due to the bolts' simultaneous exposure to shear and tensile loads, the equivalent tensile load,

$$(W_t)_{eq} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

$$(W_t)_{eq} = \frac{1}{2}\left[7519.65 + \sqrt{(7519.65)^2 + 4(3500)^2}\right]$$

$$(W_t)_{eq} = 8896.58\ N$$

The equivalent tensile load, $(W_t)_{eq} = \dfrac{\pi}{4} d_c^2 \sigma_t$

$$8896.58 = \frac{3.14}{4}\times d_c^2\times85$$

$$d_c^2 = \frac{4\times8896.58}{3.14\times85} = 133.33$$

The core diameter $(d_c) = 11.55\ mm$

From Design Data Book, we find that the standard core diameter (d_c) is 13.546 mm and the corresponding nominal diameter (d_o) is 16 mm.

5.4.3 Load in the plane containing the bolts

Fig. 5.6 illustrates the eccentric load with in the plane containing the bolts. There are two types of loads acting namely primary shear load and secondary shear load, on each threaded bolt due to eccentrically applied load W .

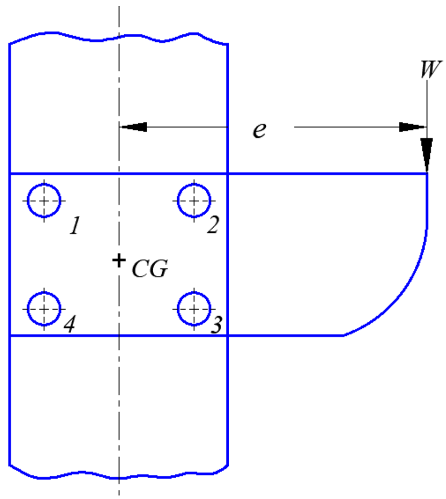


Fig. 5.6: Eccentric load in the plane containing the bolts - I

Primary shear load $W_{s1} = \frac{W}{n}$ (5.30)

Where W is eccentric load, n is the number of bolts.

Secondary shear load $W_{s2} = \frac{W\times e\times l_1}{l_1^2 + l_2^2 + l_3^2 + l_4^2}$ (5.31)

Where $l_1 =$ distance between centre of bolt (1) to the Centre of Gravity (CG)

$l_2 =$ distance between centre of bolt (2) to the CG

$l_3 =$ distance between centre of bolt (3) to the CG

$l_4 =$ distance between centre of bolt (4) to the CG

The maximum shear load on bolts 1 or 4,

$$W_{s\max} = \sqrt{W_{s1}^2 + W_{s2}^2 + 2W_{s1} \times W_{s2} \times \cos\theta} \tag{5.32}$$

Where θ is angle between W_{s1} and W_{s2}. The core diameter (d_c) can be calculated by equating the maximum shear load and maximum permissible shear force (τ).

$$W_{s\max} = \frac{\pi}{4} d_c^2 \tau \tag{5.33}$$

Example 5.7: A plate shown in Fig. 5.7 is subjected to a $15\ kN$ eccentric load (W), with the load being $450\ mm$ from the center of gravity (CG) of the bolts. The center-to-center distance between bolts 1 and 2 is $210\ mm$, and between bolts 1 and 4 is $150\ mm$. All bolts are identical, and the allowable shear stress for the bolts is $65\ MPa$. Determine the necessary bolt size.

Given Data:

Eccentric load $W = 15\ kN = 15 \times 10^3 N$

Eccentric distance $e = 450\ mm$

Distance between the bolts 1 and 2 $= 210\ mm$

Distance between the bolts 1 and 4 $= 150\ mm$

Number of bolts $n = 4$

Permissible shear stress of bolts $\tau = 65\ MPa = 65\ N/mm^2$

Find:

1. Size of the bolt

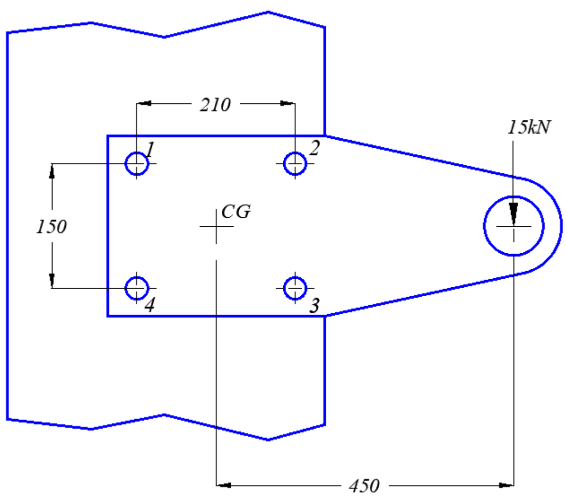


Fig. 5.7: Eccentric load in the plane containing the bolts - II

Solution:

Primary shear load on each bolt $W_{s1} = \dfrac{W}{n} = \dfrac{15\times10^3}{4} = 3750\ N$

Because each bolt is equally spaced from the center of gravity (CG) of the four bolts, each one experiences the same secondary shear load.

Distance between each bolt and the bolts' CG,

$$l_1 = l_2 = l_3 = l_4 = \sqrt{105^2 + 75^2} = 129.03\ mm$$

Secondary shear load $W_{s2} = \dfrac{W\times e\times l_1}{l_1^2 + l_2^2 + l_3^2 + l_4^2}$

$$W_{s2} = \frac{15\times10^3\times450\times129}{4\times\left(129^2\right)} = 13081\ N$$

Since the line connecting the bolt group's center of gravity to the bolt's center is perpendicular to the secondary shear load, as shown in Fig. 5.8. Consequently, the maximum shearing force on each bolt is determined by summing the primary and secondary shear loads on each bolt.

Based on Fig. 5.8's geometry, we may determine that

$\theta_1 = \theta_4 = 144^o30'$ and $\theta_2 = \theta_3 = 35^o30'$

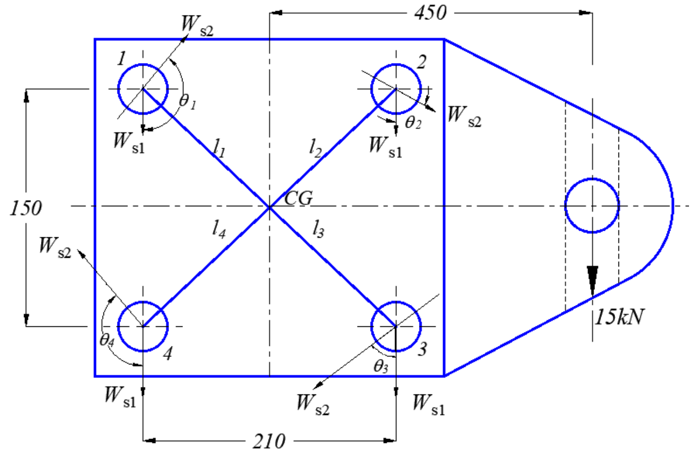


Fig. 5.8: Geometry – Shear load acting on the bolts

The maximum shear load or force on 1 and 4,

$$W_{s\max} = \sqrt{W_{s1}^2 + W_{s2}^2 + 2W_{s1}\times W_{s2}\times\cos\theta}$$

$$W_{s\max} = \sqrt{3750^2 + 13081^2 + 2\times3750\times13081\times\cos(144.5^o)}$$

$$W_{s\max} = \sqrt{3750^2 + 13081^2 + 2\times3750\times13081\times(-0.8141)}$$

$W_{s\max} = 10.261\times10^3\ N$

The maximum shear load or force on 2 and 3,

$$W_{s\max} = \sqrt{W_{s1}^2 + W_{s2}^2 + 2W_{s1}\times W_{s2}\times\cos\theta}$$

$$W_{s\max} = \sqrt{3750^2 + 13081^2 + 2\times3750\times13081\times\cos(35.5^o)}$$

$$W_{s\max} = \sqrt{3750^2 + 13081^2 + 2\times3750\times13081\times(0.8141)}$$

$$W_{s\max} = 16.28\times10^3\ N$$

The core diameter (d_c) is calculated by equating the maximum shear load and maximum permissible shear force (τ).

$$W_{s\max} = \frac{\pi}{4}d_c^2\tau \qquad \therefore 16.28\times10^3 = \frac{3.14}{4}\times d_c^2\times65$$

$$d_c^2 = \frac{4\times16.28\times10^3}{3.14\times65} = 319.06\ mm^2$$

$$d_c = 17.86\ mm$$

From Design Data Book, we find that the standard core diameter (d_c) is $20.320\ mm$ and the corresponding nominal diameter (d_o) is $24\ mm$.

5.5. FILLET WELDED JOINTS

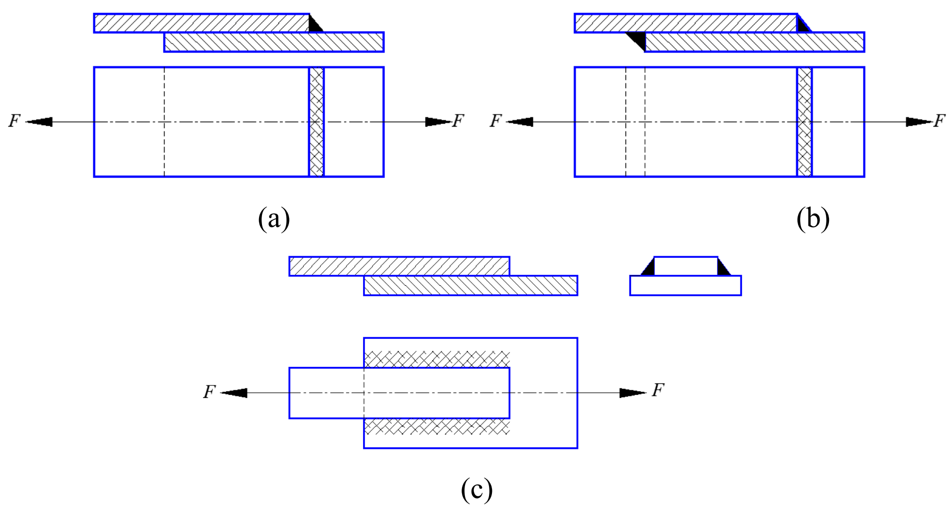


Fig. 5.9: Types of fillet joints (a) Single transverse fillet weld (b) Double transverse fillet weld (c) Double parallel fillet weld

A fillet joint, alternatively referred to as a lap joint, is a joint formed by two overlapping plates or parts. Fillet weld joins two surfaces at right angles to one another with a cross-section that is roughly triangular in shape. There are two kinds of fillet joints: parallel and transverse. When the fillet weld is oriented perpendicular to the direction of the force acting on the joint, it is called as transverse fillet weld, as depicted in Fig. 5.9 (a) and (b). Conversely, if the fillet weld is aligned parallel to the force direction, it is known as a longitudinal or parallel fillet weld, illustrated in Fig. 5.9 (c).

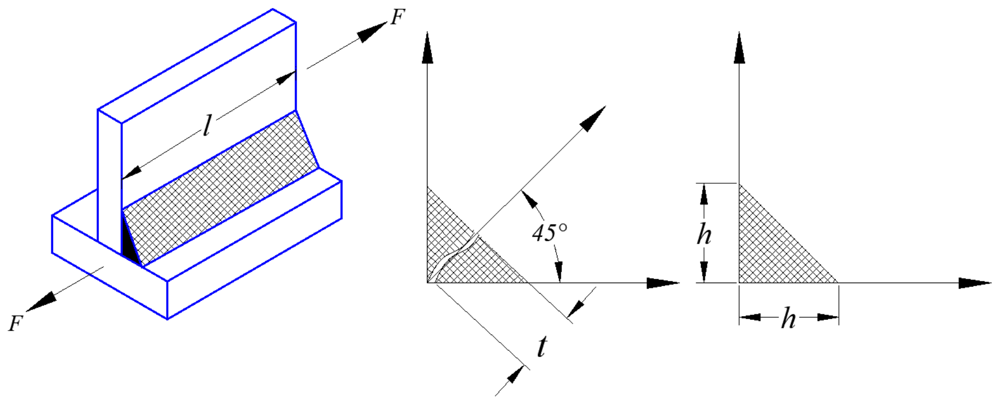
5.5.1 Design of parallel fillet welds

Fig. 5.10 depicts a parallel fillet weld that is under a tensile force F. Two terms are associated with the fillet weld's dimensions viz. leg h and throat t. The leg length specifies the size of the weld. Normally, the leg length h is equal to the plate thickness. The cross-section of the fillet weld forms a right-angled triangle with two equal sides. A leg is the length of each of the two equal sides. The throat, which is 45° to the leg dimension, is the weld's minimum cross-section. Hence,

Welded Lec 29

$$t = h\cos 45^o$$

or $$t = 0.707 \times h \tag{5.34}$$



(a) (b) (c)

Fig. 5.10: Parallel fillet weld in shear

Shear along the throat's minimum cross-section causes the fillet weld to fail. Because, the plane where the maximum shear stress is created in a parallel fillet weld is angled 45^o with respect to the leg dimension. Fig. 5.10 (b) shows the failure of the parallel fillet weld due to shear stress induced in the fillet. The throat's cross-sectional area (A) is $tl = 0.707hl$.

Welded Lec 30

The fillet weld's shear stress is determined by,

$$\tau = \frac{F}{A} = \frac{F}{0.707hl} \tag{5.35}$$

Rearranging the equation (5.35), the strength equation for parallel fillet weld is expressed as:

$$F = 0.707hl\tau \tag{5.36}$$

Where $F =$ tensile force applied on the plates (N)

$h =$ leg or thickness of the weld (mm)

$l =$ length of the weld (mm)

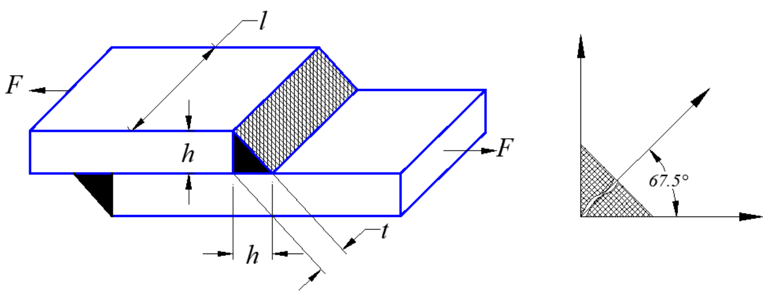
$\tau =$ permissible shear stress for the weld (N/mm^2)

Typically, the vertical plate has two welds on each of its two sides that have the same length. Hence,

$$F = 2\times(0.707hl\tau) \tag{5.37}$$

The length of each weld determined by Eq. (5.36) and (5.37) should be increased by 15 mm in order to account for the beginning and ending of the weld run. As per the American Welding Society (AWS) rule, the maximum allowable shear stress for fillet welds under a static load is $94\ N/mm^2$.

5.5.2. Design of transverse fillet welds



(a) (b)

Fig. 5.11: Failure of fillet weld

Fig. 5.11 (a) depicts a transverse fillet weld that is under a tensile force F. The weld's minimum cross section is at the throat. Consequently, the throat of the transverse fillet weld will experience the failure as a result of tensile stress.

The throat's cross-sectional area (A) is $tl = 0.707hl$. (5.38)

The fillet weld's tensile stress is determined by,

$$\sigma_t = \frac{F}{tl} \tag{5.39}$$

Substituting the value of tl from Eq. (5.38) in the Eq. (5.39),

$$\sigma_t = \frac{F}{0.707hl} \tag{5.40}$$

Rearranging the Eq. (5.40), the strength equation for transverse fillet weld is expressed as:

$$F = 0.707 h l \sigma_t \tag{5.41}$$

Where σ_t = Permissible tensile stress for the weld $\left(N/mm^2\right)$

Typically, the vertical plate has two welds on each of its two sides that have the same length. Hence,

$$F = 2\times\left(0.707 h l \sigma_t\right) \tag{5.42}$$

The cross-section of a transverse fillet weld experiences complex stresses of different types, including both shear and normal stresses. The situation is further complicated by the bending moment in the throat. Theoretically, as shown in Fig. 5.11(b), it can be demonstrated that the plane where maximum shear stress occurs in a transverse fillet weld is oriented at 67.5° to the leg dimension. To simplify the design process, the failure criterion for fillet welds is often based on shear failure. For any direction of applied load, it is assumed that the stress in the transverse fillet weld is represented as shear stress on the throat area.

Example 5.8: Two plates are joined using parallel fillet welds, as illustrated in Fig. 5.12. The plates are $10\ mm$ thick, and the allowable shear stress of the weld material is $\tau = 80\ MPa$. Determine the length of the welds needed to support a tensile force of $90\ kN$.

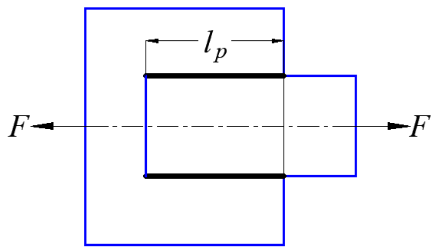


Fig. 5.12: Parallel fillet weld - I

Give Data:

Thickness of the plate $t = 10\ mm$

Axial tensile force $F = 90\ kN = 90\times10^3\ N$

Allowable shear stress $\tau = 80\ MPa = 80\ N/mm^2$

Find:

1. Weld length l

Solution:

(i) Weld Length:

From Eq. (5.37), $F = 2\times\left(0.707 h l \tau\right)$

$90000 = 2\times\left(0.707\times10\times l\times80\right)$ (Assume size of the weld $h = 10\ mm$)

$\therefore \quad l = 79.56\ mm$

The length of each weld should be increased by 15 mm in order to account for the beginning and ending of the weld run. Hence,

$$l = 79.56 + 15 = 94.56\ mm \simeq 95\ mm$$

Example 5.9: Find the safe load that can be applied to a welded bracket as shown in Fig. 5.13. The size of the weld is $6\ mm$. The allowable shear stress of weld material is $100\ N/mm^2$.

Given Data:

Size of the weld $h = 6\ mm$

Allowable shear stress of weld materials $\tau = 100\ N/mm^2$

Length of each parallel weld $l = 100\ mm$

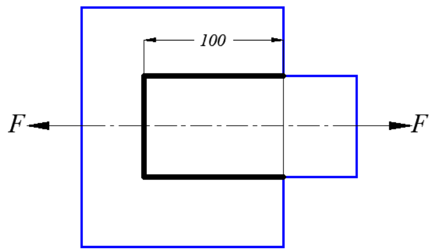


Fig. 5.13: Parallel fillet weld - II

Find:

1. Safe load F

Solution:

(i) Safe load:

From Eq. (5.37), $F = 2 \times (0.707 hl\tau)$

The length of each weld is taken as 100 mm, because the value was given in the problem is only portion to be welded. Hence, the length of the weld should not be reduced by 15 mm.

$$\text{F} = 2 \times (0.707 \times 6 \times 100 \times 100)$$

$\therefore F = 84.84\ kN$

Example 5.10: Double transverse fillet welds are employed to join two steel plates, each measuring $100\,mm$ in width and $12\,mm$ in thickness (Fig. 5.14). The maximum tensile stress allowed for both the plates and the welding material is $90\ N/mm^2$. If the weld strength is the same as the plate strength, determine the necessary weld length to carry maximum load.

Given Data:

Width of the plate $b = 100\ mm$

Thickness of the plate $t = 12\ mm$

Maximum tensile stress $\sigma_t = 90\ N/mm^2$

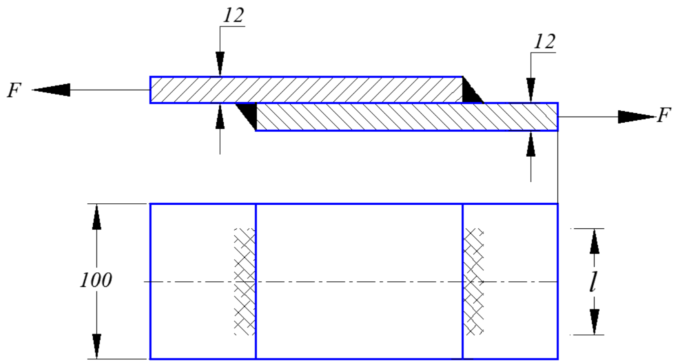


Fig. 5.14: Double transverse weld

Find:

1. Weld length l

Solution:

(i) Tensile force on plates:

Since, the plates are subjected to tensile stress, the maximum permissible tensile load acting on the plates is written by,

$$F = (bt)\sigma_t = (100 \times 12) \times 90 = 108\,000\,N$$

(ii) Weld Length:

From Eq. (5.42), $F = 2 \times (0.707 hl\sigma_t)$

$$108000 = 2 \times (0.707 \times 12 \times l \times 90)$$

$$\therefore\ l = 70.72\ mm$$

Adding $15 mm$ for starting and stopping of the weld run, the required length of the weld is given by, $l = 70.72 + 15 = 85.72 mm$ or $90\ mm$.

Example 5.11: A plate measuring 70 mm in width and 13 mm in thickness is connected to another plate using a single transverse weld and a double parallel fillet weld, as illustrated in Fig. 5.15. The joint is subjected to a maximum tensile force of 65 kN. The maximum tensile and shear stresses are 60 MPa and 70 MPa respectively. Calculate the length of each parallel fillet weld.

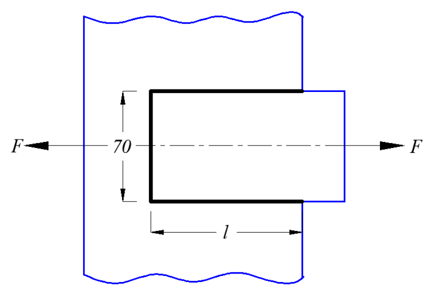


Fig. 5.15: Single transverse weld and a double parallel fillet weld

Given Data:

Width of the plate $b = 70\ mm$

Thickness of the plate $t = 13\ mm$

Tensile load $W = 65\ kN$

Maximum tensile stress

$$\sigma_t = 60\ MPa = 60\ N/mm^2$$

Maximum shear stress $\tau = 70\ MPa = 70\ N/mm^2$

Find:

1. Length of each parallel fillet weld l

Solution:

(i) Strength of transverse and parallel fillet welds:

Let, the strength of the transverse fillet weld F_1.

From Eq. (5.42), $\mathrm{F_1 = 0.707hl}\sigma_t = 0.707 \times 13 \times 70 \times 60$

$$\mathrm{F_1} = 38602.2\ \mathrm{N} \tag{i}$$

The strength of the a double parallel fillet weld W_2.

From Eq. (5.37), $F_2 = 1.414 hl\tau = 1.414 \times 13 \times l \times 70$

$$F_2 = (1287 \times l)\,\mathrm{N} \tag{ii}$$

(ii) Length of parallel fillet weld:

The total strength of the joint should be $65\ kN$. From (i) and (ii),

$$38602.2 + 1287 \times l = 65 \times 10^3$$

$\therefore \quad l = 20.5$ mm

Adding $15\ mm$ for starting and stopping of the weld run, the length of the parallel fillet weld is,

$l = 20.5 + 15 = 35.5$ mm or 40 mm

5.6. MERITS AND DEMERITS OF SCREWED AND WELDED JOINTS

5.6.1. Merits of screwed joints

1. Screwed joints exhibit great reliability
2. Assembly and disassembly of screwed joints is easy
3. There is a large selection of screwed joints that can be used in different working environments
4. The cost of the screws is relatively cheap
5. Cheap production procedures can be used as a result of standardization
6. It may be used to transmit power ex. lead screw

5.6.2. Demerits of screwed joints

1. Under certain conditions, the concentration of stress in the threaded sections varies
2. Screwed joints are weaker than welded joint

3. The threaded section of screws becomes weak due to stress concentration
4. Machine vibrations cause screwed joints to become loose

5.6.3. Merits of welded joints

1. Reducing labor costs and production time, welding is a quick and effective method of joining two pieces of metal
2. The welded joint's strength is considered as 100%.
3. Completely leak-proof joint is provided by welded joints
4. No need to drill a hole in the parent components as like screwed joints
5. Strength of welded joint is similar to that of parent parts, load carrying capacity of welded components is high
6. Ability to join different shapes, such as sheets, rods, and plates etc.

5.6.4. Demerits of welded joints

1. Welded joints are more expensive
2. A welded joint is permanent and does not permit the disassembly of joined parts
3. Residual stress develops inside the joined structures
4. Dimensions inaccuracy is caused by jointed structure deformation
5. When welded joints are utilized for extended periods of time in an environment that vibrates, they are more likely to fail
6. It can be challenging to check for flaws in welded joints, and a complex testing technique requires more expensive testing equipment

5.7. ERGONOMICS AND AESTHETIC CONSIDERATIONS IN DESIGN

5.7.1. Ergonomics of design

The English word "ergonomics" originates from the Greek terms 'ergon,' meaning 'work,' and 'nomos,' meaning 'natural laws.' Ergonomics refers to the study of the relationship between humans and machines, applying anatomical, physiological, and psychological principles to solve problems that arise from that relationship. The following are the subjects of ergonomic studies derived from design considerations:

Ergonomic

1. Designing a driver's seat using anatomical considerations
2. Instrument dial and display panel layout for operators' accurate perception
3. Design of hand wheels and levers
4. Energy used in manual labor and footwork
5. Climate, lighting, and noise levels in a machine shop environment

To find the ideal hand or foot pressure, size for levers and hand wheels, or the optimal driver's seat dimensions, ergonomists have conducted studies.

5.7.2. Man – Machine relationship

A man-machine system is characterized as an operational combination of one or more humans and one or more machines that work together to produce the intended outputs from given inputs using a predetermined process or method while staying within environmental limits. The environment can provide obstacles such as heat discomfort, noise, poor lighting, incorrect postures and gestures, and so on. These obstacles reduce input, which lowers efficiency. These must be promptly recognized in order to be avoided or addressed appropriately. A few instances of a basic man-machine system are a driver and his car, a lathe operator, a typewriter and its operator, and a programmer and his personal computer. In machine design, the machine is viewed as an entity unto itself. Ergonomists, on the other hand, view a man-machine joint system as a closed loop, as seen in Fig. 5.16.

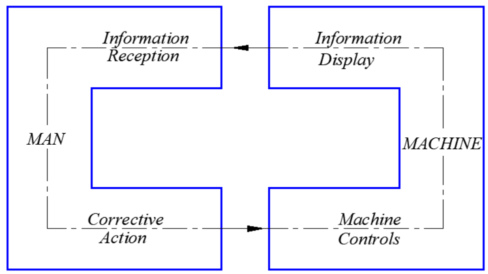


Fig. 5.16: Man – Machine closed loop system

The operator obtains information about the machine's functioning using display instruments. He will use the levers or controls if he thinks a correction is required. This will therefore change the machine's performance, which display panels will show.

In this closed-loop system, the operator and machine come into touch at two points: the display instruments that provide information to the operator, and the controls that allow the operator to modify the machine.

Three categories are used to categorize the visual display instruments:

1. Displays that provide numerical readings, including energy meters, voltmeters, speedometers, etc.
2. Displays that indicate the current status, such as a red lamp indicator
3. Displays that show predefined settings, such as a lever for a two-speed electric motor that may be adjusted to run at 1440 rpm, 720 rpm, or the "off" position.

Lever-type indicators are used for setup, while moving scale or dial-type instruments are utilized for quantitative measurements. The primary goal of display design is to minimize operator fatigue, as they must be constantly observed. The following are the ergonomic factors that display designers take into account:

1. The dial indicator's scale ought to be split into appropriate number segments, such as 0–10–20–30, rather than 0–5–30–55
2. There should be as few subdivisions as possible between numbered divisions
3. The following should be the size of the indicator's letters and numbers:

Height of letter or number should be more than or equal to the ratio of reading distance and 200

4. For stationary dials, vertical figures should be utilized, whereas radially oriented figures are appropriate for rotating dials
5. To reduce parallax error, the pointer should have a knife-edge and a mirror in the dial

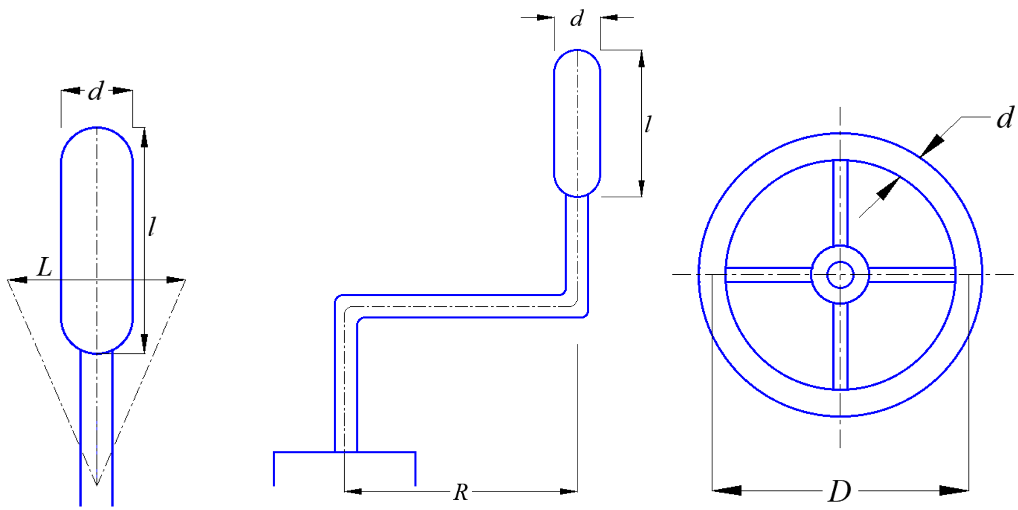
5.7.3. Design of equipment for control

The machines are operated by controls such as levers, cranks, hand wheels, knobs, switches, push buttons, and pedals. They are primarily operated by hand. Levers and hand wheels are utilized on controls that demand a lot of force to operate. It is preferable to use push buttons or knobs when the operating forces are low. The following are the ergonomic factors taken into account while designing controls:

1. The controls ought to be placed logically and be easily accessible. The control operation ought to minimize motions and steer clear of unpleasant movements.
2. When a control component comes into touch with hands, its shape should match the structure of the human hand.
3. Appropriate coloration has positive psychological impacts. To draw attention, the controls on the grey machine tool background should be painted red.

Ergonomics aims to minimize the operational challenges in a human-machine system, thereby reducing both psychological and physical stress. Ergonomic research determines the shape and dimensions of specific machine components, such as cranks, hand wheels, and levers. These considerations also establish the resisting force, which is the amount of force an operator can apply without experiencing excessive fatigue. The dimensions and resisting forces for various control elements are detailed in numerous ergonomic textbooks.

In this section, we will limit the discussion to hand wheels, cranks, and levers because they are commonly needed in machine design. Fig. 5.17 illustrates the dimensions of lever, crank and hand wheel. Table 5.2 list their dimensions as well as the amount of force they are resisting.



(a) (b) (c)

Fig. 5.17: Control elements: (a) Lever (b) Crank (c) Hand Wheel

Table 5.2: Dimensions and resisting forces for some common control elements

Lever			**Crank with heavy load (more than $25\ N$)**			**Hand wheel**		
	min	max		min	max		min	max
d	40	70	d	25	75	D	175	500
l	75	-	l	75	-	d	20	50
L	-	950	R	125	500	P	-	240
P	-	90	P	-	40			

Where $d =$ handle diameter (mm), $l =$ grasp length (mm), $L =$ displacement of lever (mm), $P =$ resisting force (N), $R =$ crank radius (mm), $D =$ mean diameter of hand wheel (mm), $d_1 =$ rim diameter (mm).

5.7.4. Design of equipment for environment

To enhance the work environment, one can improve by minimizing vibration and noise, and ensuring adequate seating, desks, and ventilation/lighting. Introduce systems or tasks using procedures familiar to the users. Organizational efficiency can improve when workers are allowed to work at their own pace, reducing psychological stress. Eliminate undesirable, uncontrolled, and unaccounted aspects of the system to prevent unnecessary fatigue and errors. Poor interfaces can lead to accidents, injuries, and increased errors, causing user difficulties with cumbersome and unnatural subtask combinations. Inefficient workers produce suboptimal output, and in ergonomics, issues like injuries, poor quality, and increased human errors are systemic, not just individual problems. Redesigning systems rather than solely relying on management can provide a solution to these issues.

5.7.5. Design of equipment for safety

Designing safe and ergonomic machinery and equipment begins with identifying potential hazards and risks they may present to users and the environment. Hazards encompass anything capable of causing harm, including moving parts, sharp edges, electricity, noise, heat, chemicals, or radiation. Risks refer to the probability and potential severity of harm resulting from exposure to these hazards. Various methods, such as checklists, inspections, audits, observations, interviews, surveys, or incident reports, can be employed to identify hazards and assess associated risks effectively.

The next step involves applying applicable design principles and standards aimed at eliminating or minimizing hazards and risks. Design principles serve as overarching guidelines to facilitate the creation of user-friendly, efficient, and effective machinery and equipment. For instance, these principles include simplifying tasks, automating processes, reducing forces, adjusting heights, providing feedback, and ensuring visibility. On the other hand, design standards consist of specific requirements intended to ensure compliance with safety regulations, codes, and best practices in ergonomics. Examples of design standards include the provision of guards, locks, labels, alarms, emergency stops, and personal protective equipment (PPE).

The third step involves engaging users and stakeholders in the design process. Users are those who will operate, maintain, or interact with the machinery and equipment, while stakeholders include individuals with an interest or influence in the design, such as managers, customers, suppliers, or regulators. By actively involving users and stakeholders, valuable insights, feedback, and suggestions can be gathered to enhance the safety and ergonomics of the machinery and equipment. Various methods can be employed to engage users and stakeholders, including focus groups, workshops, prototypes, simulations, or trials.

The fourth step involves testing and evaluating the machinery and equipment design before implementation. This process ensures that the design aligns with safety and ergonomic goals, and helps identify and address any issues or concerns. Various methods like inspections, measurements, analyzes, or experiments can be employed for testing and evaluation. Additionally, it is important to gather and analyze data on the performance, usability, and user satisfaction of the machinery and equipment.

The last step involves monitoring and reviewing the machinery and equipment design after implementation. This process helps ensure ongoing safety and ergonomic effectiveness while identifying any emerging risks or changes. Monitoring and review methods can include audits, surveys, reports, and indicators. It's essential to update and enhance the design based on feedback, data, and insights gained from this process.

5.7.6. Aesthetic considerations in design

Aesthetic

Every product has a specific function. To the satisfaction of the customer, it must carry out certain tasks. This functional requirement alone necessitates that the product and humans come into contact. A car's functional requirement is to be able to transport four people at 60 km/h. Some city people wish to travel 15 km in 15 minutes to go to their office. Thus, they buy an automobile car. A home refrigerator's unique purpose is to keep fruits and vegetables fresh for a week. One housewife in the city is unable to buy fresh veggies on a daily basis from the market. She buys the refrigerator as a result. It is evident that these functional specifications unite individuals and products.

When multiple products on the market possess identical features such as affordability, durability, and efficiency, the buyer is drawn to the product that appeals to them the most. The product's external look is a crucial component that influences sales in the market in addition to adding elegance and shine to the item. This is especially true for consumer durables such as televisions, appliances, and automobiles.

A distinct field known as ‘industrial design’ has emerged as a result of the increasing recognition of the importance of aesthetic considerations in product design. Developing novel, aesthetically acceptable forms and shapes is the responsibility of an industrial designer. As a result, the industrial designer has evolved into the hardware industry's fashion designer. Similar to fashion, a product's external appearance evolves significantly over time. The step, stream, taper, shear, and sculpture are the five fundamental forms. The step form resembles a multistory building or ‘skyscraper’ in shape. This has to do with shapes that emphasize vertical lines as opposed to horizontal ones. Aircraft structures and automobiles both use the streamline or stream form. The taper shape is made up of interlocking tapered plinths or cylinders with tapered blocks. The square-looking shear form is appropriate for free-standing engineering objects. The sculpture form is made up of hyperboloids, paraboloids, and ellipsoids. While step and shear forms work well for fixed objects, sculpture and stream forms are best suited for moving goods like automobiles.

There is a connection between the product's look and functional requirements. Functional considerations frequently lead to visually beautiful shapes. Aerodynamics research for effortless speed led to the evolution of the Boeing's streamlined design. Rigidity and strength requirements lead to a high-capacity hydraulic press's sturdy appearance and balanced dimensions. Instead of focusing on aesthetics, chromium plating is used to increase corrosion resistance on the components of household appliances.

The appropriate color choice is a crucial factor in product aesthetics. The operator's customary thoughts should align with the color selection. Numerous hues are connected to certain states of mind and circumstances. Morgan has proposed an interpretation for the colors shown in Table 5.3.

Table 5.3: Meaning of color

Color	Meaning
Red	Danger-Hazard-Hot
Orange	Possible danger
Yellow	Caution
Green	Safety
Blue	Caution-Cold
Grey	Dull

Form and color are not the only two elements that affect a product's outward appearance. It is the result of several elements working together, including noise, materials, manufacturing processes, individual component motion, rigidity and resilience, tolerances, and surface finishes. The industrial designer should choose a form that complements the product's utilitarian needs. It is important to consider the costs and accessibility of surface-treatment procedures such as painting, plating, anodizing, and blackening before finalizing the product's outside design.

UNIT SUMMARY

- Fasteners are mechanical devices used to join or affix two or more components together securely
- *Stresses in screwed fasteners:* Stresses resulting from tightening forces, stresses caused by external forces, and stresses arising from the combination of tightening and external forces
- The bolts are subjected to combination of shear and tensile load, then the equivalent tensile load $(W_t)_{eq} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$
- Primary shear load of threaded bolt due to eccentrically applied load $W_{s1} = \frac{W_{el}}{n}$
- Secondary shear load of threaded bolt due to eccentrically applied load $W_{s2} = \frac{W_{el} \times e \times l_1}{l_1^2 + l_2^2 + l_3^2 + l_4^2}$
- The maximum shear load or force of threaded bolt due to eccentrically applied load $W_{s\max} = \sqrt{W_{s1}^2 + W_{s2}^2 + 2W_{s1} \times W_{s2} \times \cos\theta}$
- *Resilience* is the capacity of a material to absorb energy when elastically deformed and to release that energy when the load is removed
- *Transverse fillet weld:* If the fillet weld is oriented perpendicular to the direction of the force acting on the joint, it is called a transverse fillet weld
- *Parallel fillet weld:* If the fillet weld is aligned parallel to the direction of the force applied to the joint, it is called a longitudinal or parallel fillet weld
- The strength equation for parallel fillet weld is $W = 0.707hl\tau$
- The strength equation for transverse fillet weld is $W = 0.707hl\sigma_t$
- *Ergonomics:* Ergonomics is defined as the study of the relationship between a man and a machine, including the application of anatomical, physiological, and psychological principles to resolve issues arising from that relationship

EXERCISES

Multiple Choice Questions

1. What is the value of initial load for making a joint fluid tight?

   (a) $P_i = 2840 + d_o$ (b) $P_i = 2840d_o$

   (c) $P_i = 1840d_o$ (d) $P_i = 840d_o$

2. The core and nominal diameters are donated as d_c and d_o. The relationship between core diameter and nominal diameter is

   (a) $d_c = 0.84d_o$ (b) $d_c = d_o / 0.84$

   (c) $d_c = 0.84 + d_o$ (d) $d_c = 0.84 - d_o$

3. A screw's specifications are determined by its

   (a) major diameter (b) core diameter

   (c) pitch circle diameter (d) inner diameter

4. The value of K for Soft copper gasket is

   (a) 0 (b) 2.5

   (c) 1.5 (d) 0.5

5. The capacity of a material to absorb energy during elastic deformation and release that energy when it is no longer under load is known as

   (a) kinetic energy (b) resilience

   (c) strain energy (d) none of these above

6. The strength equation for parallel fillet weld is

   (a) $W = 0.707hl$ (b) $W = 0.707h\tau$

   (c) $W = 0.707hl\tau$ (d) $W = 0.707l\tau$

7. In a single start thread, the lead is ________ its pitch.

   (a) double than (b) triple than

   (c) half than (d) equal to

8. Acme thread's angle is

   (a) 40° (b) 65°

   (c) 29° (d) 60°

9. The screw thread has a nominal diameter of 20 mm. Determine the core diameter.

   (a) 16.8 mm (b) 20 mm

   (c) 30 mm (d) 20.8 mm

10. Determine the nominal diameter of a coarse-threaded eye bolt needed to lift a 10 kN load, if the tensile stress is limited to 100 MPa.

(a) 13.28 mm (b) 12.28 mm
(c) 11.28 mm (d) 10.28 mm

11. In Greek terms, Ergon means
(a) safety (b) work
(c) natural laws (d) comfort

12. 'Ergonomics' is related to human
(a) safety (b) comfort
(c) both (a) and (b) (d) none of the above

13. To regulate the rotation beyond a full 360 degrees, we employ
(a) knob (b) selector
(c) crank (d) wheel

14. In aesthetic consideration in design, which color is used for safety?
(a) red (b) green
(c) yellow (d) blue

15. In aesthetic consideration in design, which color is used for caution?
(a) orange (b) grey
(c) yellow (d) blue

Answers to Multiple Choice Questions

1. (b) 2. (a) 3. (a) 4. (d) 5. (b) 6. (c) 7. (d) 8. (c) 9. (a) 10. (c) 11. (b) 12. (c) 13. (c) 14. (b) 15. (a)

Short and Long Answer Type Questions

1. Identify the types of thread forms that are commonly used.
2. List the different applications of screw threads.
3. Specify the materials used for bolts and screws.
4. How is a screw thread designated? Give an example.
5. What is the meaning of bolt M24x2?
6. What size of hole must be drilled in M34 bolt to make it uniform strength?
7. Mention the details of screw threads M18 × 1·5 – 9h.
8. Determine the safe tensile load for a bolt of M20, if the safe tensile stress is 80 N/mm^2.
9. Define the terms (a) major diameter, (b) pitch and (c) lead related to screw thread nomenclature.
10. Explain the various stresses induced in a bolt subjected to initial tightening of the nut.
11. Discuss on bolts of uniform strength.
12. Compare the welded joint with bolted joints.
13. What is parallel fillet weld?

14. List the benefits of using welded joints.
15. Identify and illustrate the types of welded joints using simple sketches.
16. Transverse fillet welds are preferred over parallel fillet welds. Why is that?
17. Express the strength of a parallel fillet weld in terms of permissible shear stress, weld leg size, and joint length.
18. A steam engine cylinder with an effective diameter of 250 mm operates under a steam pressure of 1.2 N/mm². The cylinder cover is secured with 6 bolts, each tightened to an initial load 1.5 times the steam load. To ensure a leak-proof joint, a copper gasket with a stiffness factor of 0.5 is used. Determine the required bolt size so that the stress in the bolts does not exceed 100 N/mm².
19. The cylinder head of a steam engine is attached to the cylinder flange with 10 bolts. The effective diameter of the cylinder is 350 mm, and the maximum internal pressure is 10 bar. The bolts are made of plain carbon steel with a tensile yield strength of 500 N/mm² and a safety factor of 6. Determine the required bolt size, taking into account initial tightening and a stiffness factor of 0.6.
20. A cast iron cylinder head is secured to a cylinder with a 500 mm bore using 8 stud bolts. With a maximum internal pressure of 2 MPa, the stiffness of the cylinder head is three times that of each bolt. What initial tightening load is required to ensure a leak-proof seal at the maximum pressure? Additionally, recommend a suitable bolt for this application.
21. In a bolted joint used for a pressure vessel, each bolt is tightened with an initial preload of $7\,kN$. The external load per bolt varies from 2 to $10\,kN$. Determine the size of the bolts, using the following:

    Type of gasket used: Hard copper gasket

    Yield strength : $240\ MPa$

    Endurance limit : $180\ MPa$

    Factor of safety : 2

    Fatigue stress concentration factor : 1.45

    Load correction factor : 0.7

    Neglect size and surface correction factors.
22. Figure 5.18 shows a bracket designed to support a traveling crane, which is attached to a steel column with four identical bolts at points P, P', Q and Q'. The bracket is subjected to a maximum vertical load of $4\ kN$, positioned $240\ mm$ from the column face. The bolts are made of $40C8$ $\left(S_{yt} = 380\ \text{N/mm}^2\right)$ steel, and the factor of safety is 5. Calculate the major diameter of the bolts based on the maximum principal stress. Assume $d_c = 0.8d$.

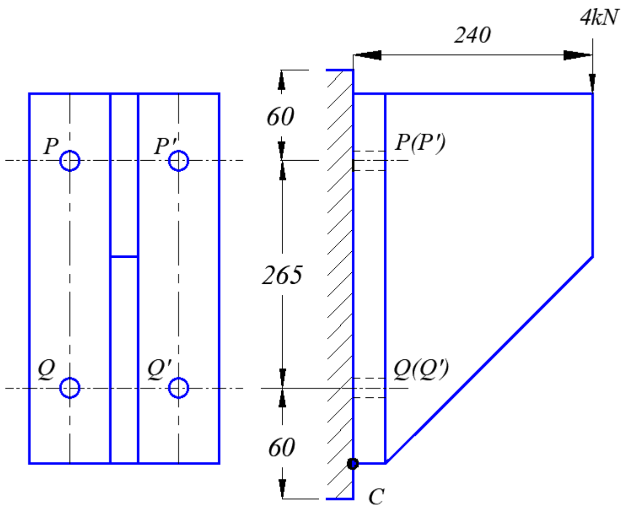


Fig. 5.18: Bracket - II

23. Given the data for the cast iron bracket shown in Fig. 5.19: $l_1 = 75\ mm$, $l_2 = 225\ mm$, $l = 300\ mm$, $F = 10\ kN$, where the bracket is fixed to the horizontal column using four identical steel bolts of type 30C8 $\left(S_{yt} = 400\ \text{N/mm}^2\right)$, two at point P and two at point Q, and with a factor of safety of 5 and a given $d_c = 0.8d$, calculate the nominal diameter of the bolts.

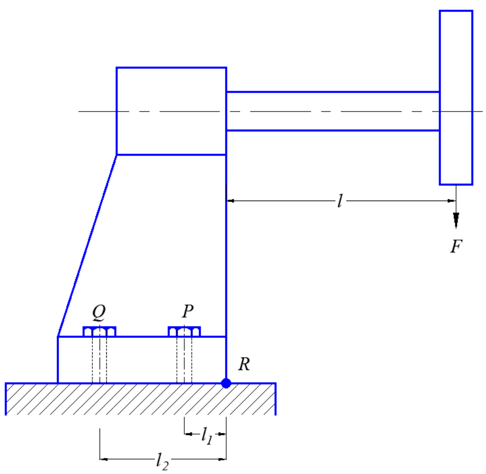


Fig. 5.19: Cast iron bracket

24. Given a mild steel plate with a thickness of $10\,mm$, which is joined to another plate using a single transverse weld and double parallel fillet welds, calculate the required plate width and weld lengths if the joint is subjected to a direct tensile force of $50\ kN$. Assume the permissible shear stress for the weld is $80\ MPa$, the permissible tensile stress is $90\ MPa$, and the permissible tensile stress for the plate material is $60\ MPa$.

25. Calculate the required length of welds to support a $50\ kN$, load between $12\,mm$ thick plates, with the plates joined using (i) two parallel fillet welds and (ii) two transverse fillet welds. The permissible tensile and shear stresses are $90\ MPa$ and $60\ MPa$, respectively.

26. A 100 mm wide, 10 mm thick plate is to be welded to another plate using double parallel fillet welds. With a static load of 80 kN applied to the plates, determine the required length of the weld, given that the allowable shear stress is 55 MPa.

27. Determine the length of welds required to transmit a load of 55 kN between 12 mm thick plates, when the plates are to be joined by (i) two parallel fillet welds (ii) two transverse fillet welds. Assume permissible shear stress of the weld material is $80\ MPa$.

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NPTEL VIDEOS

1. Lecture – 24 - Design of Welded Joints II - Design of Machine Elements - I by Prof. G. Chakrborty, Department of Mechanical Engineering, IIT Kharagpur.

Welded Joints II

2. Lecture – 25 - Design of Joints with eccentric loading - Design of Machine Elements - I by Prof. G. Chakrborty, Department of Mechanical Engineering, IIT Kharagpur.

Ecc. Loading

APPENDIX

Table A1: Design dimensions of screw threads, bolts and nuts according to IS: 4218 (Part III) 1976 (Reaffirmed 1996)

Designation	Pitch mm	Major or nominal diameter Nut and Bolt (d = D) mm	Effective or pitch diameter Nut and Bolt (d_p) mm	Minor or core diameter (d_c) mm		Depth of thread (bolt) mm	Stress area mm^2
				Bolt	Nut		
Coarse series							
M 0.4	0.1	0.4	0.335	0.277	0.292	0.061	0.074
M 0.6	0.15	0.6	0.503	0.416	0.438	0.092	0.166
M 0.8	0.2	0.8	0.67	0.555	0.584	0.123	0.295
M 1	0.25	1	0.838	0.693	0.729	0.153	0.46
M 1.2	0.25	1.2	1.038	0.893	0.929	0.158	0.732
M 1.4	0.3	1.4	1.205	1.032	1.075	0.184	0.983
M 1.6	0.35	1.6	1.373	1.171	1.221	0.215	1.27
M 1.8	0.35	1.8	1.573	1.371	1.421	0.215	1.7
M 2	0.4	2	1.74	1.509	1.567	0.245	2.07
M 2.2	0.45	2.2	1.908	1.648	1.713	0.276	2.48
M 2.5	0.45	2.5	2.208	1.948	2.013	0.276	3.39
M 3	0.5	3	2.675	2.387	2.459	0.307	5.03
M 3.5	0.6	3.5	3.11	2.764	2.85	0.368	6.78
M 4	0.7	4	3.545	3.141	3.242	0.429	8.78
M 4.5	0.75	4.5	4.013	3.58	3.688	0.46	11.3
M 5	0.8	5	4.48	4.019	4.134	0.491	14.2
M 6	1	6	5.35	4.773	4.918	0.613	20.1
M 7	1	7	6.35	5.773	5.918	0.613	28.9
M 8	1.25	8	7.188	6.466	6.647	0.767	36.6
M 10	1.5	10	9.026	8.16	8.876	0.92	58.3
M 12	1.75	12	10.863	9.858	10.106	1.074	84
M 14	2	14	12.701	11.546	11.835	1.227	115

Designation	Pitch mm	Major or nominal diameter Nut and Bolt (d = D) mm	Effective or pitch diameter Nut and Bolt (d_p) mm	Minor or core diameter (d_c) mm		Depth of thread (bolt) mm	Stress area mm^2
				Bolt	Nut		
M 16	2	16	14.701	13.546	13.835	1.227	157
M 18	2.5	18	16.376	14.933	15.294	1.534	192
M 20	2.5	20	18.376	16.933	17.294	1.534	245
M 22	2.5	22	20.376	18.933	19.294	1.534	303
M 24	3	24	22.051	20.32	20.752	1.84	353
M 27	3	27	25.051	23.32	23.752	1.84	459
M 30	3.5	30	27.727	25.706	26.211	2.147	561
M 33	3.5	33	30.727	28.706	29.211	2.147	694
M 36	4	36	33.402	31.093	31.67	2.454	817
M 39	4	39	36.402	34.093	34.67	2.454	976
M 42	4.5	42	39.077	36.416	37.129	2.76	1104
M 45	4.5	45	42.077	39.416	40.129	2.76	1300
M 48	5	48	44.752	41.795	42.587	3.067	1465
M 52	5	52	48.752	45.795	46.587	3.067	1755
M 56	5.5	56	52.428	49.177	50.046	3.067	2022
M 60	5.5	60	56.428	53.177	54.046	3.374	2360
Fine series							
M 8 × 1	1	8	7.35	6.773	6.918	0.613	39.2
M 10 × 1.25	1.25	10	9.188	8.466	8.647	0.767	61.6
M 12 × 1.25	1.25	12	11.184	10.466	10.647	0.767	92.1
M 14 × 1.5	1.5	14	13.026	12.16	12.376	0.92	125
M 16 × 1.5	1.5	16	15.026	14.16	14.376	0.92	167
M 18 × 1.5	1.5	18	17.026	16.16	16.376	0.92	216
M 20 × 1.5	1.5	20	19.026	18.16	18.376	0.92	272
M 22 × 1.5	1.5	22	21.026	20.16	20.376	0.92	333
M 24 × 2	2	24	22.701	21.546	21.835	1.227	384
M 27 × 2	2	27	25.701	24.546	24.835	1.227	496

Designation	Pitch mm	Major or nominal diameter Nut and Bolt (d = D) mm	Effective or pitch diameter Nut and Bolt (d_p) mm	Minor or core diameter (d_c) mm		Depth of thread (bolt) mm	Stress area mm^2
				Bolt	Nut		
M 30 × 2	2	30	28.701	27.546	27.835	1.227	621
M 33 × 2	2	33	31.701	30.546	30.835	1.227	761
M 36 × 3	3	36	34.051	32.319	32.752	1.84	865
M 39 × 3	3	39	37.051	35.319	35.752	1.84	1028

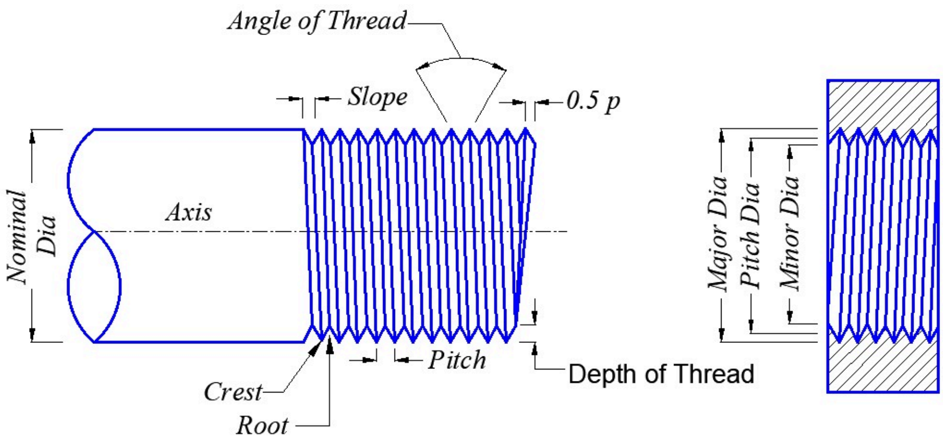

Fig. A1: Terminologies in screw threads

CO AND PO ATTAINMENT TABLE

Course Outcomes (COs) for this course can be mapped with the Program Outcomes (POs) after the completion of the course and a correlation can be made for the attainment of POs to analyze the gap. After proper analysis of the gap in the attainment of POs necessary measures can be taken to overcome the gaps.

Course Outcomes	**Expected Mapping with Program Outcomes** *(1- Weak Correlation; 2- Medium Correlation; 3- Strong Correlation)*						
	PO-1	**PO-2**	**PO-3**	**PO-4**	**PO-5**	**PO-6**	**PO-7**
CO-1							
CO-2							
CO-3							
CO-4							
CO-5							

INDEX

DESIGN OF MACHINE ELEMENTS

A KUMARAVEL
M KATHIRSELVAM

This book gives detailed description about Introduction to Design, Simple machine part design, Antifriction Bearings, Shafts, Keys, Couplings and Spur Gears, Power Screws, Springs, Fasteners, Ergonomics & Aesthetic consideration in design. Logical organization and a wide variety of problems with their step by step solutions makes this book a perfect offering on the subject.

Salient Features

- Content of the book aligned with the mapping of Course Outcomes (COs), Programs Outcomes (Pos) and Unit Outcomes (UOs).
- In the beginning of each unit outcomes are listed to make the student understand what is expected out of him/her after completing that unit.
- Book provides lots of recent information, interesting facts, QR Codes for E-resources, etc.
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- Figures and tables are inserted to improve clarity of the topics.
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